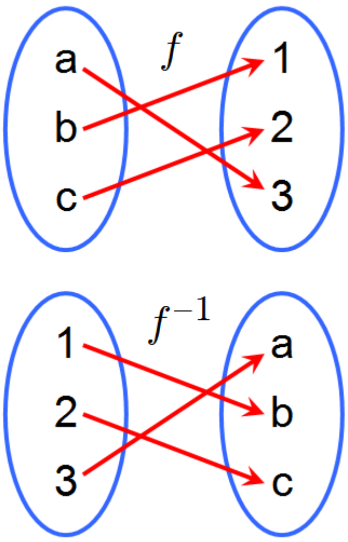
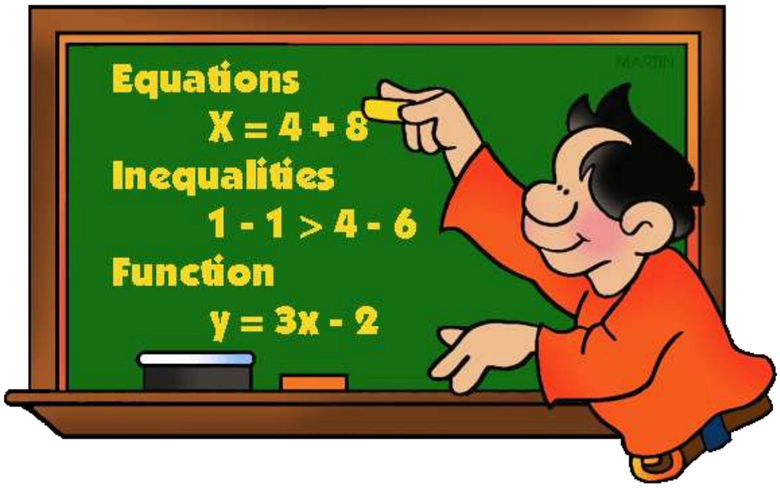
FUNCTIONS





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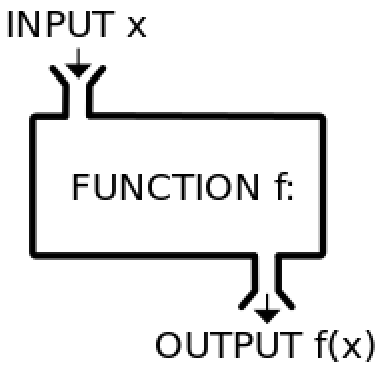


Revision of Solving Polynomial Equations

(i) "one term in x"

Examples
Solve:

(a) $2-7x=12$

(b) $4x^3=16$

(c) $\frac{17x}{3}-5=12$

(ii) "more than one term in x"

Method:
1) Get the right hand side to equal zero (= 0)
2) Eliminate all denominators (where necessary)
3) Factorise Left Hand Side
4) Use Null Factor Law (NFL)

Factorising

- **Quadratic (** ax^2+bx+c **)**
  - **If** b^2-4ac **(the discriminant, Δ) is:**
    - **Perfect square, use "criss-cross"**
    - **Not a perfect square, but positive use the quadratic formula,**

      $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
    - **Negative, NO REAL solution**

$\Delta < 0$,
$\Delta = 0$,
$\Delta > 0$,

- **Cubic (** ax^3+bx^2+cx+d **)**
  - **Use grouping (if possible)**
  - **Factor theorem and long division**

- **Quartic (** $ax^4+bx^3+cx^2+dx+e$ **)**
  - **Use substitution, eg let** $a=x^2$
  - **Factor theorem**

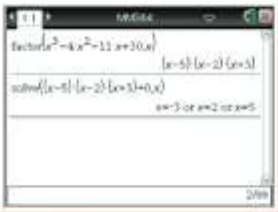
Examples:

Solve the following for x:

1) $3x^2 - 7x + 2 = 0$

Using the TI-Nspire

Use **factor** (menu > **Algebra** > **Factor**) and **solve** (menu > **Algebra** > **Solve**) as shown.



2) $x^3 + 4x^2 = 6 - x$

3) $(3x+2)^2 + 12(3x+2) - 13 = 0$

4) $x^4 - 2x^2 + 1 = 0$

5) $x^3 + x^2 - 4x - 4 = 0$

iii) Iteration

Use iteration to find the solutions to correct to 1 decimal point.

$x^2 - x - 5 = 0$

- **Questions:** 22 question sheet. (see Below)
- **Check using *solve* command on Ti-NSpire**

Solving Polynomial Equations – “20 Question Sheet”

- “one term in x”
- “>1 term in x”

Solve the following:

Question		***Hint***
1.	$5-3x=16$	-
2.	$1-\frac{1}{x^2}=0$	*Eliminate the denominator*
3.	$3x^5=96$	-
4.	$7x^5=200+6x^5$	*Collect like terms, answer to 4 d.p*
5.	$\frac{2x}{3}-\frac{3}{4}=\frac{5}{6}$	*Eliminate denominators in one go*
6.	$x-5+\frac{4}{x}=0$	-
7.	$2x^2=8x+64$	-
8.	$x^2+8x+8=0$	*(i) exact answer* *(ii) correct to 3 d.p.*
9.	$-2x^2-6x=1$	*Correct to 3 d.p.*
10.	$5x^2+2x+1=0$	-
11.	$x^3+4x^2+x=6$	*Make RHS=0, then factor theorem*
12.	$4x^3+12x^2-x-3=0$	*Grouping is quicker*
13.	$x^4+x^2-2=0$	*Let a = x^2*
14.	$x^4-14x^2=-1$	*Make RHS=0, let a =x^2, ans. to 3 d.p.*
15.	$x^4+x^3-7x^2-x+6=0$	*Grouping two and three*
16.	$(3x+1)^2=3x+1$	*Let a =3x+1*
17.	$x^2+1+\frac{2}{x}=0$	*Eliminate denominator then factor theorem*
18.	$x^2=\frac{1}{1+2x^2}$	*Eliminate denominator then let a=x^2*
19.	$6(x^3-x^5)=0$	*Eliminate constant, HCF*
20.	$\frac{1}{3}(x+1)^4(5-2x)+2=2$	*Eliminate constants*

For the following use iteration to find solutions correct to 1 decimal place (1 d.p)

21. $x^2+x-7=0$

22. $x^3+x-5=0$

Completing The Square

Completing the square allows a quadratic of the form $y = ax^2 + mx + n$, to be written in the Turning Point form, $y = a(x-b)^2 + c$.

Examples: Express the following in the Turning Point form $y = a(x-b)^2 + c$.

1) $y = x^2 + 4x + 1$

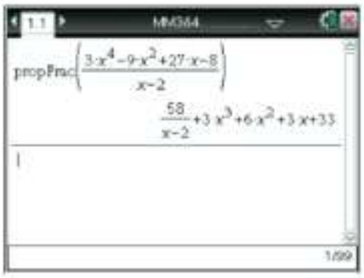
2) $y = x^2 - 3x + 1$

3) $y = -3x^2 - 12x - 7$

- **Ex 4A Q 2, 4 (Only Complete the Square)**
- **CAS Calculator: *expand* command to check answers & the *Complete The Square* command**

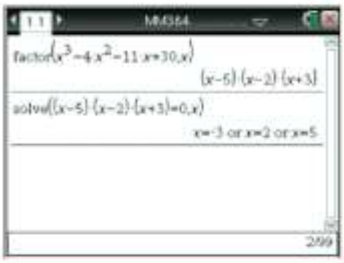
Using the TI-Nspire

Use **propFrac** from (menu) > **Algebra** > **Fraction Tools** > **Proper Fraction** as shown.



Using the TI-Nspire

Use **factor** ((menu) > **Algebra** > **Factor**) and **solve** ((menu) > **Algebra** > **Solve**) as shown.



Factor Theorem

Examples

1) Without performing long division, find the remainder when $2x^3 - 8x + 11$ is divided by $(x+3)$.

$Let \quad P(x) = 2x^3 - 8x + 11$

2) Find "a", given that when $x^3 - 2x + a$ is divided by $(x-2)$ the remainder is 7.

$Let \quad P(x) = x^3 - 2x + a$

Rational Root Theorem

- Sometimes there are no integer solutions to a polynomial, but there maybe rational solutions.
- e.g. if $P(x) = 2x^3 - x^2 - x - 3$, we can show $P(1) \neq 0$, $P(-1) \neq 0$, $P(3) \neq 0$, $P(-3) \neq 0$.
- So there is no integer solution.
- So next we try $P\left(\frac{3}{2}\right)$, $P\left(-\frac{3}{2}\right)$, $P\left(\frac{1}{2}\right)$ & $P\left(-\frac{1}{2}\right)$ and will discover $P\left(\frac{3}{2}\right) = 0$ therefore $(2x - 3)$ is a factor of $P(x)$.
- Unsure of signs then solve the equation $2x - 3 = 0$ for .

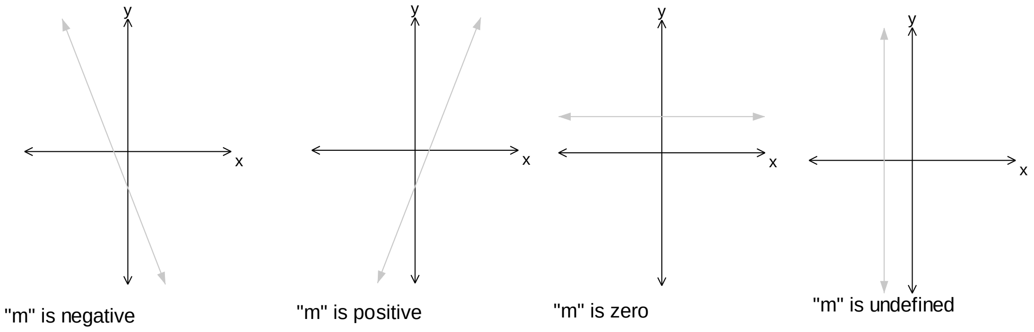
Example: Use the Rational-root theorem to help factorise $P(x) = 3x^3 + 8x^2 + 2x - 5$

$P(x) = 3x^3 + 8x^2 + 2x - 5 \qquad P(1) = \qquad P(-1) = \qquad P(5) = \qquad P(-5) = \qquad P(-\frac{5}{3}) =$
$So \quad (\qquad) \ is \ a \ factor.$

- **Ex4D Q 2, 4, 7, 8, 10, 11, 16, 17, 20**

Straight Lines/Simultaneous Equations

- The gradient of a straight line is always constant.
- Gradient $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$



- Distance between 2 points: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ (Pythagoras)
- Midpoint, *M*, of two points is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- If m_1 is the gradient of a straight line and m_2 is the gradient of another straight line…
    - If the two lines are parallel then $m_1 = m_2$
    - If the two lines are perpendicular then $m_1 \times m_2 = -1 \quad or \quad m_1 = \frac{-1}{m_2}$

Equation of a straight line:

- To find the equation of a straight line, you need:
    - The gradient (m) and the Y-intercept (c), then use $y = mx + c$
    - The gradient (m) and the coordinates of one point on the line (x_1, y_1) then use $y = mx + c$ or $y - y_1 = m(x - x_1)$.
    - The coordinates of two points on the line (x_1, y_1) and (x_2, y_2), and then use $m = \frac{y_2 - y_1}{x_2 - x_1}$, then use $y = mx + c$ or $y - y_1 = m(x - x_1)$.

