## Survey of Calculus

**Exercise**: Find $\lim_{x\to 3} g(x)$, where $g(x) = \dfrac{x^3 - 3x^2}{x - 3}$.

**Solution**: There are two ways we can evaluate this limit:

First, we could use a table. The function $g(x)$ is undefined when $x = 3$, since the value $x = 3$ makes the denominator 0. But we're looking for the $y$-value as $x$ gets *close* to $x = 3$. So we can test values that are close to 3, but not equal to 3.

| $x$ | 2.9 | 2.99 | 2.999 | 3 | 3.001 | 3.01 | 3.1 |
|---|---|---|---|---|---|---|---|
| $g(x)$ | 8.41 | 8.94 | 8.994 | | 9.006 | 9.06 | 9.61 |

We can see that although $g(x)$ is undefined at 3, as $x$ nears 3, $g(x)$ approaches 9, so $\lim_{x\to 3} g(x) = 9$.

Another method to solve this limit is by using algebra. Factoring the numerator, we see $x^3 - 3x^2 = x^2(x - 3)$. So we can simplify $g(x)$ to

$$g(x) = \frac{x^2 - 3x^2}{x - 3} = \frac{x^2(x - 3)}{(x - 3)} = x^2.$$

Note: This is provided $x \neq 3$. If $x = 3$, then the function is undefined at that point, and is not equal to $x^2$.

This means that the graph of $g(x)$ is *almost* the same as the graph as $x^2$, but there is a hole at $x = 3$. Since we are looking for values as $x$ approaches 3, we look for values of the function that are close to, but not equal to 3. Therefore, the limit is

$$\lim_{x\to 3} g(x) = \lim_{x\to 3} x^2 = 9.$$