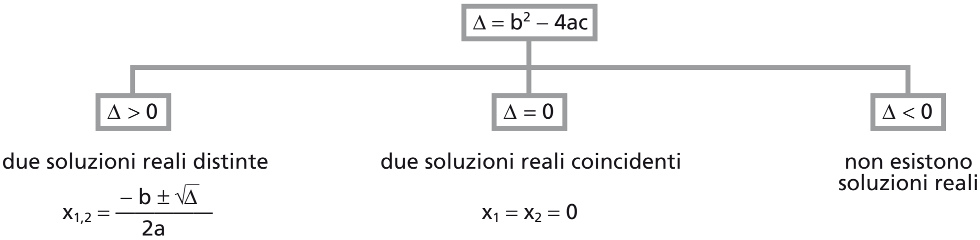
RICHIAMI DI ALGEBRA

LE EQUAZIONI DI SECONDO GRADO

Un'equazione di secondo grado è riconducibile alla forma normale: $ax^2 + bx + c = 0,\ a \neq 0$

- $b = 0,\ c \neq 0$ (equazione pura) $\rightarrow\ ax^2 + c = 0\ \rightarrow\ x^2 = -\dfrac{c}{a}$
  - se $-\dfrac{c}{a} < 0$: impossibile
  - se $-\dfrac{c}{a} > 0\ \rightarrow\ x_{1,2} = \pm\sqrt{-\dfrac{c}{a}}$
- $c = 0,\ b \neq 0$ (equazione spuria) $\rightarrow\ ax^2 + bx = 0\ \rightarrow\ x(ax + b) = 0\ \rightarrow\ x_1 = 0,\ x_2 = -\dfrac{b}{a}$
- $b = c = 0$ (equazione monomia) $\rightarrow\ ax^2 = 0\ \rightarrow\ x_1 = x_2 = 0$
- $b \neq 0,\ c \neq 0$ (equazione completa). Il discriminante è $\Delta = b^2 - 4ac$.



Formula ridotta: b pari $\rightarrow x_{1,2} = \dfrac{-\dfrac{b}{2} \pm \sqrt{\dfrac{\Delta}{4}}}{a}$.

LE DISEQUAZIONI DI SECONDO GRADO

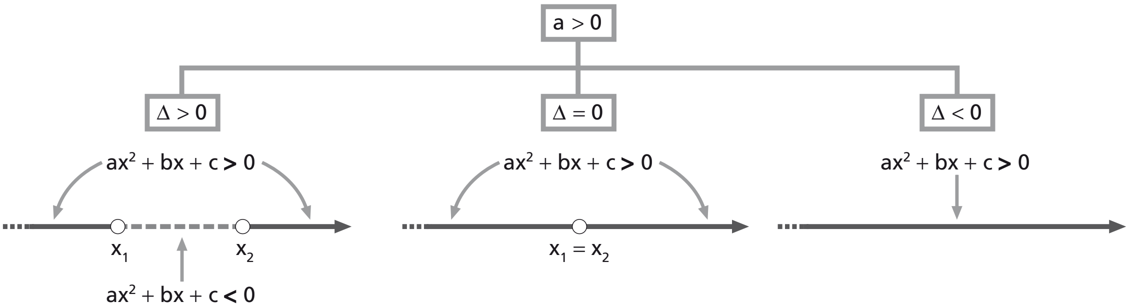
Per risolvere le disequazioni $\boldsymbol{ax^2 + bx + c > 0}$ e $\boldsymbol{ax^2 + bx + c < 0}$ (con $a > 0$), si considera l'equazione associata

$ax^2 + bx + c = 0$.

Se $\boldsymbol{\Delta > 0}$, la disequazione:
- $ax^2 + bx + c > 0$ è verificata dai valori esterni all'intervallo individuato dalle radici dell'equazione associata;
- $ax^2 + bx + c < 0$ è verificata dai valori interni.

Se $\boldsymbol{\Delta = 0}$, la disequazione:
- $ax^2 + bx + c > 0$ è sempre verificata tranne che per il valore della radice doppia dell'equazione associata;
- $ax^2 + bx + c < 0$ non è mai verificata.

Se $\boldsymbol{\Delta < 0}$, la disequazione:
- $ax^2 + bx + c > 0$ è sempre verificata;
- $ax^2 + bx + c < 0$ non è mai verificata.



LE EQUAZIONI E LE DISEQUAZIONI CON IL VALORE ASSOLUTO

$|A(x)| = k$
- se $k < 0$: non ha soluzione
- se $k \geq 0$: $A(x) = \pm k$

$|A(x)| < k$
- se $k > 0$: $-k < A(x) < k \rightarrow \begin{cases} A(x) > -k \\ A(x) < k \end{cases}$
- se $k \leq 0$: non ha soluzione

$|A(x)| > k$
- se $k > 0$: $A(x) < -k \vee A(x) > k$
- se $k = 0$: $A(x) \neq 0$
- se $k < 0$: sempre verificata

LE EQUAZIONI E LE DISEQUAZIONI IRRAZIONALI

$\sqrt[n]{A(x)} = B(x)$
- se n è dispari: $A(x) = [B(x)]^n$
- se n è pari: $\begin{cases} A(x) \geq 0 \\ B(x) \geq 0 \\ A(x) = [B(x)]^n \end{cases}$

$\sqrt[n]{A(x)} < B(x)$
- se n è dispari: $A(x) < [B(x)]^n$
- se n è pari: $\begin{cases} A(x) \geq 0 \\ B(x) > 0 \\ A(x) < [B(x)]^n \end{cases}$

$\sqrt[n]{A(x)} > B(x)$
- se n è dispari: $A(x) > [B(x)]^n$
- se n è pari: $\begin{cases} B(x) < 0 \\ A(x) \geq 0 \end{cases} \vee \begin{cases} B(x) \geq 0 \\ A(x) > [B(x)]^n \end{cases}$

LE PROPRIETÀ DELLE POTENZE

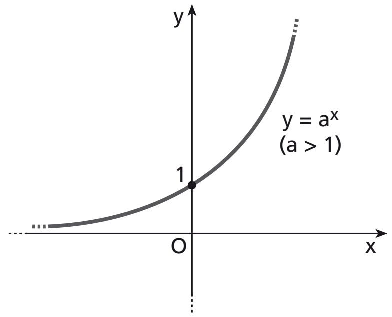
- $a^m \cdot a^n = a^{m+n}$
- $a^m : a^n = a^{m-n}$ con $a \neq 0$
- $(a^m)^n = a^{m \cdot n}$
- $a^m \cdot b^m = (a \cdot b)^m$
- $a^m : b^m = (a : b)^m$ con $b \neq 0$
- $a^{-n} = \dfrac{1}{a^n}$ con $a \neq 0$