Example: Find the equation of the line passing through (-2, -3) and (2, 5).

$Let \quad (x_1, y_1) = (-2, -3) \quad and \quad (x_2, y_2) = (2, 5)$

- **Ex2C Q 1, 2, 3, 5, 10, 11, 12, 13, 14, 16, 23**

Simultaneous Equations

- **3 situations**
  - **No solutions**
  - **Infinitely many solutions**
  - **A unique solution**

1. No solution
   - Means the lines are parallel
   - They have the same gradient but a different Y-intercept
   - e.g. $2x+y=2 \quad \& \quad 2x+y=5$
2. Infinitely many solutions
   - Means you have the same line
   - e.g. $2x+y=2 \quad \& \quad 4x+2y=4$
3. A unique solution
   - Means the lines are different and meet at one point only.
   - e.g. $2x+y=2 \quad \& \quad x+y=5$

Example 1: Explain why the following pair of simultaneous equations have no solutions

$2x+3y=6 \quad \& \quad 4x+6y=24$

Example 2:

Consider the system of simultaneous equations given by:

$$mx+y=2$$
$$2x+(m-1)y=m$$

Find the value(s) of m for which there is no solution.

Note: For a unique solution the determinant $\neq 0$. For the above example:

$$
\boxed{\begin{aligned} m(m-1)-2 &\neq 0 \\ &\ldots \\ (m-2)(m+1) &\neq 0 \\ m &\neq -1, 2 \end{aligned}}
$$

=> the values of *m* for which there is a unique solution, $m \in R\backslash\{-1, 2\}$

Simultaneous Linear Equations Worksheet

1. Consider the system of simultaneous linear equations given by

(a) $\begin{aligned} mx+12y &= 24 \\ 3x+my &= m \end{aligned}$ (b) $\begin{aligned} (m+1)x+2y &= 0 \\ 4x+(m-1)y &= m \end{aligned}$

Find the value(s) of *m* for which there is a unique solution.

2. Consider the system of simultaneous linear equations given by

(a) $\begin{aligned} (m-1)x+5y &= 7 \\ 3x+(m-3)y &= 0.7m \end{aligned}$ (b) $\begin{aligned} (m-3)x-5y &= -2 \\ 2x-(2m+2)y &= -4 \end{aligned}$

Find the value(s) of *m* for which there are infinitely many solutions.

3. Consider the system of simultaneous linear equations given by

(a) $\begin{aligned} mx+2y &= 6 \\ 3x+(m-1)y &= 6 \end{aligned}$ (b) $\begin{aligned} 5x+(m-3)y &= 1 \\ mx+2y &= m \end{aligned}$

Find the value(s) of *m* for which there is no solution.

Answers:

1 (a) $m \in R \setminus \{-6, 6\}$ (b) $m \in R \setminus \{-3, 3\}$

2 (a) $m = 6$ (b) $m = 4$

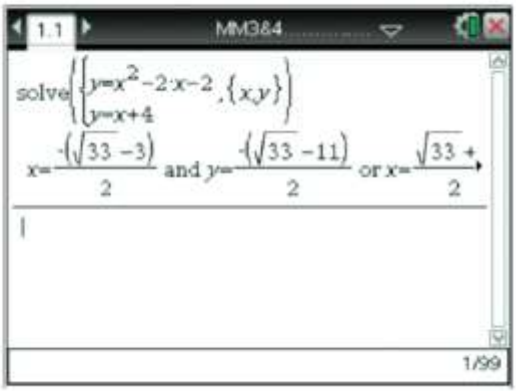
3 (a) $m = -2$ (b) $m = -2, 5$

- **Ex 2F 3, 4, 5, 6**

Using the TI-Nspire

Use the simultaneous equation template (menu > **Algebra** > **Solve System of Equations** > **Solve System of Equations**) and complete as shown.

Use the **up arrow** (▲) to move up to the answer and use the **right arrow** (▶) to display the remaining part of the answer.

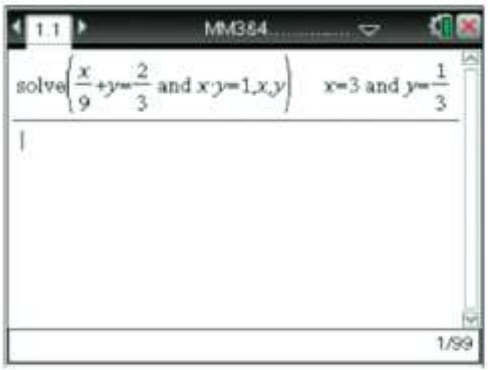


Using the TI-Nspire

Use menu > **Algebra** > **Solve** to solve as shown.

Note that the multiplication sign is required between x and y.

The simultaneous equation solver template (menu > **Algebra** > **Solve System of Equations** > **Solve System of Equations**) could also have been used in this example.



Sets

Notation

- A set is a collection of objects
- The objects are known as elements
  - If x is an element of A, $x \in A$
  - $y \notin A \Rightarrow y$ is not an element of A or
$2 \notin$ the set of odd numbers
- If something is a subset of A, for example B, $B \subseteq A$. (Boys in the Year 12 Methods class is an example of a subset)
- If 2 sets have common elements, it is called an intersection ($\cap$) ie $A \cap B$.
- $\varnothing$ it the empty set.
- $\cup$, union, $A \cup B$ is the set of elements that are either in A or B.
- The **set difference** of two sets *A* and *B* is given by $A \backslash B = \{x: x \in A, x \notin B\}$. Means what's in *A* but not in *B*.

Example

$If \quad A = \{1,2,3,7\} and \quad B = \{3,4,5,6,7\}$

$(i) Find \quad (a) A \cap B \quad (b)\ A \cup B \quad (c)\ A \setminus B \quad (d)\ B \setminus A$

$(a)\ A \cap B =$

$(b)\ A \cup B =$

$(c)\ A \setminus B =$

$(d)\ B \setminus A =$

(ii) True or False

(a) $3 \in$ A

(b) $5 \in$ A

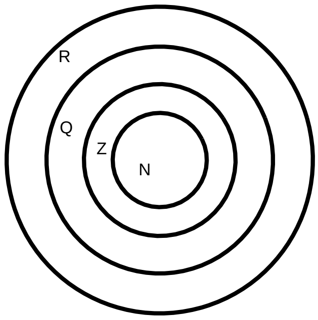
(c) $6 \notin$ B

(d) $\{4,5\} \subseteq$ A

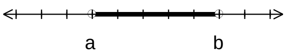
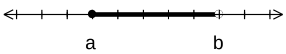
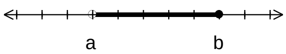
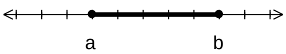
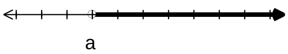
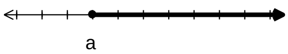
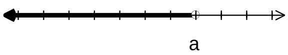
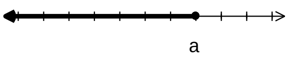
(e) $\{4,5\} \subseteq$ B

Sets of Numbers

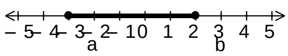
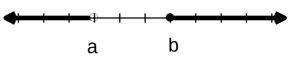
- **N**, the set of Natural Numbers {1, 2, 3, 4, …..} is a subset of...
- **Z**, the set of Integers {….,-2, -1, 0 , 1, 2, ….} is a subset of…
- **Q**, the set of Rational numbers, numbers which can be expressed in the form $\frac{m}{n}$ is a subset of…
- **R**, the set of Real numbers
- $N \subseteq Z \subseteq Q \subseteq R$
- **Q'**, is the set of irrational numbers, eg, $\sqrt{2}, \pi, e$



Subsets of the Real numbers

Set	Interval	Number Line
$\{x: a < x < b\}$	(a, b)	
$\{x: a \leq x < b\}$	[a, b)	
$\{x: a < x \leq b\}$	(a, b]	
$\{x: a \leq x \leq b\}$	[a, b]	
$\{x: x > a\}$	(a, ∞)	
$\{x: x \geq a\}$	[a, ∞)	
$\{x: x < a\}$	(-∞, a)	
$\{x: x \leq a\}$	(-∞, a]	

Example: Complete:

	Set	Interval	Number Line
A	$\{x: x>2\}$		
B		[-2,3]	
C			
D		(-∞, 5]	
E			
F		R^+	
G	$\{x: x < 0\}$		
H		$R \setminus \{0\}$	

- **Exercise 1A Q 1, 2, 3, 4, 5, 6, 7, 8, 9**

Relations and Functions

Definition of a function

- Any relation in which no two ordered pairs have the same first element (ie x – value).
    - The x value is only used once
    - {(1,2), (2,4), (3,6), (4,8)} is a function
    - {(-2,0), (-1, -3), (-1, 3), (0,-2), (0,2)} is not a function
- A function is a relation with one-to-one correspondence or many-to-one correspondence.
- Eg of that:
- Functions are a subset of relations (one-to-many or many-to-many)
- If a relation is represented graphically, apply a "**vertical line**" test to decide whether it is a function or not
    - Cuts the graph once – function
    - Cuts the graph more than once - not a function
- The first elements of the ordered pair in a function makes the set called the *DOMAIN*.
- The second elements make the set called the *RANGE*.
- Some other terms used: Image (y), pre-image (x), $(x, y) \in f$

