Taak theorie verloop van functies

Virga Jessecollege **Toets WISKUNDE**
Hasselt **leerkracht: Karel Appeltans**
Datum: **schooljaar .....**
Klas: V **Afdeling:**
Naam:..... **Aantal uren wiskunde: 6**

1	Show that $$2x - \sin(x) + x^3 + 2 = 0$$ has one and exactly one solution. Identify the theorem(s) you are using.
2	True or False: Every polynomial of degree three has exactly one local maximum and one local minimum.
3	On the axes below, sketch the graph of a function f which satisfies: • f is continuous for all x • $f'(x) > 0$ and $f''(x) < 0$ for x in the interval $(1, \infty)$, • $f'(x) > 0$ and $f''(x) < 0$ for x in the interval $(-\infty, 1)$ • f is not differentiable at $x = 1$.
4	: Let $f(x)$ be a continuous function on the open interval (a, b). Which of the following four statements are always true? Select all that apply. (A) If $\lim_{x\to a+} f(x) = -\infty$ and $\lim_{x\to b-} f(x) = \infty$, then there is at least one zero of $f(x)$ inside (a, b). (B) If $f(x)$ has a local minimum at c in (a, b), then $f'(c) = 0$. (C) $f(x)$ must have both maximum and minimum inside (a, b). (D) If $f''(c) > 0$ for some point c in (a, b), then $f(x)$ has a local minimum at c. ☐ A ☐ D ☐ B ☐ C ☐ None of these answers are correct.
5	Bereken a, m en b zodat de functie $f: x \to \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$ voldoet aan de voorwaarden van de middelwaardestelling van Lagrange in [0,2]

6

Let $g(x)$ be a function with domain $\{x \mid x \neq 2\}$ that is twice differentiable and satisfies the following conditions:

i) $\lim_{x\to\infty} g(x) = 1, \quad \lim_{x\to-\infty} g(x) = 0,$

ii) $g(1) = 2, \quad \lim_{x\to2^+} g(x) = \infty, \quad \lim_{x\to2^-} g(x) = -\infty,$

iii) $g'(x) > 0$ when $x < 1$ and $g'(x) < 0$ when $x > 1$ and $x \neq 2$,

iv) $g''(x) > 0$ when $x < -1$ and when $x > 2$,

v) $g''(x) < 0$ when $-1 < x < 2$.

Graph the function $g(x)$ on the axes given below.

7

10. Verify that $f(x) = x^2 - 4x + 1$ satisfies the three hypotheses of Rolle's theorem on the interval $[0, 4]$. Then find all numbers c that satisfy the conclusion of Rolle's theorem.

8

4.(10 pts) Prove that $2x - 1 - \sin x = 0$ has exactly one real root.

9

VI) Verificare che la funzione seguente soddisfi le ipotesi del teorema di Lagrange sull'intervallo $[-1, 2]$ per ogni valore del parametro

$$f(x) = \begin{cases} \sqrt{1-x^2} \text{ per } -1 \leq x \leq 0 \\ ax^3 + 1 \text{ per } 0 < x \leq 1 \\ a(3x-2) + 1 \text{ per } 1 < x \leq 2 \end{cases}$$

10

). Data la funzione

$$f(x) = \begin{cases} x^3, & \text{se } 0 \leq x \leq 1, \\ x^2 - kx + k, & \text{se } 1 < x \leq 2, \end{cases}$$

determinare il parametro k in modo che nell'intervallo $[0,2]$ sia applicabile il teorema di Lagrange e trovare il punto di cui la tesi del teorema assicura l'esistenza.

11

(15 points) Let $y = x^5 + 2x^3 + 5x + 2$ on the interval $(-1, 1)$. Show that at least one tangent line to the curve is parallel to the line $y = 8x + 3$. If you use any theorems, you must state them and show that their conditions are satisfied.

12

If a function f is continuous for all x and if f has a local maximum at $(-1, 4)$ and a local minimum at $(3, -2)$, which of the following statements *must* be true?

(A) The graph of f has an inflection point somewhere between $x = -1$ and $x = 3$.

(B) $f'(-1) = 0$.

(C) The graph of f has a horizontal asymptote.

(D) The graph of f has a horizontal tangent line at $x = 3$.

(E) The graph of f intersects both axes.

Verklaar jullie antwoord

13

. The derivative of a function f is given by

$$f'(x) = (x-1)x(x+1)^2(x+2)^3.$$

The function f has a local maximum only at

A. $x = -2$

B. $x = 0$

C. $x = -1$ and $x = 1$

D. $x = -1$ and $x = 0$

E. $x = -2$ and $x = 0$

14

. The first derivative of a function $f(x)$ is given by $f'(x) = x(x-1)^2(x-2)^3(x-3)^4(x-4)^5$. How many local minima of f are there on $(-\infty, \infty)$?

