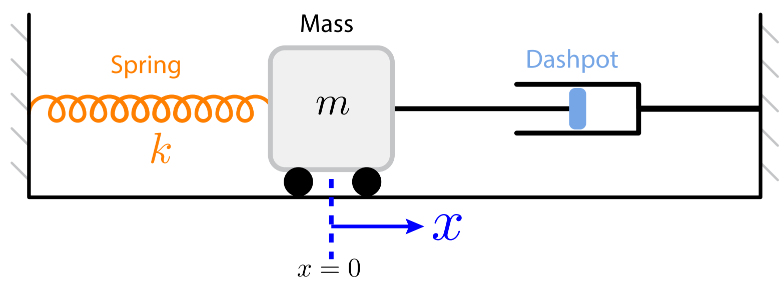
18.03 Recitation 5

Spring-mass-dashpot with different damping conditions

1.



Consider the spring-mass-dashpot system in which $m = 1$ (kg), and $k = 16$ (Newton per meter). The position $x(t)$ (meters) of the mass at t (seconds) is governed by the DE

$$\ddot{x} + b\dot{x} + 16x = 0. \tag{1}$$

We will investigate the effect of b, the damping constant, on the solutions to the DE.

(a) What is the characteristic polynomial?

(b) For what values of b are the roots of the characteristic equation real and distinct?

(c) Assume b is within the range you found in the previous problem. Write down the exponential solutions, $x_1(t)$ and $x_2(t)$, corresponding to the two smaller and larger roots respectively.

(d) Find the position $x(t)$ of the mass at t (seconds) if at $t = 0$, the mass is 1 meter to the right of the equilibrium position and is moving to the left at $b/2$ meters per second. (Use $x > 0$ when the mass is to the right of the equilibrium position.)

(e) As above, assume a value b such that the characteristic polynomial has two distinct real roots. How many times can the mass pass through the equilibrium position $x = 0$?

(f) For what values of b are the roots of the characteristic equation not real?

(g) Assume a value of b such that the characteristic polynomial has two distinct non-real roots. Find the position $x(t)$ of the mass at t (seconds) for the same initial conditions considered previously: at $t = 0$, the mass is 1 meter to the right of the equilibrium position and is moving to the left at $b/2$ meters per second.

(h) Consider the solution $x(t)$ for the initial value problem in the previous question. What is the length of the time interval τ (in seconds) between each time the mass returns to the equilibrium position? This fixed time interval is known as the **pseudoperiod** of the oscillation of the mass.

2. Recall that in a general spring-mass-dashpot system, the position of the mass is modeled by

$$m\ddot{x} + b\dot{x} + kx = 0, \qquad m > 0,\ k, b \geq 0. \tag{2}$$

What can you say about the real part of the roots of the characteristic polynomial of the system?