I PRODOTTI NOTEVOLI E LA SCOMPOSIZIONE IN FATTORI

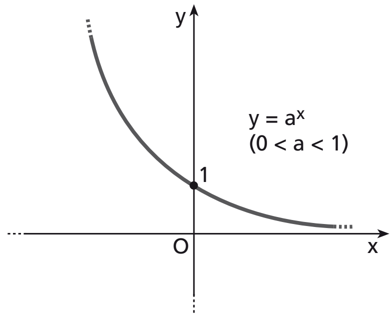
- $(A + B)(A - B) = A^2 - B^2$
- $(A \pm B)^2 = A^2 \pm 2AB + B^2$
- $(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$
- $(A \pm B)^3 = A^3 \pm 3A^2B + 3AB^2 \pm B^3$
- $A^3 \pm B^3 = (A \pm B)(A^2 \mp AB + B^2)$

LA FUNZIONE ESPONENZIALE E LA FUNZIONE LOGARITMO

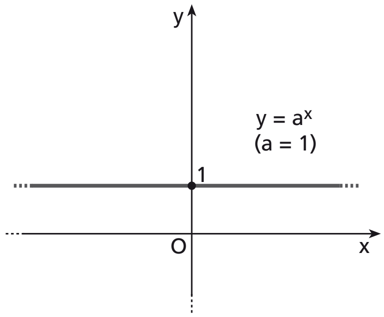
La funzione esponenziale



a.
- C.E.: $\mathbb{R}$;
- codominio: $\mathbb{R}^+$;
- funzione crescente in $\mathbb{R}$;
- corrispondenza biunivoca;
- $a^x \to 0$ per $x \to -\infty$;
- $a^x \to +\infty$ per $x \to +\infty$.

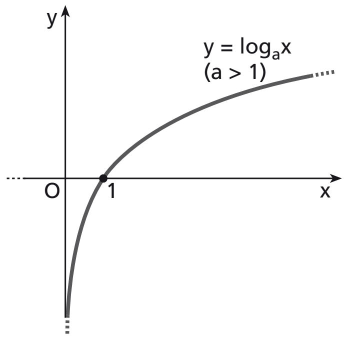


b.
- C.E.: $\mathbb{R}$;
- codominio: $\mathbb{R}^+$;
- funzione decrescente in $\mathbb{R}$;
- corrispondenza biunivoca;
- $a^x \to 0$ per $x \to +\infty$;
- $a^x \to +\infty$ per $x \to -\infty$.

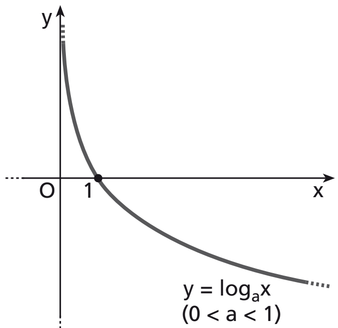


c.
- C.E.: $\mathbb{R}$;
- codominio: {1};
- funzione costante;
- funzione non iniettiva.

La funzione logaritmo



a.
- C.E.: $\mathbb{R}^+$;
- codominio: $\mathbb{R}$;
- funzione crescente in $\mathbb{R}^+$;
- corrispondenza biunivoca;
- $\log_a x \to -\infty$ per $x \to 0$;
- $\log_a x \to +\infty$ per $x \to +\infty$.



b.
- C.E.: $\mathbb{R}^+$;
- codominio: $\mathbb{R}$;
- funzione decrescente in $\mathbb{R}^+$;
- corrispondenza biunivoca;
- $\log_a x \to +\infty$ per $x \to 0$;
- $\log_a x \to -\infty$ per $x \to +\infty$.

Logaritmo di un prodotto

$$\log_a (b \cdot c) = \log_a b + \log_a c, \quad (b > 0, c > 0)$$

Logaritmo di un quoziente

$$\log_a \frac{b}{c} = \log_a b - \log_a c, \qquad (b > 0, c > 0)$$

Logaritmo di una potenza

$$\log_a b^n = n \cdot \log_a b, \quad (b > 0)$$

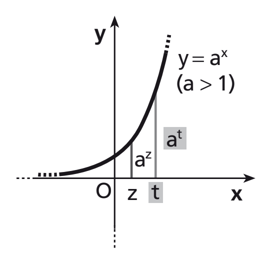
Cambiamento di base nei logaritmi

$$\log_a b = \frac{\log_c b}{\log_c a} \qquad a > 0, b > 0, c > 0 \quad a \neq 1, c \neq 1$$

Disequazioni esponenziali

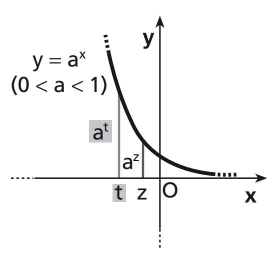
$a > 1$

$a^t > a^z \iff t > z$



$0 < a < 1$

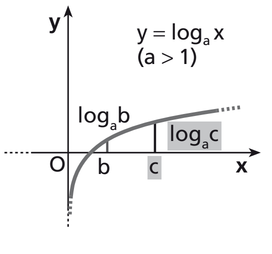
$a^t > a^z \iff t < z$



Disequazioni logaritmiche

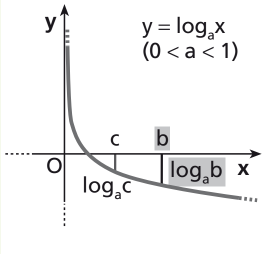
$a > 1$

$\log_a b < \log_a c \iff b < c$



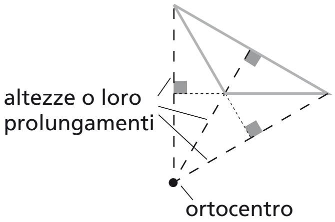
$0 < a < 1$

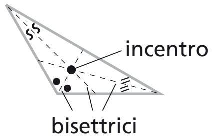
$\log_a b < \log_a c \iff b > c$



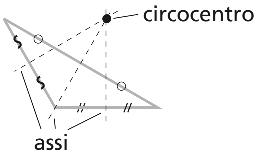
RICHIAMI DI GEOMETRIA

I punti notevoli di un triangolo

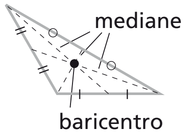




L'incentro è il centro della circonferenza inscritta.



