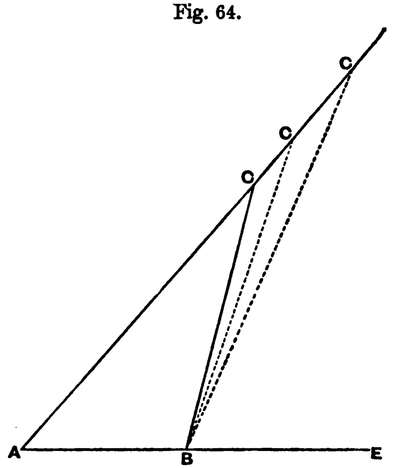
H F G and H G F, respectively, equal to the angles C A B and C B A, and inasmuch as the point H, where the sides F H and G H meet, is necessarily determined by this operation, as well as the angle F H G, we have the triangle F G H exactly similar to the triangle A B C.

62.

As it is important in practice, as already said, that angles should be exactly measured, it is not enough to observe them, even with the most perfect instruments: means must be found to verify their measure, to correct it, if necessary. There are simple and easy methods of doing so. Returning to the triangle A B C, we feel that the magnitude of the angle C must depend on the magnitude of the angles A and B, for if these angles were increased or diminished, the position of the lines C A, B C, would be changed, and consequently the angle C contained by those lines. If then that angle depends on the angles A and B, we may assume that the number of degrees contained in the angles A and B should determine the number of degrees contained in the angle C, and this should serve to verify the operation performed in determining the angles A and B; for we may be sure that we have correctly measured the angles A and B, if, in afterwards measuring the angle C, we find in it the number of degrees which it should have relatively to the angles A and B.

Fig. 64.

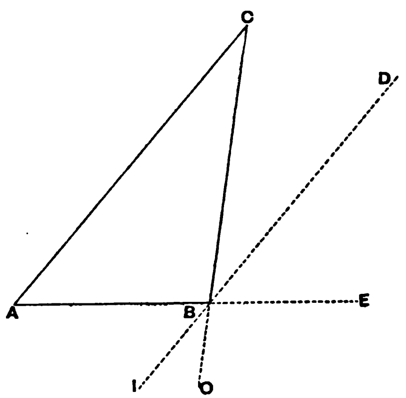


To find how from the angles A and B we can determine the angle C, let us examine what would happen to this angle, if the lines A C, B C, either approached towards, or receded from, each other. Let us suppose, for instance, that B C, turning round the point B, recedes from A B, approaching B E, it is clear that while B C turns, the angle B becomes opened continually, and that on the contrary the angle C becomes more and more contracted. This at first sight would lead us to assume that the diminution of the angle C would equal the augmentation of the angle B, and that therefore the sum of the three angles A B C would be always the same, whatever might be the inclination of the lines A B, B C, to the line A E.

63.

Now, this assumed inference carries with it its own demonstration, for if we draw I D parallel to A C, we see first that the angles A C B and C B D, called *alternate angles*, are equal: which

Fig. 65.



is evident, seeing that as the lines A C and I B are parallel, they must be equally inclined to C B O, and therefore the angle I B O must equal also the angle A C B. But the angle I B O is also equal to the angle C B D, because the line I D cannot be more inclined to C O on the one side than on the other, therefore the angle D B C, equal to the angle I B O, is equal to the angle A B C, its alternate angle.

64.

We see, in the second place, that the angle C A E is equal to the angle D B E because of the parallels C A and D B. Therefore the three angles of the triangle could be put side by side with a common vertex at the point B, and it is seen that the three angles D B E, C B D, and C B A, which are respectively equal to the three angles C A B, A C B, and C B A, are together equal to two right angles (Par. 57).

As all that has been said applies to any triangle whatever, we are assured of this general property, that the sum of the three angles of a triangle is constantly the same, and that it is equal to that of two right angles, or, what comes to the same thing, to 180 degrees.

65.

Therefore, to determine the value of the third angle of a triangle, when we have measured two, we subtract from 180 degrees the number of degrees contained in the two angles together, a property which gives a very convenient mode of verifying the measurement of the angles of a triangle, and which serves many other purposes, as will be seen in the sequel.

We may satisfy ourselves here by deducing the more immediate consequences of this law.

66.

A triangle cannot have more than one right angle; and it is still more obvious that it cannot have more than one obtuse angle.

67.

If one of the three angles of a triangle is a right angle, the sum of the two others is always equal to a right angle.

These two propositions are so clear that they have no need of demonstration.

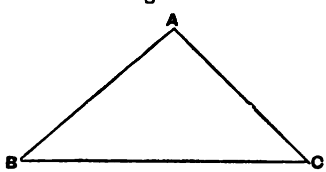
68.

If we prolong one of the sides of the triangle A B C, for instance, the side A B, the exterior angle C B E alone is equal to the two interior opposite angles B C A and C A B, together. For if to the angle C B A be added either the two angles B C A and C A B, or the angle C B E, the sum remains still equal to 180 degrees, or to two right angles (Par. 64).

69.

Knowing one of the angles of an isosceles triangle A B C, we know the two others. Let us have the angle at the vertex A. It is clear that if we subtract the number of degrees contained in this angle from 180 degrees, which is the sum of the three angles of the triangle, half the remainder will measure each of the angles B and C at the base.

Fig. 66.



If one of the two angles B or C at the base were known, the double of its value subtracted from 180 degrees would give the angle A at the vertex.

70.

As an equilateral triangle is only an isosceles triangle, of which anyone of the sides may be taken as base, its three angles are necessarily equal, and each contains 60 degrees, the third part of 180 degrees.

71.

Hence is readily deduced the method of describing a hexagon or polygon of six sides (promised in Par. 24).

For, in order to find a line that sets off a sixth part of the circumference, such line must be the chord of an arc of 60 degrees, the sixth part of 360 degrees, or the whole circum-