Notation for description of a Function

- $f : R \to R, \quad f(x) = 3x^2$
- f is the name of the function (use f, g, h)
- R is the domain (be careful for restrictions of the domain)
- $\to R$ is the possible values that the domain can map onto (it is not the actual range)
- $f(x) = 3x^2$ represents the rule

Example: Rewrite the following using the function notation

$\{(x, y) : h(x) = 5x^2 + 7, x > -2\}$

Example: For the function with the rule $g(x) = 4x^2 + 5$, evaluate:

(i)	$g(2)$	(ii)	$g(-3)$	(iii)	$g(0)$
(iv)	$g(a)$	(v)	$g(x+h)$	(vi)	$g(x)=9$

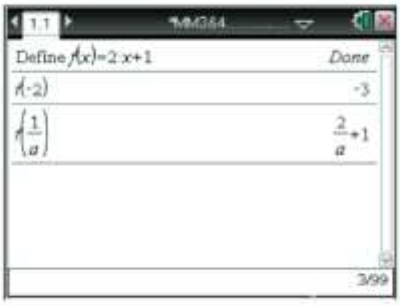
- **Ex1B Q 1, 2 cef, 3, 4, 5, 6, 7, 8, 9cde,10, 11, 12abc, 13, 14, 15, 16**

Using the TI-Nspire

Function notation can be used with a CAS calculator.
Use (menu) > **Actions** > **Define** to define the function $f(x) = 4x - 3$.
Type $f(-3)$ followed by **enter** to evaluate $f(-3)$.
Type $f(\{1, 2, 3\})$ followed by **enter** to evaluate $f(1)$, $f(2)$ and $f(3)$.

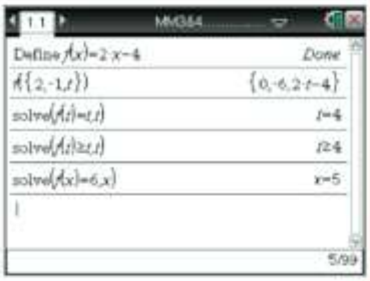
Using the TI-Nspire

Use (menu) > **Actions** > **Define** to define the function $f(x) = 2x + 1$.
Type $f(-2)$ followed by **enter** to evaluate $f(-2)$.
Type $f\left(\frac{1}{a}\right)$ followed by **enter** to evaluate $f\left(\frac{1}{a}\right)$.



Using the TI-Nspire

Use (menu) > **Actions** > **Define** to define the function and (menu) > **Algebra** > **Solve** to solve as shown.
The symbol $\geq$ can be found using (ctrl) + (=) and select $\geq$ or use (ctrl) + (menu) > **Symbols**.
On the Clickpad (grey handheld) you can use (ctrl) + (>) or (>) (=).



Using the TI-Nspire

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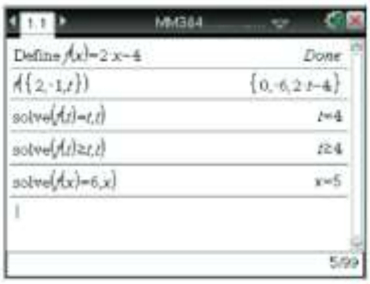


<u>One to One Functions</u>

- **VERTICAL LINE TEST –** to see if we have a function
- **HORIZONTAL LINE TEST –** to see if we have a one-to-one function
- **Examples:**
    - Parabola
    - Cubic
    - Exponential

<u>Implied domains</u>

- Often the domain is not stated for a function.
- Assume the domain is to be as large as possible (i.e. select from *R*)
- Examples:
    - $y = 3x^2$, the implied domain is _____
    - $y = \sqrt{x}$, the implied domain is _____
    - $y = \sqrt{x-2}$,
    - $y = \sqrt{2-x}$,
    - $y = \frac{1}{x}$,
    - $y = \frac{4}{2-x}$,
    - $y = \sqrt{x^2 - 7x + 12}$, look at graph of parabola,
    - $y = \frac{1}{x^2 - 7x + 12}$,

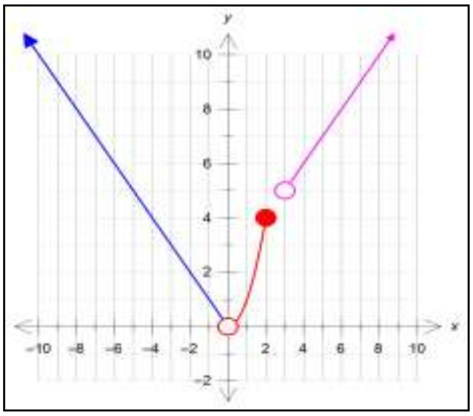
- **Ex 1C Q 1, 2, 3, 4, 5, 6, 7, 8**

<u>Hybrid functions (Piecewise functions)</u>

- A function which has <u>different rules</u> for <u>different subsets</u> of the domain.

Example: Sketch the graph of:

$$f(x)=\begin{cases}-x, x<0\\ x^2, 0<x\le 2\\ x+2, x>3\end{cases}$$ and state the domain and range.



Domain =
Range =

For the above function find:

(a) $f(-4)$ (b) $f(1)$ (c) $f(2a)$

(a) $f(-4)=$
$=$

(b) $f(1)=$
$=$

(c)
(i)
(ii)
(iii)
$f(2a)=\begin{cases} \\ \\ \end{cases}$

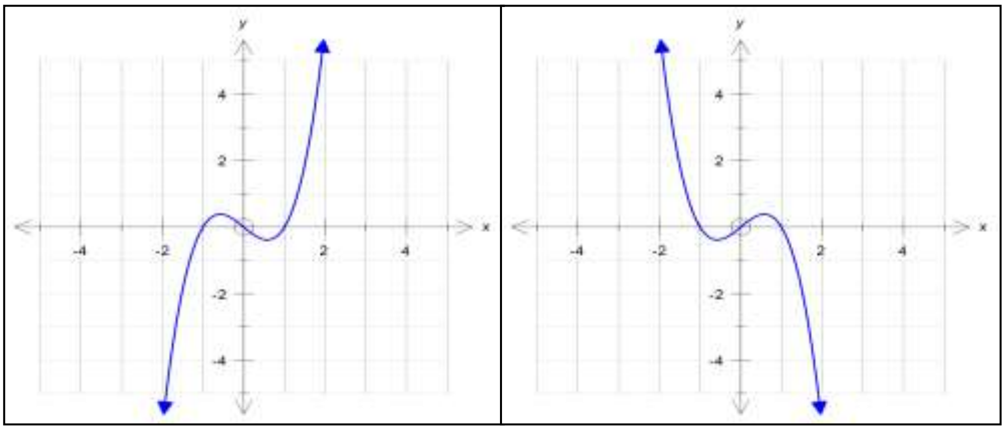
- *Ti-NSpire* : From the templates, select the hybrid with 3 choices. (see p18 of text).

Odd & Even Functions

- An odd function is defined by:

$$f(-x) = -f(x)$$
$$if \quad f(x) = x^3 - x$$
$$\Rightarrow f(-x) = (....)^3 - (....)$$
$$f(-x) = \qquad = \qquad = \qquad \therefore$$

This is the TEST to see if a function is ODD or EVEN



- Can also consider an Odd function has 180° rotational symmetry about the origin.

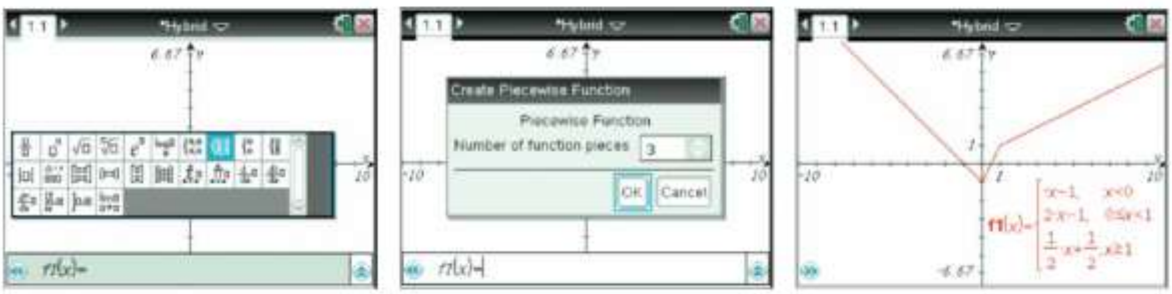
- An even function is defined by:

$$f(-x) = f(x)$$
$$if \quad f(x) = x^2 - 1$$
$$\Rightarrow f(-x) = (....)^2 - .....$$
$$f(-x) = \qquad = \qquad \therefore$$

- **Ex 1C Q 9 , 10 , 11 , 12, 13, 14, 15, 16, 17, 18, 19**

Plotting hybrid functions

In a Graphs application with the cursor in the entry line, select the hybrid (piecewise) function template, (ctrl + on the Clickpad) and enter as shown.



Sums and Products of Functions

Example: If $f(x)=\sqrt{x-2}$ and $g(x)=\sqrt{4-x}$ find:

(a) $(f+g)(x)$

(b) $(f+g)(3)$

(c) $(fg)(x)$

(d) $(fg)(3)$

$(a)\quad (f+g)(x)=f(x)+g(x)=$

Implied domain for $f=[\qquad)$ **and the implied domain for** $g=(\qquad]$

$\Rightarrow$ **only defined domain for** $f+g=[\qquad]$

$(b)\quad (f+g)(3)=$

$(c)\quad (fg)(x)=f(x)\times g(x)=$

$(d)\quad (fg)(3)=$

Graphing by Additions of Ordinates

This involves the addition of the *y*-values of the given equations.