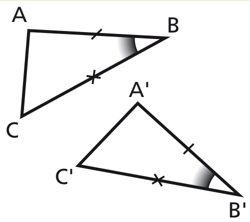
Il circocentro è il centro della circonferenza circoscritta.



Il baricentro divide ogni mediana in due parti di cui quella contenente il vertice è doppia dell'altra

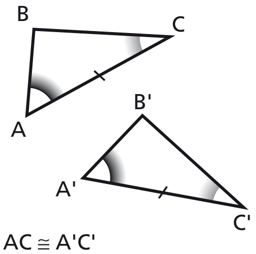
I criteri di congruenza dei triangoli

1° criterio



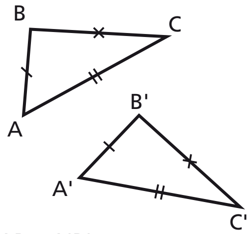
AB ≅ A'B'
BC ≅ B'C' ⇨ ABC ≅ A'B'C'
B̂ ≅ B̂'

2° criterio



AC ≅ A'C'
Â ≅ Â' ⇨ ABC ≅ A'B'C'
Ĉ ≅ Ĉ'

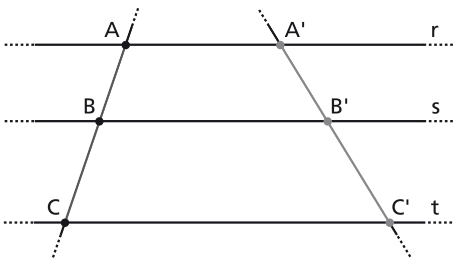
3° criterio



AB ≅ A'B'
BC ≅ B'C' ⇨ ABC ≅ A'B'C'
AC ≅ A'C'

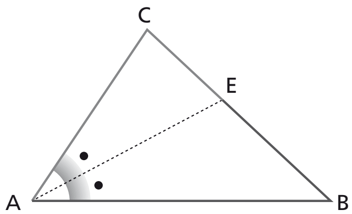
Il teorema di Talete

Teorema di Talete



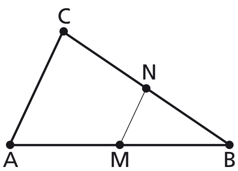
r // s // t ⇒ AB : BC = A'B' : B'C'

Teorema della bisettrice di un angolo interno di un triangolo



BE : CE = AB : AC

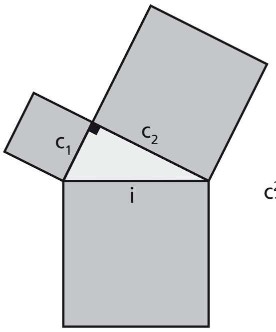
Conseguenza del teorema di Talete



AM ≅ MB ⇨ MN // AC
CN ≅ NB ⇨ MN ≅ $\frac{1}{2}$ AC

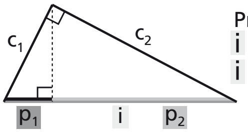
L'equivalenza e la similitudine

Il teorema di Pitagora



$c_1^2 + c_2^2 = i^2$

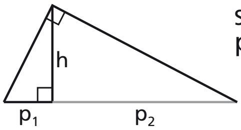
I teoremi di Euclide



Primo teorema di Euclide

$i : c_1 = c_1 : p_1$

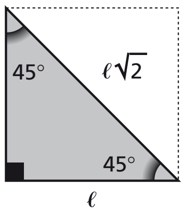
$i : c_2 = c_2 : p_2$

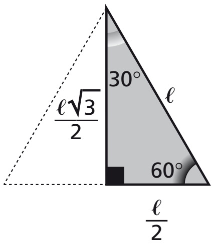


Secondo teorema di Euclide

$p_1 : h = h : p_2$

Relazioni fra i lati di triangoli notevoli

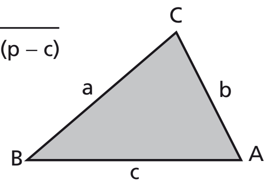




Formula di Erone

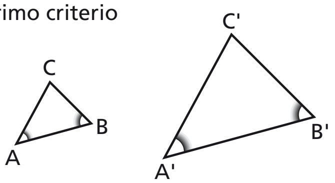
$\mathscr{A} = \sqrt{p \cdot (p - a) \cdot (p - b) \cdot (p - c)}$

