Area under a curve for $f(x)=x^n$

Pre-Knowledge: Indefinite Integration

Using the integration applet:

Initial Settings
- Lower=0
- Upper = 1
- Rectangles =2
- $f(x)=x^2$

Investigation
1. Select the lower checkbox and increase the number of rectangles.
2. What do you notice about the rectangles and the curve as the number of rectangles increases?
3. Now select the upper checkbox and repeat increasing the number of rectangles.
4. The rectangles are clearly an approximation to the actual area. Where must the value of the actual area lie?
5. What do you think the actual area is? Compare your answer with the actual area by clicking the area checkbox. Note the answer.

Reset the boxes and sliders. Repeat the above with increasing powers of *n* until you are ready to make a conjecture for a formula for the area under a curve (Hint: Use your knowledge of integration). Test your formula on some other values of *n*.

Use the Geogebra applet to answer the following:

What if the function changes to $f(x)=ax^n$ with a being some constant. Does your formula still work?

Use the applet to show that the area under the curve of $f(x)=2x^2$ from 0 to 1 is not the same as the area under the function $f(x)=x^2$ from 0 to 2.

What should be the upper limit for the area under $f(x)=x^2$ to be the same as the area under the curve $f(x)=2x^2$ from 0 to 1?

What if a function has an asymptote e.g. $f(x) = \frac{1}{x^2}$? Does your formula still work?

What about $f(x) = \frac{1}{x}$? Does your formula work now?