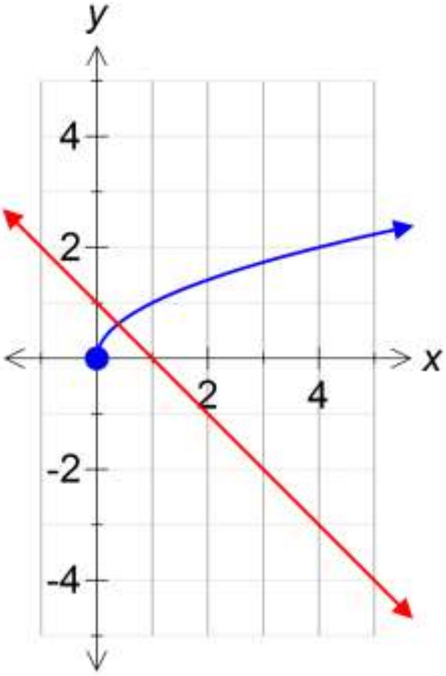
For example, if $f(x)=\sqrt{x}$ and $g(x)=1-x$ the graph of $y=f(x)+g(x)$ is obtained by adding the *y*-values for every value of *x* for which both curves simultaneously exist.

For $f(x)=\sqrt{x}$ the domain is [0,∞)

For $g(x)=1-x$ the domain is (-∞,∞)

Therefore, the values of *x* for which both curves are defined simultaneously is given by
[0,∞)

Sketch the two graphs above, on graph paper, see blackboard for specific instructions.



Adding the *y*-values is straight forward as long as you know the equations of the graphs. However, you need to be able to add two graphs without this information.

Hints: when using the addition of ordinates.

1. Look for regions where both graphs are positive
(ie both lie above the *x*-axis)
(this means that when you add the *y*-values, you will obtain a larger positive *y*-value)

2. Look for regions where both graphs are negative (ie both lie below the *x*-axis)
(this means that when you add the *y*-values, you will obtain a more negative *y*-value)

3. Consider the regions where the graphs differ in sign and then be discerning in where the sum of the two values lie.

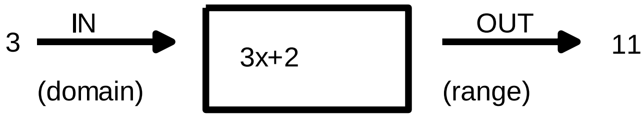
4. Look for asymptotic behaviour.

If you are asked to find $f(x)-g(x)$, it is easier to sketch $f(x)+(-g(x))$,that is, reflect $g(x)$in the *x*-axis and continue as above.

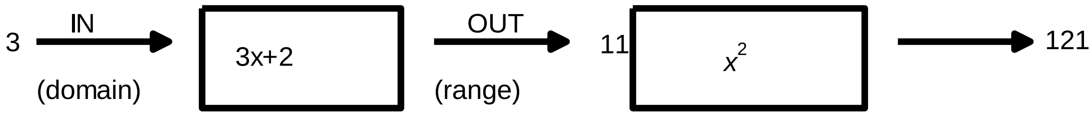
- **Ex1D Q 1, 2, 3, 4, 6, 8a, 10, 12**

Composite functions

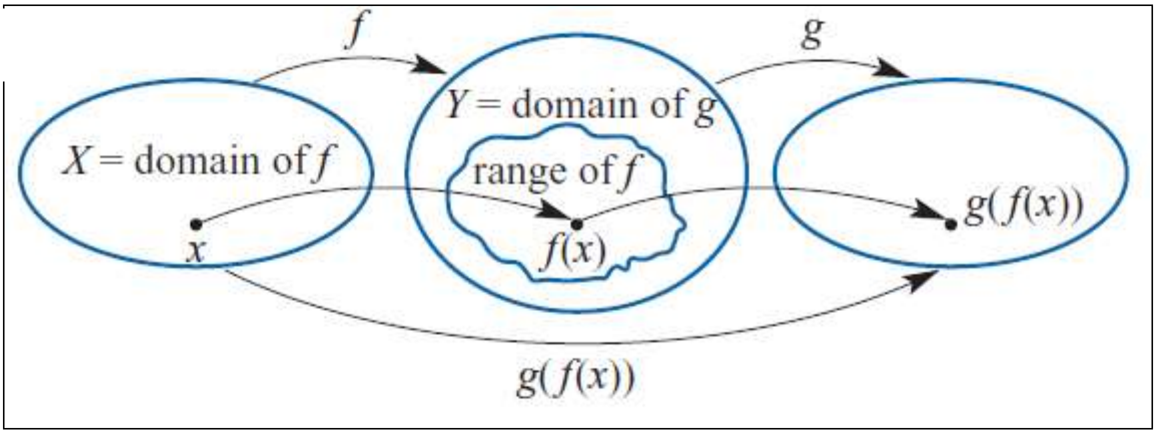
- Think of a function machine, eg $f(x)=3x+2$ and find $f(3)$.



- What happens if we use 2 machines, eg $f(x)=3x+2$ and $g(x)=x^2$



- New function has been defined, $h(x)$, $h(x)=(3x+2)^2$
- $h(3)=121$ or $h(3)=g(f(3))=g(11)$
- $h(-2)=g(f(-2))=g(-4)=16$
- $\Rightarrow h$ is said to be the composition of g with f.
- $h=g\circ f$ or $h(x)=g(f(x))$
- The domain of h or $h(x)$ is the domain of f.
- Consider $f(x)=x-3$ and $h(x)=\sqrt{x}$. When $x=4$, $x=2$



Example: Find both $f\circ g$ and $g\circ f$, stating the domain and range of each, if
$f:R\to R,\ f(x)=2x-1$ and $g:R\to R,\ g(x)=3x^2$.

	Domain	Range
f		
g		

$f\circ g =$
$=$
$=$
$dom\ f\circ g =$
$ran\ f\circ g =$

$g\circ f =$
$=$
$dom\ g\circ f =$
$ran\ g\circ f =$

- $f\circ g$ is defined since ________________
- $g\circ f$ is defined since _________________

Example 2: If $g(x) = 2x - 1, x \in R$ and $f(x) = \sqrt{x}, x \geq 0$

(a) state which of $f \circ g$ and $g \circ f$ is defined

(b) state the domain and rule of the defined.

	Domain	Range
f		
g		

(a)

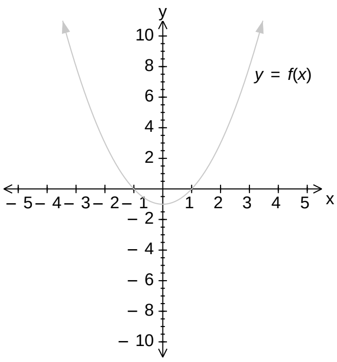
(b)

Example 3: for $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$,

(a) Is (i) $g \circ f$ defined, (ii) $f \circ g$ defined?

(b) Determine a restriction for f, f^*, so that $g \circ f^*$ is defined.

	Domain	Range
f		
g		



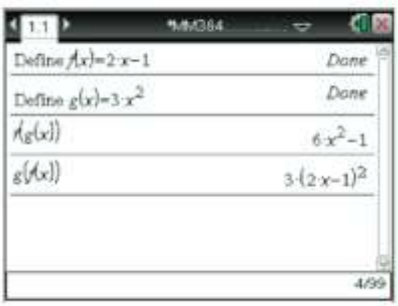
(a)

(b)

- **Ex1E Q 1a - e, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12**

Using the TI-Nspire

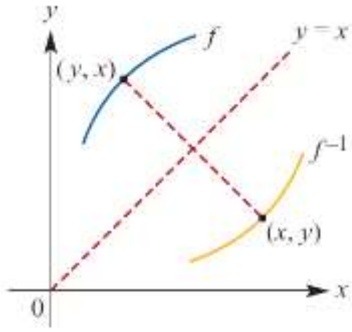
Define $f(x) = 2x - 1$ and $g(x) = 3x^2$. The rules for $f \circ g$ and $g \circ f$ can now be found using $f(g(x))$ and $g(f(x))$.



Inverse Functions

The Inverse of a function $y = f^{-1}(x)$

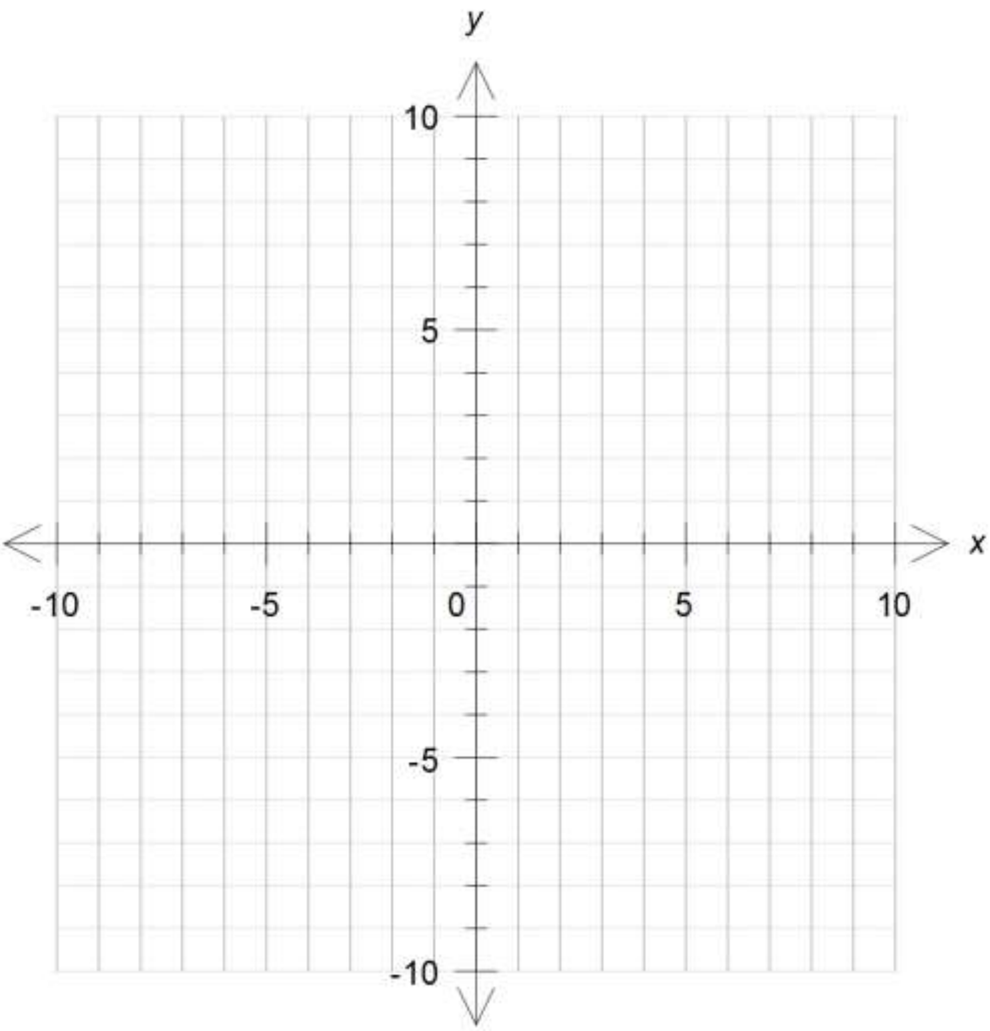
For the function $y = f(x)$



- The graph of its inverse $y = f^{-1}(x)$ is found by reflecting the original in the line $y = x$.
- The rule of its inverse is found by swapping x for y (and then making y the subject of the equation.