con $p = \dfrac{a + b + c}{2}$



Criteri di similitudine dei triangoli

Primo criterio

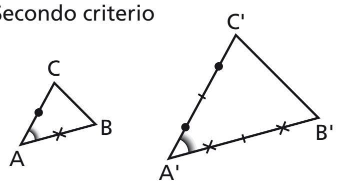


$\hat{A} \cong \hat{A}'$ $\hat{B} \cong \hat{B}'$

⇩

$ABC \approx A'B'C'$

Secondo criterio

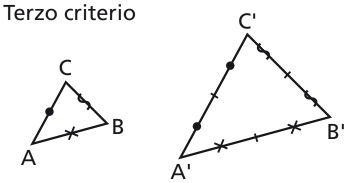


$\hat{A} \cong \hat{A}'$ $AB : A'B' = AC : A'C'$

⇩

$ABC \approx A'B'C'$

Terzo criterio



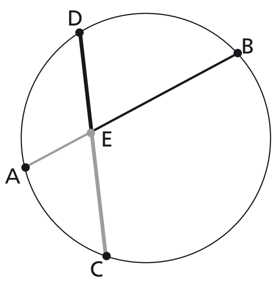
$AB : A'B' = AC : A'C' = BC : B'C'$

⇩

$ABC \approx A'B'C'$

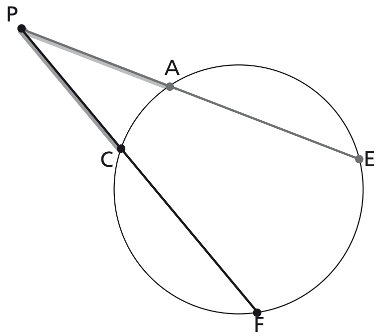
La similitudine nella circonferenza

Teorema delle corde secanti



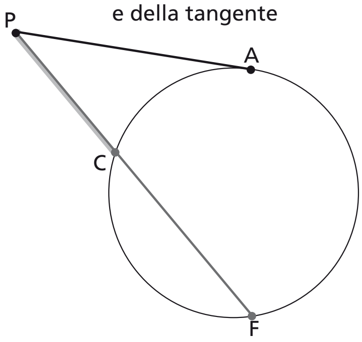
$AE : CE = ED : EB$

Teorema delle secanti



$PF : PE = PA : PC$

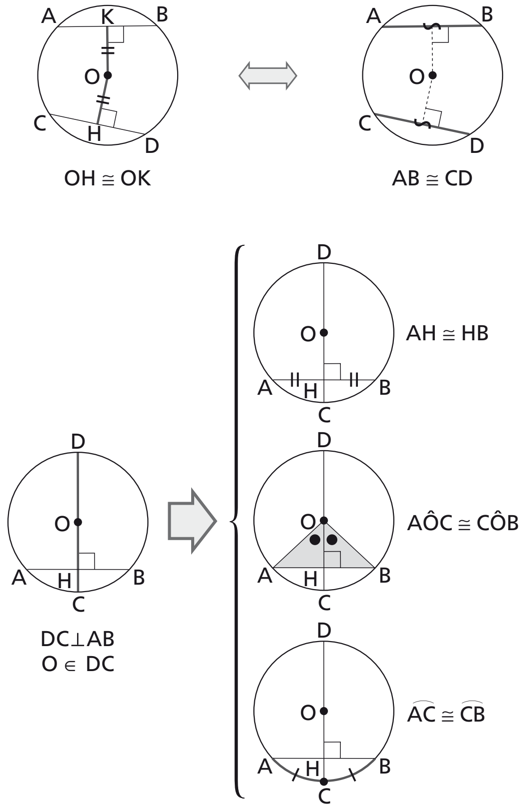
Teorema della secante e della tangente



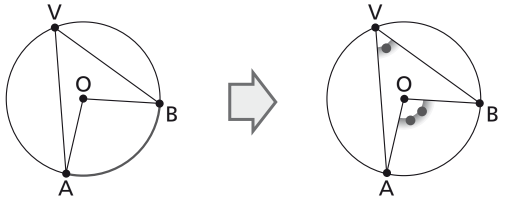
$PF : PA = PA : PC$

La circonferenza

I teoremi sulle corde

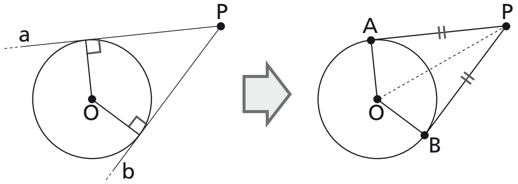


Angoli alla circonferenza e angoli al centro



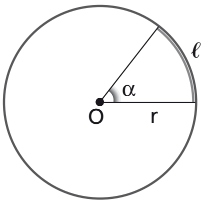
Ogni angolo alla circonferenza è la metà dell'angolo al centro corrispondente.

Tangente a una circonferenza da un punto esterno



Se da un punto esterno a una circonferenza si conducono le rette tangenti, i segmenti di tangente risultano congruenti

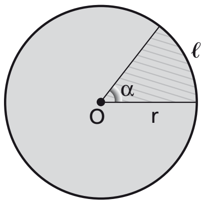
La lunghezza della circonferenza e l'area del cerchio



$$c = 2\pi r$$

$$\ell = \frac{\alpha}{180}\pi r$$

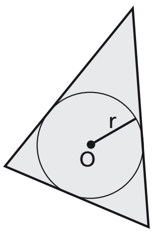
a. Misure della circonferenza (*c*) e dell'arco di angolo al centro α (ℓ).



$$C = \pi r^2$$

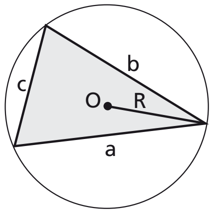
$$S = \frac{\alpha}{360}\pi r^2 = \frac{1}{2}\ell r$$

b. Misure dell'area del cerchio (*C*) e dell'area del settore circolare di angolo al centro α (*S*).



$$r = \frac{\mathcal{A}}{p}$$

a. Raggio del cerchio inscritto nel triangolo.

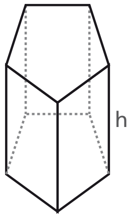


$$R = \frac{abc}{4\mathcal{A}}$$

b. Raggio del cerchio circoscritto al triangolo.