A. 0

B. 1

C. 2

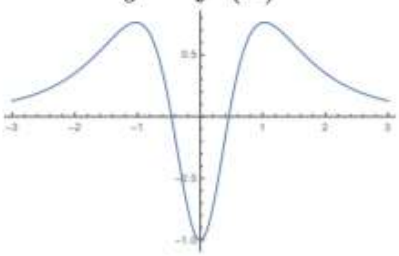
D. 3

E. We need the formula for $f(x)$ to determine the value and to see whether it is a local minimum. So we can NOT determine how many local minima there re on $(-\infty, \infty)$ from the given data.

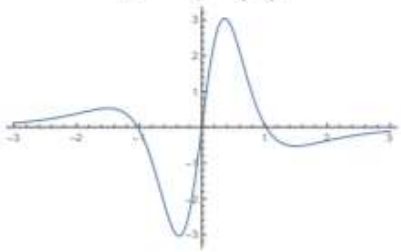
15

7. The graphs of $y = f'(x)$ and $y = f''(x)$ are given below.

$y = f'(x)$

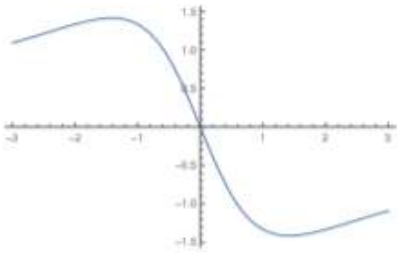


$y = f''(x)$

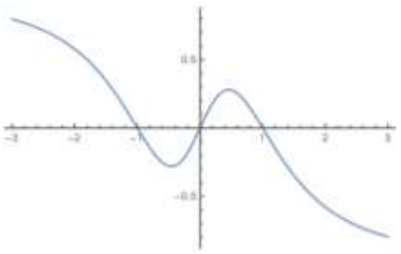


Which of the following then can be the graph of $y = f(x)$.

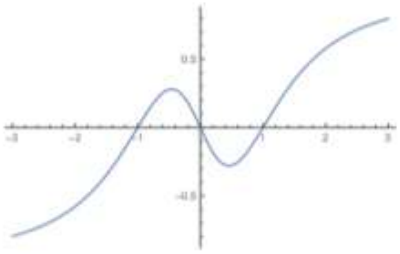
A.



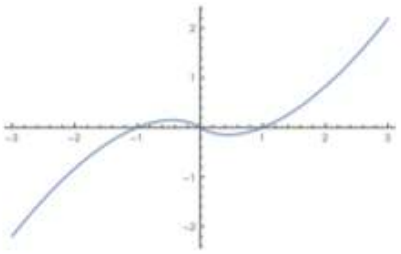
B.



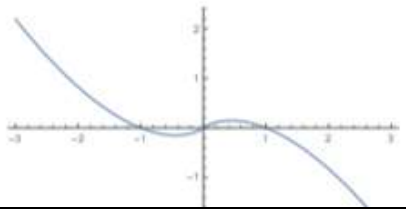
C.



D.



E.



16

The derivative of the function f is given by

$$f'(x) = (x-4)^5(x-2)^2x^3(x+2)^7(x+4)^4$$

Let M be the number of values of x at which local maximum is achieved, and m is the number of values of x at which local minimum is achieved. What is $M - m$?

A. -2

B. -1

C. 0

D. 1

E. 2

17

. (10 points) Suppose that the function $f(x)$ has an unknown formula, but the following properties are known.

(a) Domain= $[0, 4]$.

(b) $f(0) = 0, f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 2$.

(c) $f'(x) > 0$ for $x < 3$, and $f'(x) < 0$ for $x > 3$.

(d) $f''(x) > 0$ for $0 < x < 1$ and $x > 2$, and $f''(x) < 0$ for $1 < x < 2$.

Sketch a possible graph of $y = f(x)$.

18

. [5 points] On the axes provided below, sketch the graph of a single function $y = f(x)$ satisfying all of the following conditions:

(i) the function $f(x)$ is defined on $-6 < x < 6$ and continuous on $-6 < x < 3$,

(ii) the average rate of change of $f(x)$ on $[-5, -3]$ is equal to 2,

(iii) $f'(x) = -\frac{1}{2}$ for $1 < x < 3$,

(iv) $f(x) = f(-x)$ for $-3 \leq x \leq 3$,

(v) $f(x)$ is concave up and decreasing for $4 < x < 6$.

19

Let f be a function such that $f'(1) = 0$ and $f''(x) = x^4 - a^2x^2 - 2a - 1$. Which of the following conditions always guarantees that f has a local minimum at $x = 1$?

(A) $a < -2$ (B) $-2 < a < 0$ (C) $a > 0$ (D) $-2 < a < 2$ (E) $-2 < a < 1$ (F) $a < 2$

20

T **F** The absolute minimum value of $1 - x^2$ on $[-1, 2]$ is -3.

T **F** If f has a local minimum at c and f is differentiable at c, then $f''(c) > 0$.

T **F** An inflection point of f always occurs at a critical number of f.

T **F** If f has an absolute maximum at c and f is differentiable at c, then $f'(c) = 0$.

T **F** If $f'(c) = 0$, then f has a local extremum at c.

Juist of fout. Verklaar jouw antwoord

21

( ) For the function $f(x) = ax^4 - 3x^3$ with constant $a > 0$, use the techniques of calculus to answer the following. Show your work and proper justification for your answers.