Example: For the function
$f : [-1, 3] \to R \text{ where } f(x) = 3x - 2$

a. Sketch the graph of f;
b. Sketch the graph of $y = x$;
c. Using the line $y = x$ as the "mirror" reflect the graph of f in it;
d. Find the domain and range for f and its inverse f^{-1};
e. Find the rule for $f^{-1}(x)$;
f. Fully define $f^{-1}(x)$.

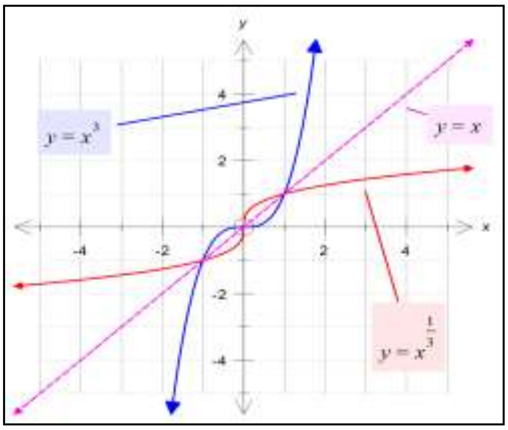


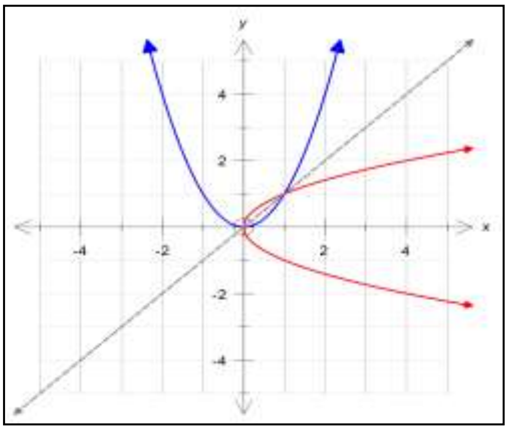
To find the rule for the inverse we swap the x and y in the original equation.

$fully\ defined: \quad f^{-1}:[\quad , \quad] \rightarrow R,\ f^{-1}(x) =$

- The domain of f = range f^{-1} and range f = domain f^{-1}
- If the graphs intersect, then the points of intersection MUST also be on the line $y = x$.
- So the points of intersection can be found in 3 ways:
  - $f(x) = f^{-1}(x)$
  - $f(x) = x$
  - $f^{-1}(x) = x$

- It is usually quicker and easier to use one of the last two.
- All functions have inverses, but the inverses may not be functions (they may only be relations).
- e.g. compare $y = x^3$ *and* $y = x^2$

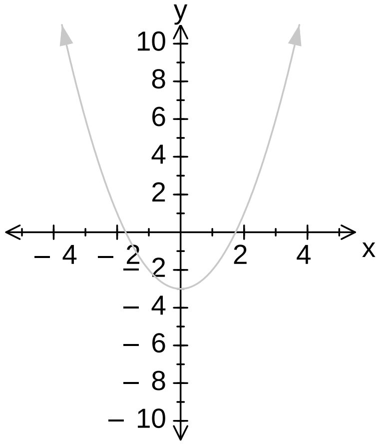




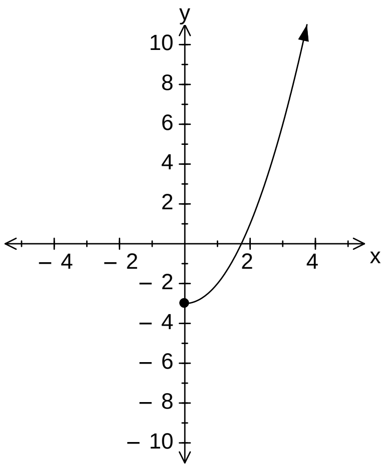
- Original is a function — Original is a function
- Inverse is a function — Inverse is not a function
- ***A function f, has an inverse function, written f^{-1} only if f is a one-to-one function.***
- ***i.e. a horizontal and a vertical line only crosses the graph of f once.***

It is possible to restrict the domain on a function, so it will have an inverse function, e.g. $f(x)=x^2$, the domain can be restricted in many ways, e.g $R^+ \cup \{0\}, [2,10), (-\infty,0], [-5,-1]$

Example: Restrict the domain of $g(x)=x^2-3$, so that we have an inverse function $g^{-1}(x)$. Find the two possible $g^{-1}(x)$, where the domain is as large as possible.

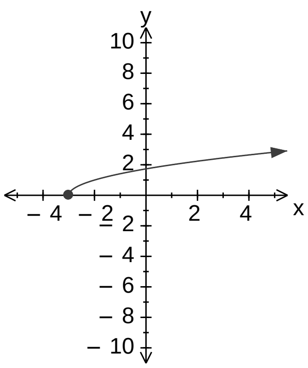


- Let's choose the RHS of the curve, i.e $x \geq 0$

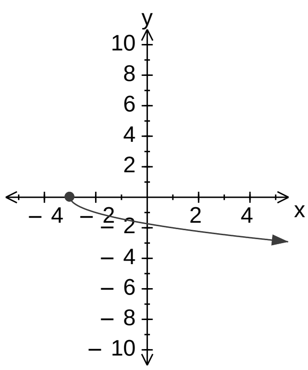


$$g(x)=x^2-3$$
$$Let\ y=x^2-3$$
$$swap\ x\ \&\ y$$
$$x=$$
$$=$$
$$=$$
$$y= \qquad or \qquad y=$$

which one is g^{-1}?



$domain: [-3,\infty)$
$range: [0,\infty)$



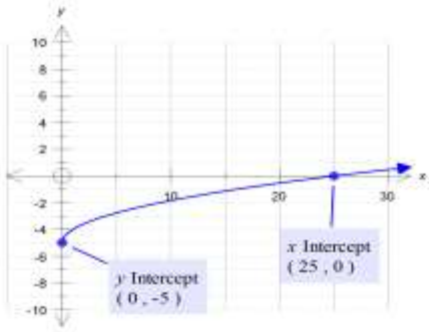
$domain: [-3,\infty)$
$range: (-\infty,0]$

- As the $dom\ g = ran\ g^{-1}$ & $ran\ g = dom\ g^{-1}$ then $g^{-1}(x)=+\sqrt{x+3}$.
- $\Rightarrow g^{-1}: [-3,\infty) \to R,\ g^{-1}(x)=\sqrt{x+3}$
- If we used the LHS of the curve, then $\Rightarrow g^{-1}: [-3,\infty) \to R,\ g^{-1}(x)=-\sqrt{x+3}$

Example: If $h: S \to R,\ h(x) = \sqrt{x} - 5,$ find:

(a) S (b) h^{-1} (c) $h^{-1}(-2)$ (d) $h^{-1}(-7)$

(a) **S is the domain,** $dom\ h = [\quad , \quad)$ and $ran\ h = [\quad , \quad)$



(b)

$h(x) = \sqrt{x} - 5$

$Let\ y = \sqrt{x} - 5$

$swap\ x\ \&\ y$

$= \sqrt{\ } -$

$= \sqrt{\ }$

$=$

$\Rightarrow h^{-1}(x) =$

$h^{-1}: [\qquad) \to R,\ h^{-1}(x) =$

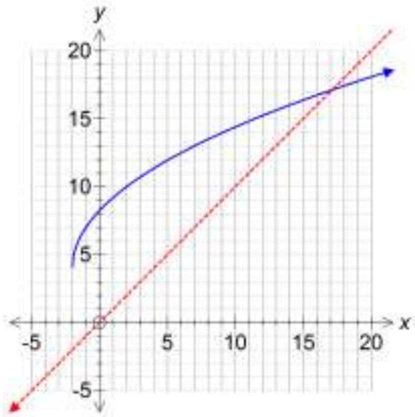
(c) $h^{-1}(-2) =$

(d) $h^{-1}(-7)$

Example

Find the inverse of the function with rule $f(x) = 3\sqrt{x+2} + 4$ and sketch both functions on one set of axes, clearly showing the exact coordinates of intersection of the two graphs.

Solution



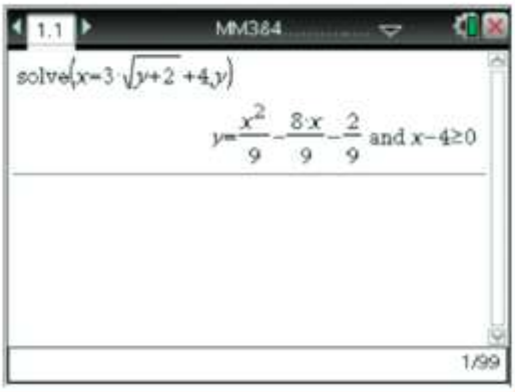
Note: The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.
The graph of $y = f^{-1}(x)$ is obtained from the graph of $y = f(x)$ by applying the transformation $(x, y) \to (y, x)$.
In this particular example, it is simpler to solve $f^{-1}(x) = x$ to solve the point of intersection.
Graphical Calculator can be used to find the inverse of a function

- Define the function
- Solve(*f(y)=x, y)*

- **Ex1F Q 1, 2, 3, 4, 6, 7, 8aceg, 9, 10, 11, 12, 13, 14, 15**

Using the TI-Nspire

To find the rule for the inverse of $y = 3\sqrt{x+2} + 4$, enter **solve** $(x = 3\sqrt{y+2} + 4, y)$.