Formule di geometria solida

PRISMA RETTO

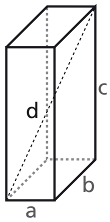


$A_\ell = 2p \cdot h$

$A_t = A_\ell + 2A_b$

$V = A_b \cdot h$

PARALLELEPIPEDO RETTANGOLO



$A_b = ab$

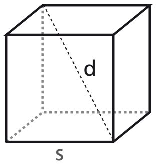
$A_\ell = 2(ac + bc)$

$A_t = 2(ac + ab + bc)$

$V = a \cdot b \cdot c$

$d = \sqrt{a^2 + b^2 + c^2}$

CUBO



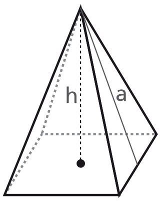
$A_b = s^2$

$A_t = 6s^2$

$V = s^3$

$d = s\sqrt{3}$

PIRAMIDE RETTA

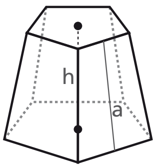


$A_\ell = p \cdot a$

$A_t = A_\ell + A_b$

$V = \frac{1}{3} A_b \cdot h$

TRONCO DI PIRAMIDE RETTA

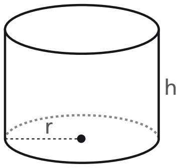


$A_\ell = (p + p') \cdot a$

$A_t = A_\ell + A_b + A'_b$

$V = \frac{1}{3} h \left(A_b + A'_b + \sqrt{A_b \cdot A'_b}\right)$

CILINDRO



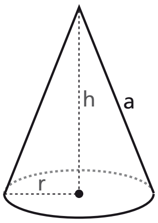
$A_b = \pi r^2$

$A_\ell = 2\pi r \cdot h$

$A_t = 2\pi r (h + r)$

$V = \pi r^2 \cdot h$

CONO



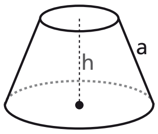
$A_b = \pi r^2$

$A_\ell = \pi r a$

$A_t = \pi r (a + r)$

$V = \frac{1}{3} \pi r^2 \cdot h$

TRONCO DI CONO



$A_b = \pi r^2$

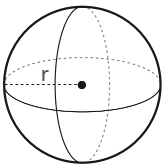
$A'_b = \pi r'^2$

$A_\ell = \pi a (r + r')$

$A_t = A_\ell + A_b + A'_b$

$V = \frac{1}{3} \pi h (r^2 + r'^2 + r \cdot r')$

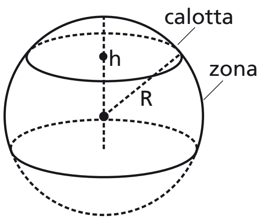
SFERA



$A = 4\pi r^2$

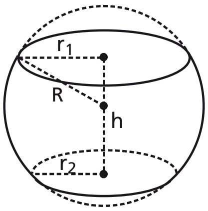
$V = \frac{4}{3} \pi r^3$

CALOTTA E ZONA SFERICA



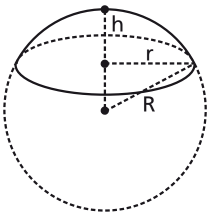
$S = 2\pi R h$

SEGMENTO SFERICO A DUE BASI



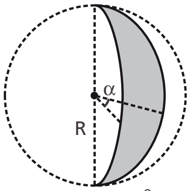
$V = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3 + \pi r_1^2 \frac{h}{2} + \pi r_2^2 \frac{h}{2}$

SEGMENTO SFERICO A UNA BASE



$V = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3 + \pi r^2 \frac{h}{2} = \frac{1}{3}\pi h^2 (3R - h)$

FUSO SFERICO

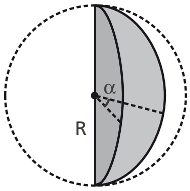


$S_f = 2R^2 \alpha^{rad} = \frac{\alpha°}{90°} \pi R^2$

α^{rad}: ampiezza del diedro in radianti

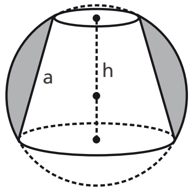
$\alpha°$: ampiezza del diedro in gradi

SPICCHIO SFERICO



$V_s = \frac{2}{3} \alpha^{rad} R^3 = \frac{\alpha°}{270°} \pi R^3$

ANELLO SFERICO



$V_a = \frac{1}{6} \pi a^2 h$

GEOMETRIA ANALITICA

La **distanza fra due punti** $A(x_A; y_A)$ e $B(x_B; y_B)$ è data da: $\overline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$.

Il **punto medio** del segmento AB è $M(x_M; y_M)$ con: $x_M = \dfrac{x_A + x_B}{2}$, $\quad y_M = \dfrac{y_A + y_B}{2}$.

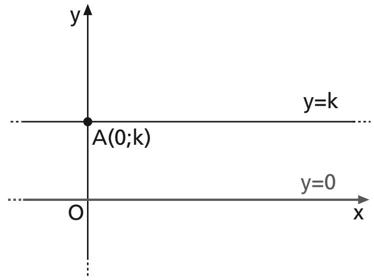
Il **baricentro di un triangolo** di vertici $A(x_A; y_A)$, $B(x_B; y_B)$, $C(x_C; y_C)$ è $G(x_G; y_G)$ con:

$$x_G = \frac{x_A + x_B + x_C}{3}, \quad y_G = \frac{y_A + y_B + y_C}{3}.$$

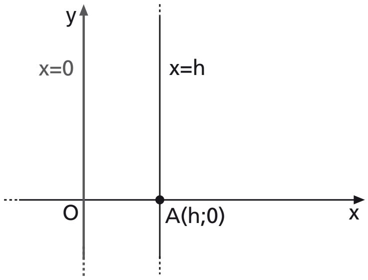
La **distanza di un punto** $P(x_0; y_0)$ **da una retta** r di equazione $ax + by + c = 0$ è uguale a: $d = \dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.

Il piano cartesiano e la retta

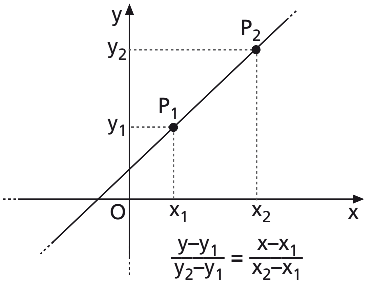
L'equazione di una retta



a. Retta parallela all'asse *x*.

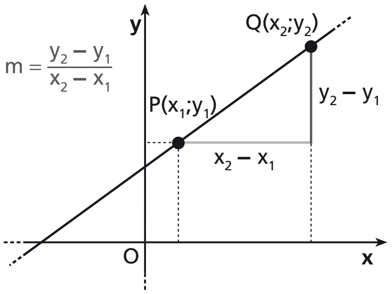


b. Retta parallela all'asse *y*.

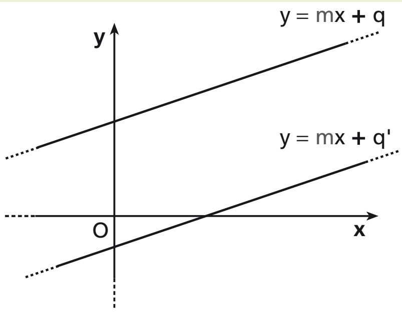


c. Retta non parallela agli assi passante per i punti $P_1(x_1; y_1)$ e $P_2(x_2; y_2)$.

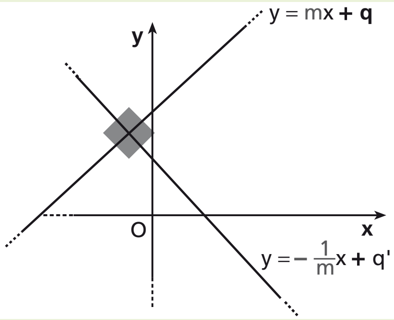
Coefficiente angolare



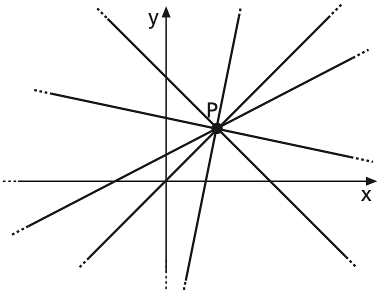
Rette parallele



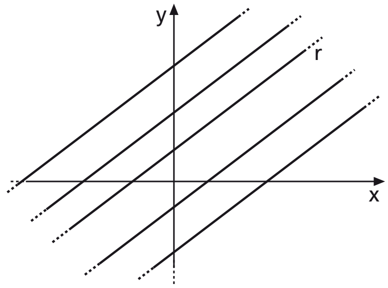
Rette perpendicolari



I fasci di rette



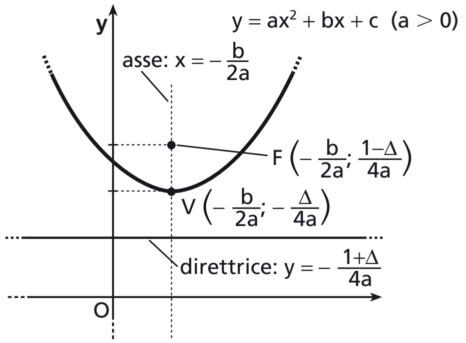
a. Fascio proprio di rette per un punto *P*: insieme di tutte le rette del piano passanti per *P*. *P* è detto **centro del fascio.**



b. Fascio improprio di rette parallele a una retta r.