(a) Determine all critical points of f. Classify each as a local maximum, a local minimum, or neither.

22

Consider the function defined by $f(x) = x^3 - 7x - 10$.

(a) (1 point) Use the Intermediate Value Theorem to show that a zero exists on the interval $[-1, 4]$.

(b) (2 points) Find the number in $(-1, 4)$ that satisfies the conclusion of the Mean Value Theorem.

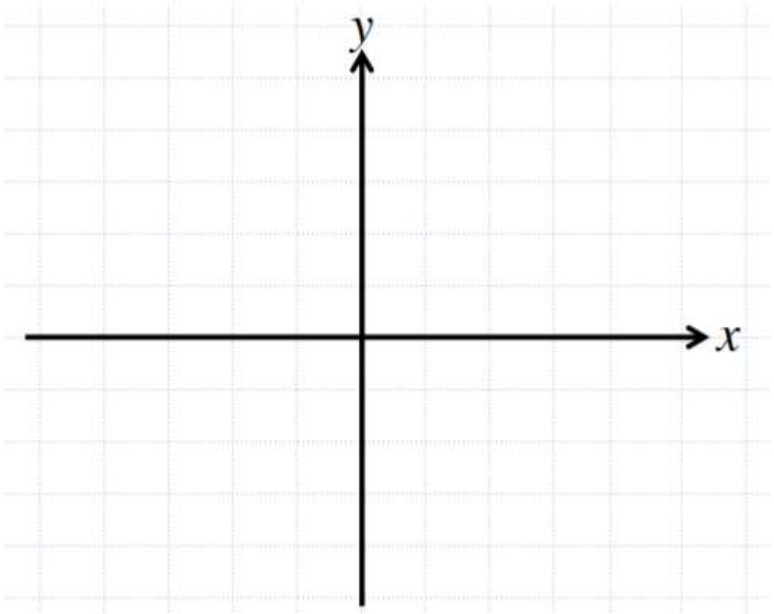
(c) (1 point) Use Rolle's Theorem to show that there is a number c in $(-1, 3)$ such that $f'(c) = 0$.

23

(c) Sketch a graph of a continuous function $y = f(x)$ that satisfies the following:

- $f(-5) = f(0) = 0$;
- $f'(x) < 0$ for $-5 < x < -3$ and $1 < x < 5$;
- $f'(x) > 0$ for $-3 < x < 1$;
- $f''(x) < 0$ for $-2 < x < 3$;
- $f''(x) > 0$ for $-5 < x < -2$ and $3 < x < 5$;

Clearly label and local extrema and points of inflection.



24

T **F** If c is a critical number of f, then $f'(c) = 0$.

T **F** If $f''(x) > 0$ at all points on the x-axis and $f'(c) = 0$, then f has its absolute minimum at c.

T **F** If $f'(x) > 0$ on $[a, b]$, then f is increasing on $[a, b]$.

T **F** If $f''(x) > 0$ on $[a, b]$, then f is concave up on $[a, b]$.

T **F** If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

Juist of Fout, verklaar jullie antwoord

25

The graph below is the **derivative**, $f'(x)$, of a continuous function f whose domain is all real numbers. Use this graph to answer Questions 19 and 20.

19. For what x-value(s) does f have an inflection point?

(a) $x = 0$ only

(b) $x = 2$ only

(c) $x = 0,\ 2$ only

(d) $x = 0,\ 3$ only

(e) $x = -1,\ 1,\ 4$ only

20. On what interval(s) is $f(x)$ decreasing?

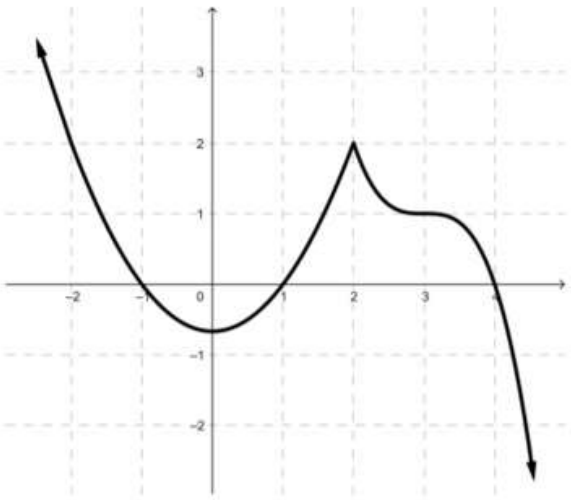
(a) $(-1, 1),\ (4, \infty)$

(b) $(-\infty, -1),\ (1, 4)$

(c) $(0, 2)$

(d) $(-\infty, 0),\ (2, \infty)$

(e) $(-\infty, -1),\ (1, 2),\ (2, 4)$



26

(6 points) Using the graph below of $y = f(x)$, list the following quantities in increasing order (from smallest to largest). No justification is necessary.

$f'(2)$ $\quad f(0)$ $\quad f'(-0.9)$ $\quad$ The number 1 $\quad f''(0)$ $\quad f''(1)$