Power Functions

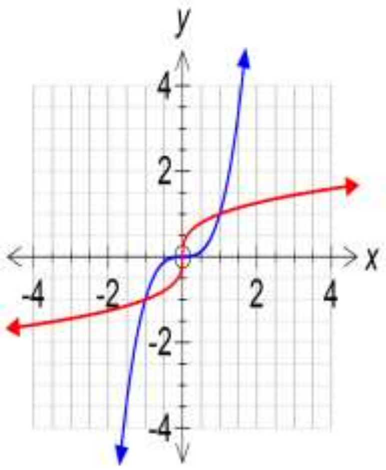
- Power functions are of the form: $f(x) = x^p; p \in Q$ (i.e. *p* is rational)

Strictly increasing and strictly decreasing functions

- A function f is said to be *strictly increasing* when $a < b$ implies $f(a)<f(b)$ for all *a* and *b* in its domain.
- If a function is strictly increasing, then it is a one-to-one function and has an inverse that is also strictly increasing.

Example 1: The function $f : R \to R, f(x) = x^3$ is strictly increasing with zero gradient at the origin.

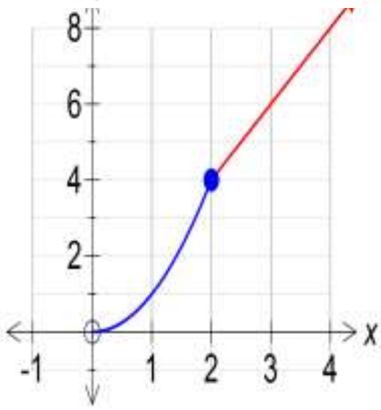
The inverse function $f^{-1} : R \to R, f^{-1}(x) = x^{\frac{1}{3}}$, is also strictly increasing, with a vertical tangent of undefined gradient at the origin.



Example 2: The hybrid function *g* with domain [0, ∞) and rule:

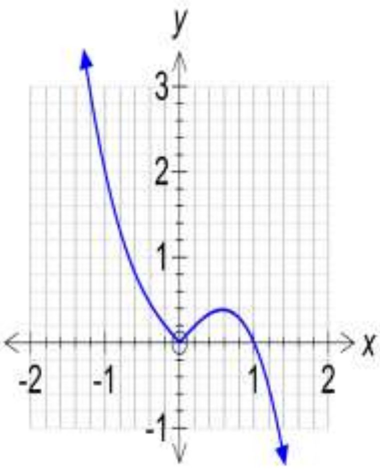
$$g(x) = \begin{cases} x^2 & 0 \le x \le 2 \\ 2x & x > 2 \end{cases}$$

is strictly increasing, and is not differentiable at *x* = 2.



Example 3: Consider $h : R \to R, h(x) = |x| - x^3$

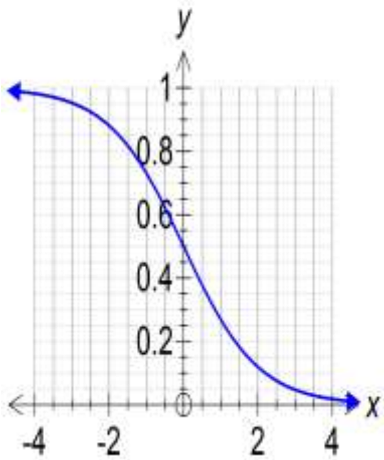
H is not strictly increasing,
But is strictly increasing over the *interval* $\left[0, \frac{1}{\sqrt{3}}\right]$.



Strictly Decreasing

A function f is said to be *strictly decreasing* when $a < b$ implies $f(a) > f(b)$ for all a and b in its domain.

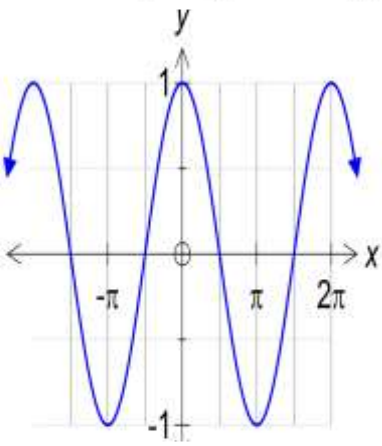
A function is said to be strictly decreasing over an interval when $a < b$ implies $f(a) > f(b)$ for all a and b in its interval.



Example 4: The function $f: R \rightarrow R,\ f(x) = \dfrac{1}{e^x + 1}$

The function is strictly decreasing over R.

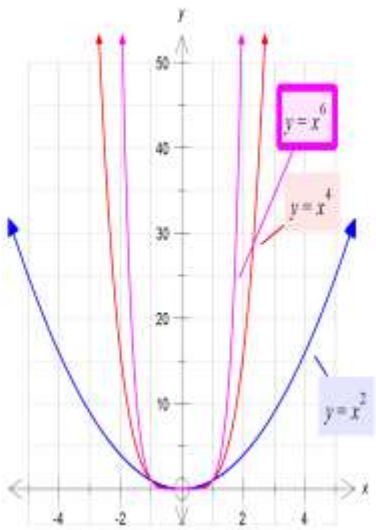
Example 5: The function $g: R \rightarrow R, g(x) = \cos(x)$



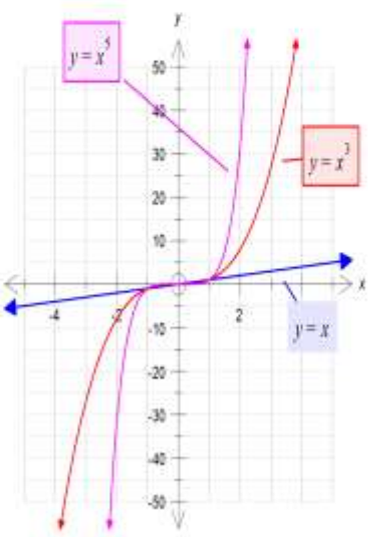
g is not strictly decreasing.

But g is strictly decreasing over the interval $[0, \pi]$.

Power functions with positive integer index



- Functions of the form: $f(x) = x^p; p = 1, 2, ...$
- 2 groups: the even powers and the odd powers.
- **Even powers**, $f(x) = x^2, x^4, x^6, ...$
    - All have the "U-shaped" graph
    - Domain: R
    - Range: $R^+ \cup \{0\}$ *or* $[0, \infty)$
    - Strictly increasing for x……
    - Strictly decreasing for x……
    - As $x \rightarrow \pm\infty,\ f(x) \uparrow \infty$



- **Odd powers**, $f(x) = x, x^3, x^5, ...$
    - All slope from bottom left to top right
    - Domain: R
    - Range: R
    - Strictly increasing for …….
    - F is one-to-one
    - As $x \rightarrow +\infty,\ f(x) \rightarrow \infty$ & $x \rightarrow -\infty,\ f(x) \rightarrow -\infty$

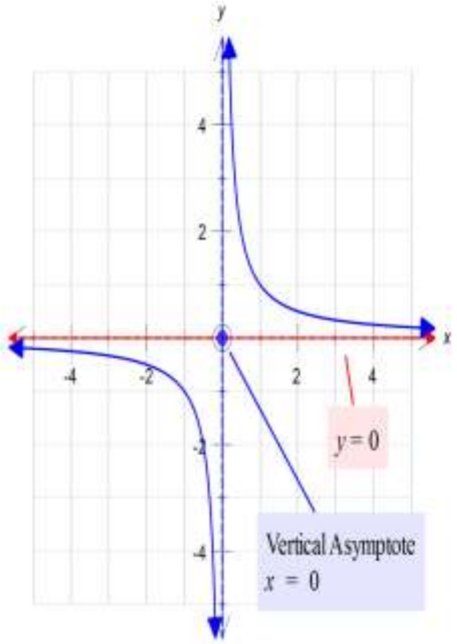
Power functions with negative integer index

- Functions of the form: $f(x) = x^p$; $p = -1, -2, \ldots$.
- 2 groups: the even powers and the odd powers.

Odd Negative Powers

Functions of the form: $f(x) = x^p$; $p = -1, -3, \ldots$

- Sketch the graph of $f(x) = \frac{1}{x}$ $or = x^{-1}$



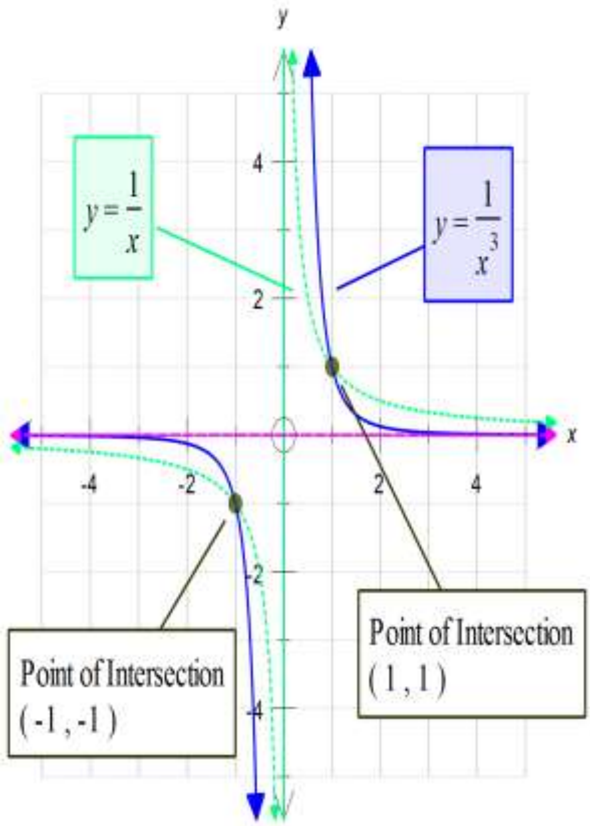
- Domain:
- Range:
- Asymptotes:
  - **Horizontal:** $y = 0$

