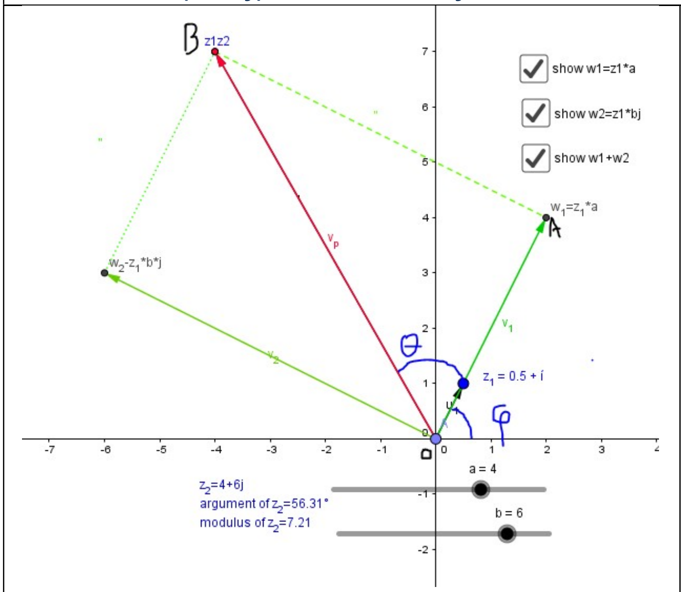
Let's multiply z_1 by $z_2=a+bj$

$z_1 \times z_2 = z_1 \times (a+bj) = z_1 \times a + z_1 \times bj = w_1 + w_2$, where w_1 $z_1 \times a$ and $w_2 = z_1 \times bj$



$w_1 = z_1 a \Leftrightarrow = |z_1| a$

$w_2 = z_1 bj \Leftrightarrow |w_2| = |z_1| b$

• Applying pythagoras theorem on AOB right angle triangle:

$OA^2 + AB^2 = OB^2$

$|w_1|^2 + |w_2|^2 = |z_1 z_2|^2 \Leftrightarrow |z_1|^2 a^2 + |z_1|^2 b^2 = |z_1 z_2|^2 \Leftrightarrow$

$|z_1|^2 (a^2 + b^2) = |z_1 z_2|^2 \Leftrightarrow |z_1|^2 |z_2|^2 = |z_1 z_2|^2 \Leftrightarrow$

$|z_1||z_2| = |z_1 z_2|$

• $\arg z_2 = \arctan(\frac{b}{a})$

• Using again the triangle OAB :

$\theta = \arctan(\frac{|w_2|}{|w_1|}) = \arctan(\frac{|z_1| b}{|z_1| a}) = \arctan(\frac{b}{a}) = \arg z_2$

• We know that $\arg z_1 = \varphi$, *which* is the angle from the positive real axis to the vector representing z_1.

Similarly thinking:

$\arg z_1 z_2 = \theta + \varphi = \arg z_1 + \arg z_2$

When we multiply two complex numbers using their polar form, we multiply their moduli and add their arguments.

If $Z_1=r_1(\cos\theta+j\sin\theta)$ and , $Z_2=r_2(\cos\phi+j\sin\phi)$ then

$Z_1 Z_2= r_1 r_2(\cos(\theta+\phi)+j\sin(\theta+\phi))$