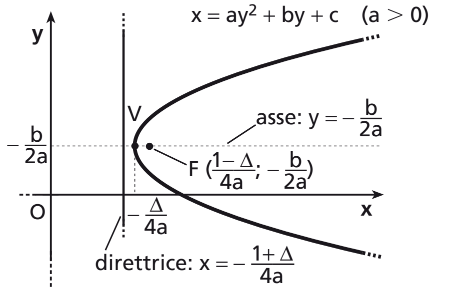
Le coniche

La parabola con asse parallelo all'asse *y*



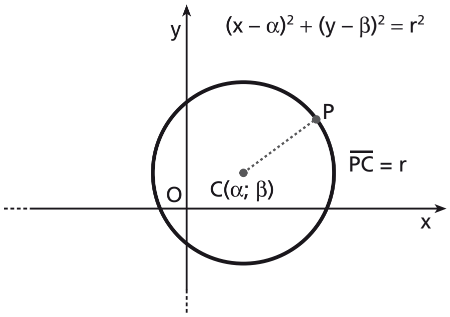
se $a < 0$ la concavità è rivolta verso il basso

La parabola con asse parallelo all'asse *x*

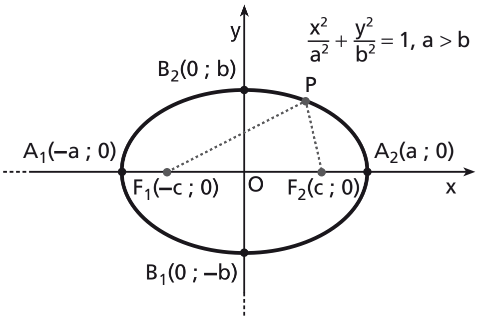


se $a < 0$ la concavità è rivolta nel verso opposto

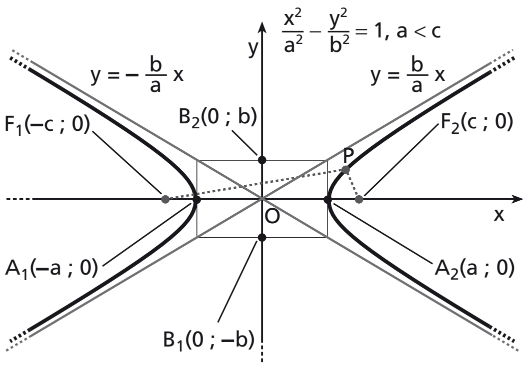
La circonferenza



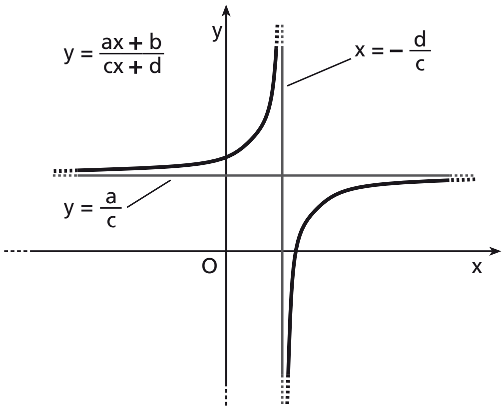
L'ellisse



L'iperbole

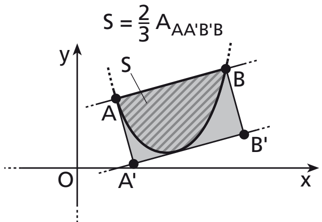


La funzione omografica

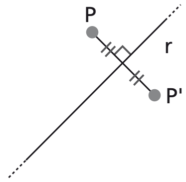


IL SEGMENTO PARABOLICO

Tracciamo la retta parallela ad *AB* e tangente alla parabola, e consideriamo su di essa le proiezioni *A′* e *B′* di *A* e *B*. L'area del segmento parabolico è uguale a $\frac{2}{3}$ dell'area del rettangolo *AA′B′B*.



LA SIMMETRIA ASSIALE



Fissata nel piano una retta r, la **simmetria assiale rispetto alla retta r** è quella isometria che a ogni punto del piano P fa corrispondere il punto P' del semipiano opposto rispetto a r, in modo che r sia l'asse del segmento PP', ossia:

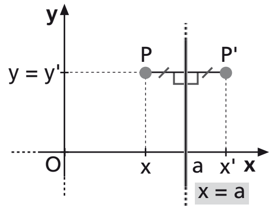
- r passa per il punto medio di PP';
- PP' è perpendicolare alla retta r.

La retta r è detta **asse di simmetria**.

Nel piano cartesiano prendiamo in esame le seguenti simmetrie assiali, fornendo le relative equazioni.

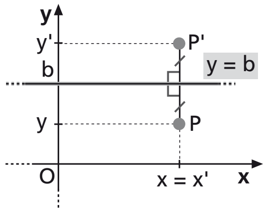
a. Simmetria con asse $x = a$ (asse parallelo all'asse y)

$$\begin{cases} x' = 2a - x \\ y' = y \end{cases}$$



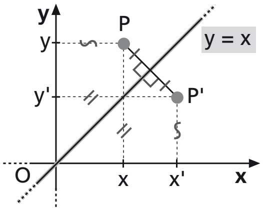
b. Simmetria con asse $y = b$ (asse parallelo all'asse x)

$$\begin{cases} x' = x \\ y' = 2b - y \end{cases}$$



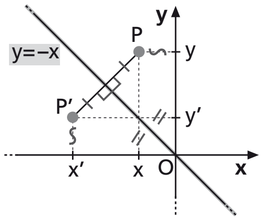
c. Simmetria con asse $y = x$ (bisettrice del primo e terzo quadrante)

$$\begin{cases} x' = y \\ y' = x \end{cases}$$



d. Simmetria con asse $y = -x$ (bisettrice del secondo e quarto quadrante)