    $As\ x \to \infty, \frac{1}{x} \to 0^+$

    $As\ x \to -\infty, \frac{1}{x} \to 0^-$
  - **Vertical:** $x = 0$

    $As\ x \to 0^+, \frac{1}{x} \to \infty$

    $As\ x \to 0^-, \frac{1}{x} \to -\infty$
- Odd function: $f(-x) = \frac{1}{-x} = -\frac{1}{x}$
- Sketch the graph of $f(x) = \frac{1}{x^3}$ $or = x^{-3}$



Even Negative Powers

Functions of the form: $f(x) = x^p; p = -2, -4, \ldots$

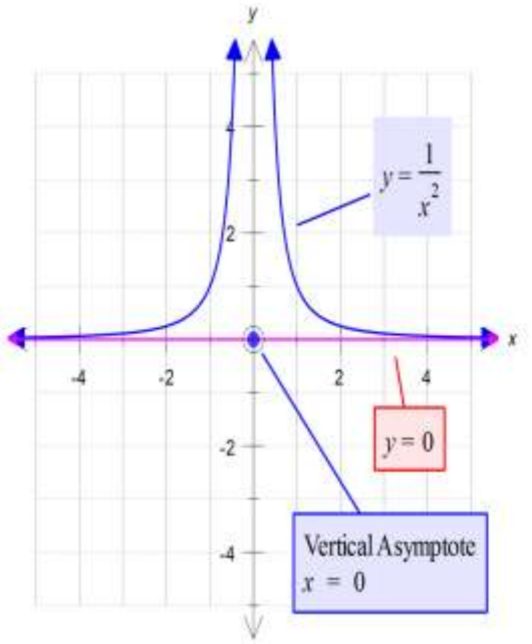
- Sketch the graph of $f(x) = \frac{1}{x^2}$ or $= x^{-2}$
- Domain:
- Range:
- Asymptotes:
  - **Horizontal:** $y = 0$

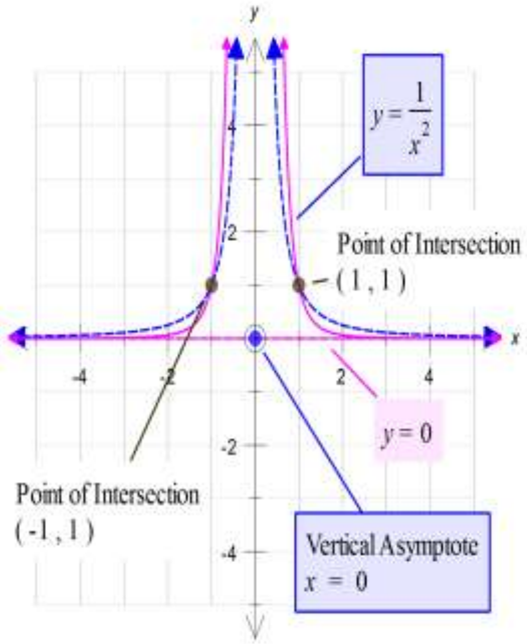
    $As\ x \to \infty, \frac{1}{x} \to 0^+$

    $As\ x \to -\infty, \frac{1}{x} \to 0^-$
  - **Vertical:** $x = 0$

    $As\ x \to 0^+, \frac{1}{x} \to \infty$

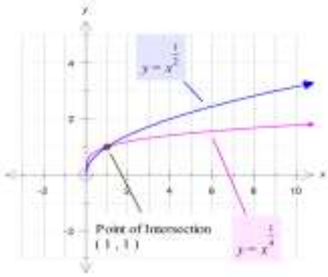
    $As\ x \to 0^-, \frac{1}{x} \to \infty$
- Even function: $f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2}$
- Sketch the graph of $f(x) = \frac{1}{x^4}$ or $= x^{-4}$



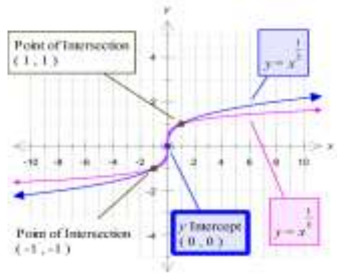


Functions with rational powers: $f(x) = x^{\frac{p}{q}}$

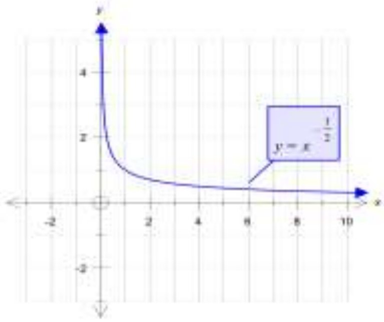
- Of the form: $f(x) = x^{\frac{1}{q}}$
- Remember $f(x) = x^{\frac{1}{q}} = \sqrt[q]{x}$
- Maximal domain is $[0, \infty)$ when q is even:



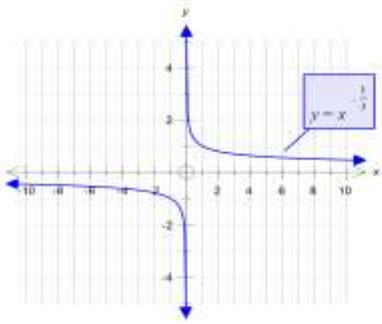
- Maximal domain is R when q is odd:



- Consider: $f(x) = \dfrac{1}{\sqrt[q]{x}} = x^{-\frac{1}{q}} = \dfrac{1}{x^{\frac{1}{q}}}$
- Domain $(0, \infty)$ if q is even and $R\backslash\{0\}$ if q is odd.
- $f(x) = \dfrac{1}{\sqrt{x}}$:
- Asymptotes: $y = 0$ and $x = 0$.

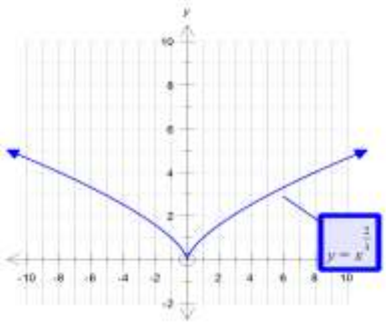


- $f(x) = \dfrac{1}{\sqrt[3]{x}}$

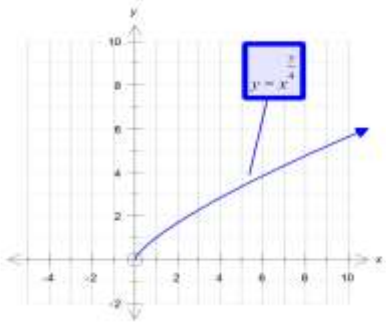


- In General: $f(x) = x^{\frac{p}{q}} = \left(\sqrt[q]{x}\right)^p$
- Always defined for $x \geq 0$, and when q is odd for all x.

- Eg: $f(x) = x^{\frac{2}{3}}$



- e.g.: $f(x) = x^{\frac{3}{4}}$



Inverses of Power Functions

Example: Find the inverse of each of the following:

a) $f : R \to R,\ f(x) = x^5$

b) $f : (-\infty, 0] \to R,\ f(x) = x^4$

c) $f : R \to R,\ f(x) = 8x^3$

d) $f : (1, \infty) \to R,\ f(x) = 64x^6$

$(a)\quad f : R \to R,\ f(x) = x^5$	$(b)\quad f : (-\infty, 0] \to R,\ f(x) = x^4$
$(c)\quad f : R \to R,\ f(x) = 8x^3$	$(d)\quad f : (1, \infty) \to R,\ f(x) = 64x^6$

- **Ex1G Q 1, 2, 3, 4, 5**

Past VCAA Exam Questions

2008

Question 6

The simultaneous linear equations

$$ax + 3y = 0$$
$$2x + (a + 1)y = 0$$

where a is a real constant, have infinitely many solutions for

A. $a \in R$
B. $a \in \{-3, 2\}$
C. $a \in R \setminus \{-3, 2\}$
D. $a \in \{-2, 3\}$
E. $a \in R \setminus \{-2, 3\}$

Question 7

The inverse of the function $f: R^+ \to R$, $f(x) = \frac{1}{\sqrt{x}} - 3$ is

A. $f^{-1}: R^+ \to R$ $\quad f^{-1}(x) = (x + 3)^2$

B. $f^{-1}: R^+ \to R$ $\quad f^{-1}(x) = \frac{1}{x^2} + 3$

C. $f^{-1}: (3, \infty) \to R$ $\quad f^{-1}(x) = \frac{-1}{(x-3)^2}$

D. $f^{-1}: (-3, \infty) \to R$ $\quad f^{-1}(x) = \frac{1}{(x+3)^2}$

E. $f^{-1}: (-3, \infty) \to R$ $\quad f^{-1}(x) = -\frac{1}{x^2} - 3$

Question 20

The function $f: B \to R$ with rule $f(x) = 4x^3 + 3x^2 + 1$ will have an inverse function for

A. $B = R$

B. $B = \left(\frac{1}{2}, \infty\right)$

C. $B = \left(-\infty, \frac{1}{2}\right]$

D. $B = \left(-\infty, \frac{1}{2}\right)$

E. $B = \left[-\frac{1}{2}, \infty\right)$

2009

Question 1

The simultaneous linear equations

$$kx - 3y = 0$$
$$5x - (k + 2)y = 0$$

where k is a real constant, have a unique solution provided

A. $k \in \{-5, 3\}$
B. $k \in R \setminus \{-5, 3\}$
C. $k \in \{-3, 5\}$
D. $k \in R \setminus \{-3, 5\}$
E. $k \in R \setminus \{0\}$

Question 3

Let $f: R \setminus \{0\} \to R$ where $f(x) = \frac{3}{x} - 4$. Find f^{-1}, the inverse function of f.

3 marks

2010

Question 7

The simultaneous linear equations $(m-1)x+5y=7$ and $3x+(m-3)y=0.7m$ have infinitely many solutions for

A. $m \in R \setminus \{0, -2\}$
B. $m \in R \setminus \{0\}$
C. $m \in R \setminus \{6\}$
D. $m = 6$
E. $m = -2$

Question 9

The function $f: (-\infty, a] \to R$ with rule $f(x) = x^3 - 3x^2 + 3$ will have an inverse function provided

A. $a \le 0$
B. $a \ge 2$
C. $a \ge 0$
D. $a \le 2$
E. $a \le 1$

Question 5

For the system of simultaneous linear equations

$$x = 5$$
$$z + y = 10$$
$$z - y = 6$$

an equivalent matrix equation is

A. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$

C. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2 \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 10 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 6 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 10 \end{bmatrix}$

Question 3

Let $f: R^+ \to R$ where $f(x) = \frac{1}{x^2}$.

a. Find $g(x) = f(f(x))$.