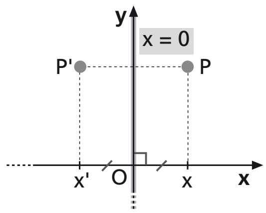
$$\begin{cases} x' = -y \\ y' = -x \end{cases}$$



e. Simmetria con asse $x = 0$ (asse y)

$$\begin{cases} x' = -x \\ y' = y \end{cases}$$

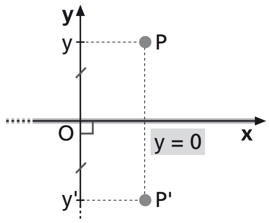
Due punti simmetrici rispetto all'asse y hanno ascisse opposte e la stessa ordinata.



f. Simmetria con asse $y = 0$ (asse x)

$$\begin{cases} x' = x \\ y' = -y \end{cases}$$

Due punti simmetrici rispetto all'asse x hanno la stessa ascissa e ordinate opposte.



GONIOMETRIA E TRIGONOMETRIA

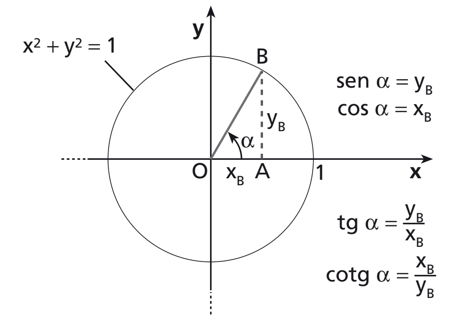
Le funzioni goniometriche

La prima relazione fondamentale

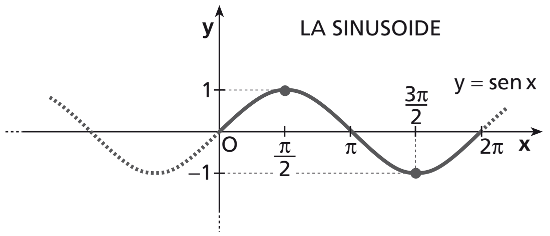
$$\text{sen}^2\,\alpha + \cos^2\alpha = 1$$

La seconda relazione fondamentale

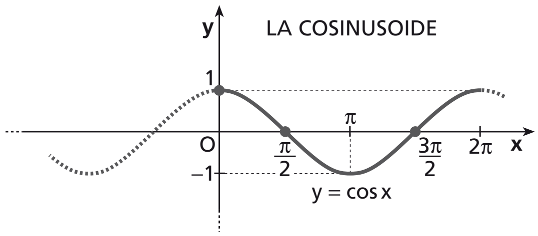
$$\text{tg}\,\alpha = \frac{\text{sen}\,\alpha}{\cos\alpha}$$



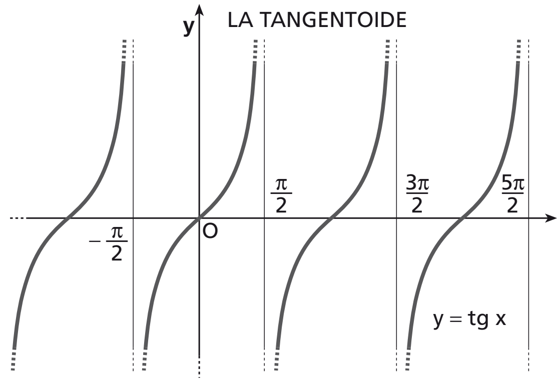
I grafici delle funzioni goniometriche



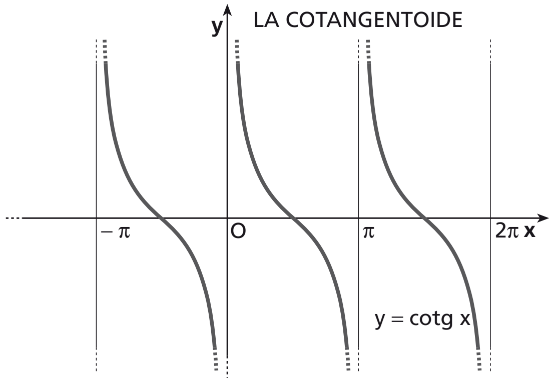
Periodicità: $\forall\alpha \in \mathbb{R}$, $\forall k \in \mathbb{Z}$ $\quad \text{sen}(\alpha + 2k\pi) = \text{sen}\alpha$



Periodicità: $\forall\alpha \in \mathbb{R}$, $\forall k \in \mathbb{Z}$ $\quad \cos(\alpha + 2k\pi) = \cos\alpha$



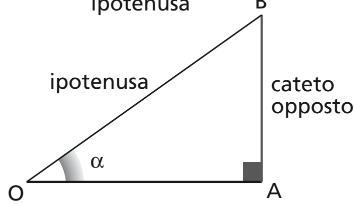
Periodicità: $\forall\alpha \in \mathbb{R}$, $\forall k \in \mathbb{Z}$ $\quad \text{tg}(\alpha + k\pi) = \text{tg}\,\alpha$



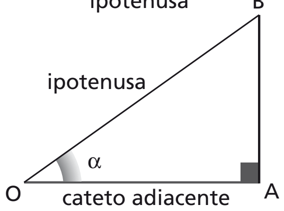
Periodicità: $\forall\alpha \in \mathbb{R}$, $\forall k \in \mathbb{Z}$ $\quad \text{cotg}(\alpha + k\pi) = \text{cotg}\,\alpha$

Seno, coseno e tangente su un triangolo rettangolo

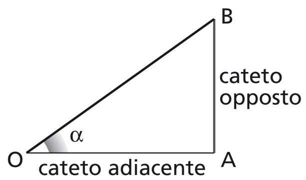
$$\text{sen}\,\alpha = \frac{\text{cateto opposto}}{\text{ipotenusa}}$$



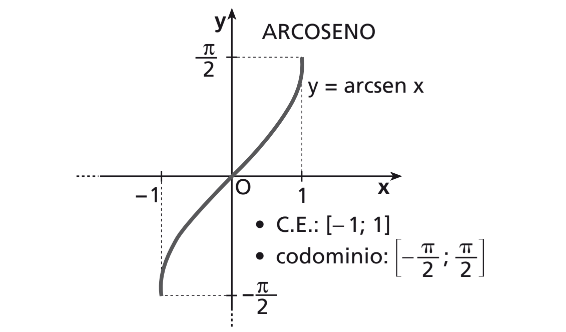
$$\cos\alpha = \frac{\text{cateto adiacente}}{\text{ipotenusa}}$$