1 mark

b. Evaluate $g^{-1}(16)$, where g^{-1} is the inverse function of g.

1 mark

2011

Question 1

The midpoint of the line segment joining $(0, -5)$ to $(d, 0)$ is

A. $\left(\frac{d}{2}, -\frac{5}{2}\right)$

B. $(0, 0)$

C. $\left(\frac{d-5}{2}, 0\right)$

D. $\left(0, \frac{5-d}{2}\right)$

E. $\left(\frac{5+d}{2}, 0\right)$

Question 2

The gradient of a line **perpendicular** to the line which passes through $(-2, 0)$ and $(0, -4)$ is

A. $\frac{1}{2}$

B. -2

C. $-\frac{1}{2}$

D. 4

E. 2

Question 3

If $x + a$ is a factor of $4x^3 - 13x^2 - ax$, where $a \in R \setminus \{0\}$, then the value of a is

A. -4

B. -3

C. -1

D. 1

E. 2

Question 5

The inverse function of $g: [2, \infty) \to R, g(x) = \sqrt{2x-4}$ is

A. $g^{-1}: [2, \infty) \to R, g^{-1}(x) = \frac{x^2+4}{2}$

B. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = (2x-4)^2$

C. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = \sqrt{\frac{x}{2}+4}$

D. $g^{-1}: [0, \infty) \to R, g^{-1}(x) = \frac{x^2+4}{2}$

E. $g^{-1}: R \to R, g^{-1}(x) = \frac{x^2+4}{2}$

Question 8

Consider the function $f: R \to R, f(x) = x(x-4)$ and the function

$$g: \left[\frac{3}{2}, 5\right) \to R, g(x) = x + 3.$$

If the function $h = f + g$, then the domain of the inverse function of h is

A. $[0, 13)$

B. $\left[-\frac{3}{4}, 10\right]$

C. $\left(-\frac{3}{4}, \frac{15}{4}\right]$

D. $\left[\frac{3}{4}, 13\right)$

E. $\left[\frac{3}{2}, 13\right)$

Question 18

The equation $x^3 - 9x^2 + 15x + w = 0$ has only one solution for x when

A. $-7 < w < 25$

B. $w \leq -7$

C. $w \geq 25$

D. $w < -7$ or $w > 25$

E. $w > 1$

Question 4

If the function f has the rule $f(x) = \sqrt{x^2 - 9}$ and the function g has the rule $g(x) = x + 5$

a. find integers c and d such that $f(g(x)) = \sqrt{(x+c)(x+d)}$

2 marks

b. state the maximal domain for which $f(g(x))$ is defined.

2 marks

Question 6

Consider the simultaneous linear equations

$$kx - 3y = k + 3$$
$$4x + (k + 7)y = 1$$

where k is a real constant.

a. Find the value of k for which there are infinitely many solutions.

3 marks

b. Find the values of k for which there is a unique solution.

1 mark

2012

Question 3

The range of the function $f:[-2, 3) \to R$, $f(x) = x^2 - 2x - 8$ is

A. R

B. $(-9, -5]$

C. $(-5, 0)$

D. $[-9, 0]$

E. $[-9, -5)$

Question 5

Let the rule for a function g be $g(x) = \log_e((x-2)^2)$. For the function g, the

A. maximal domain = R^+ and range = R

B. maximal domain = $R\backslash\{2\}$ and range = R

C. maximal domain = $R\backslash\{2\}$ and range = $(-2, \infty)$

D. maximal domain = $[2, \infty)$ and range = $(0, \infty)$

E. maximal domain = $[2, \infty)$ and range = $[0, \infty)$

Question 17

A system of simultaneous linear equations is represented by the matrix equation

$$\begin{bmatrix} m & 3 \\ 1 & m+2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ m \end{bmatrix}.$$

The system of equations will have **no solution** when

A. $m = 1$

B. $m = -3$

C. $m \in \{1, -3\}$

D. $m \in R\backslash\{1\}$

E. $m \in \{1, 3\}$

Question 3

The rule for function h is $h(x) = 2x^3 + 1$. Find the rule for the inverse function h^{-1}.

2 marks

2013

Question 2

The midpoint of the line segment that joins $(1, -5)$ to $(d, 2)$ is

A. $\left(\frac{d+1}{2}, -\frac{3}{2}\right)$

B. $\left(\frac{1-d}{2}, -\frac{7}{2}\right)$

C. $\left(\frac{d-4}{2}, 0\right)$

D. $\left(0, \frac{1-d}{3}\right)$

E. $\left(\frac{5+d}{2}, 2\right)$

Question 3

If $x + a$ is a factor of $7x^3 + 9x^2 - 5ax$, where $a \in R\backslash\{0\}$, then the value of a is

A. -4

B. -2

C. -1

D. 1

E. 2

Question 5

If $f: (-\infty, 1) \to R, f(x) = 2\log_e(1-x)$ and $g: [-1, \infty) \to R, g(x) = 3\sqrt{x+1}$, then the maximal domain of the function $f + g$ is

A. $[-1, 1)$

B. $(1, \infty)$

C. $(-1, 1]$

D. $(-\infty, -1]$

E. R

Question 21

The cubic function $f: R \to R, f(x) = ax^3 - bx^2 + cx$, where a, b and c are positive constants, has no stationary points when

A. $c > \frac{b^2}{4a}$

B. $c < \frac{b^2}{4a}$

C. $c < 4b^2a$

D. $c > \frac{b^2}{3a}$

E. $c < \frac{b^2}{3a}$

2014

Question 2

The linear function $f: D \to R, f(x) = 4 - x$ has range $[-2, 6)$.

The domain D of the function is

A. $[-2, 6)$

B. $(-2, 2]$

C. R

D. $(-2, 6]$

E. $[-6, 2]$

Question 6

The function $f: D \to R$ with rule $f(x) = 2x^3 - 9x^2 - 168x$ will have an inverse function for

A. $D = R$

B. $D = (7, \infty)$

C. $D = (-4, 8)$

D. $D = (-\infty, 0)$

E. $D = \left[-\frac{1}{2}, \infty\right)$

Question 9

The inverse of the function $f: R^+ \to R,\ f(x) = \frac{1}{\sqrt{x}} + 4$ is

A. $f^{-1}: (4, \infty) \to R \qquad f^{-1}(x) = \frac{1}{(x-4)^2}$

B. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = \frac{1}{x^2} + 4$

C. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = (x+4)^2$

D. $f^{-1}: (-4, \infty) \to R \qquad f^{-1}(x) = \frac{1}{(x+4)^2}$

E. $f^{-1}: (-\infty, 4) \to R \qquad f^{-1}(x) = \frac{1}{(x-4)^2}$

Question 17

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for

A. $a = 3$

B. $a = -3$

C. both $a = 3$ and $a = -3$

D. $a \in R\backslash\{3\}$

E. $a \in R\backslash\{-3, 3\}$

Question 18

The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for

A. $k = 6$

B. $k > 6$ or $k < -2$

C. $-2 \le k \le 6$

D. $6 - 2\sqrt{3} \le k \le 6 + 2\sqrt{3}$

E. $k = -2$

Question 2

The inverse function of $f : (-2, \infty) \to R,\ f(x) = \dfrac{1}{\sqrt{x+2}}$ is

A. $f^{-1}: R^{+} \to R \qquad f^{-1}(x) = \dfrac{1}{x^2} - 2$

B. $f^{-1}: R\backslash\{0\} \to R \qquad f^{-1}(x) = \dfrac{1}{x^2} - 2$

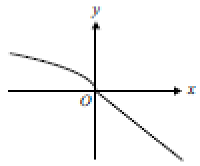
C. $f^{-1}: R^{+} \to R \qquad f^{-1}(x) = \dfrac{1}{x^2} + 2$

D. $f^{-1}: (-2, \infty) \to R \qquad f^{-1}(x) = x^2 + 2$

E. $f^{-1}: (2, \infty) \to R \qquad f^{-1}(x) = \dfrac{1}{x^2 - 2}$

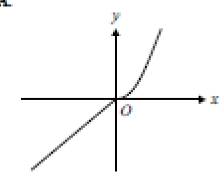
Question 5

Part of the graph of $y = f(x)$ is shown below.



The corresponding part of the graph of the inverse function $y = f^{-1}(x)$ is best represented by

A.



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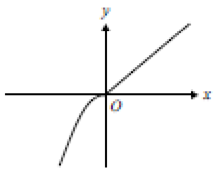
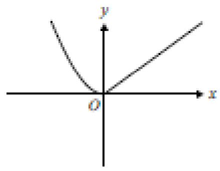
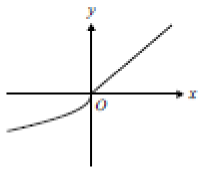


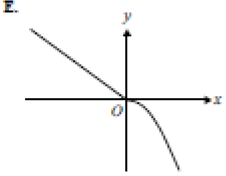
C.



D.



E.



Question 6

For the polynomial $P(x) = x^3 - ax^2 - 4x + 4$, $P(3) = 10$, the value of a is

A. -3

B. -1

C. 1

D. 3

E. 10

Question 7

The range of the function $f: (-1, 2] \rightarrow R$, $f(x) = -x^2 + 2x - 3$ is

A. R

B. $(-6, -3]$

C. $(-6, -2]$

D. $[-6, -3]$

E. $[-6, -2]$

Question 17

A graph with rule $f(x) = x^3 - 3x^2 + c$, where c is a real number, has three distinct x-intercepts.

The set of all possible values of c is

A. R

B. R^+

C. $\{0, 4\}$

D. $(0, 4)$

E. $(-\infty, 4)$

Question 21

The graphs of $y = mx + c$ and $y = ax^2$ will have no points of intersection for all values of m, c and a such that

A. $a > 0$ and $c > 0$

B. $a > 0$ and $c < 0$

C. $a > 0$ and $c > -\frac{m^2}{4a}$

D. $a < 0$ and $c > -\frac{m^2}{4a}$

E. $m > 0$ and $c > 0$