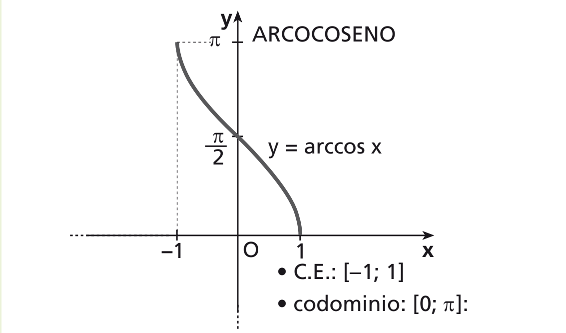


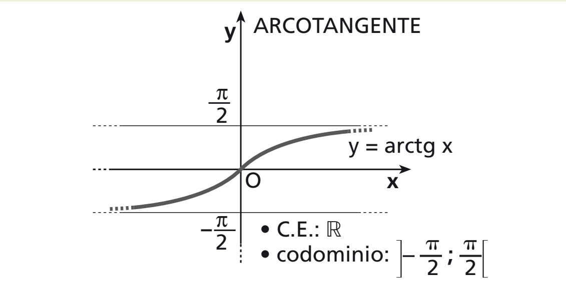
$$\text{tg}\,\alpha = \frac{\text{cateto opposto}}{\text{cateto adiacente}}$$

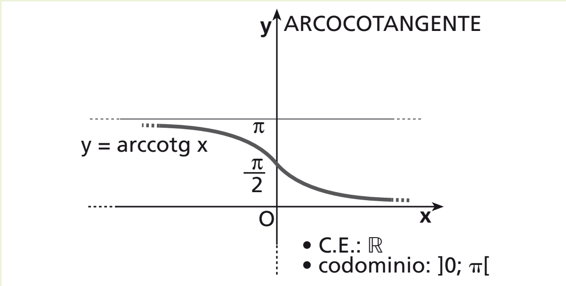


Le funzioni goniometriche inverse







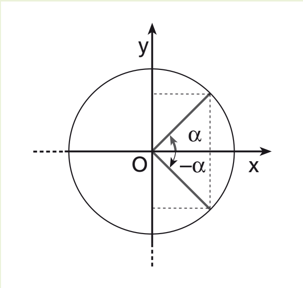


Seno, coseno, tangente e cotangente di angoli notevoli

radianti	gradi	seno	coseno	tangente	cotangente
0	0	0	1	0	non esiste
$\frac{\pi}{12}$	15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$\frac{\pi}{10}$	18°	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{5+2\sqrt{5}}$
$\frac{\pi}{8}$	22°30′	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{5}$	36°	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{5-2\sqrt{5}}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{3}{10}\pi$	54°	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{25+10\sqrt{5}}}{5}$	$\sqrt{5-2\sqrt{5}}$
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{2}{5}\pi$	72°	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$
$\frac{5}{12}\pi$	75°	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	non esiste	0

Funzioni goniometriche di angoli associati

α e $-\alpha$



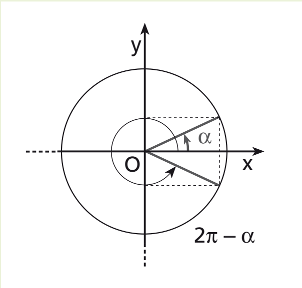
$\text{sen}(-\alpha) = -\text{sen}\,\alpha$

$\cos(-\alpha) = \cos\alpha$

$\text{tg}(-\alpha) = -\text{tg}\,\alpha$

$\text{cotg}(-\alpha) = -\text{cotg}\,\alpha$

α e $2\pi-\alpha$



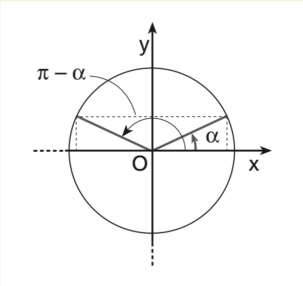
$\text{sen}(2\pi-\alpha) = -\text{sen}\,\alpha$

$\cos(2\pi-\alpha) = \cos\alpha$

$\text{tg}(2\pi-\alpha) = -\text{tg}\,\alpha$

$\text{cotg}(2\pi-\alpha) = -\text{cotg}\,\alpha$

α e $\pi-\alpha$



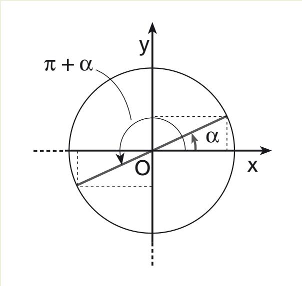
$\text{sen}(\pi-\alpha) = \text{sen}\,\alpha$

$\cos(\pi-\alpha) = -\cos\alpha$

$\text{tg}(\pi-\alpha) = -\text{tg}\,\alpha$

$\text{cotg}(\pi-\alpha) = -\text{cotg}\,\alpha$

α e $\pi+\alpha$



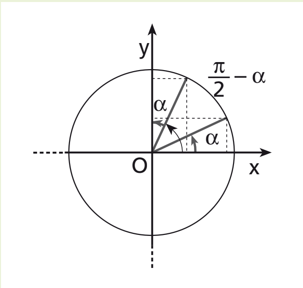
$\text{sen}(\pi+\alpha) = -\text{sen}\,\alpha$

$\cos(\pi+\alpha) = -\cos\alpha$

$\text{tg}(\pi+\alpha) = \text{tg}\,\alpha$

$\text{cotg}(\pi+\alpha) = \text{cotg}\,\alpha$

α e $\frac{\pi}{2}-\alpha$



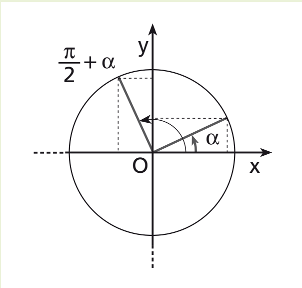
$\text{sen}\left(\frac{\pi}{2}-\alpha\right) = \cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right) = \text{sen}\,\alpha$

$\text{tg}\left(\frac{\pi}{2}-\alpha\right) = \text{cotg}\,\alpha$

$\text{cotg}\left(\frac{\pi}{2}-\alpha\right) = \text{tg}\,\alpha$

α e $\frac{\pi}{2}+\alpha$



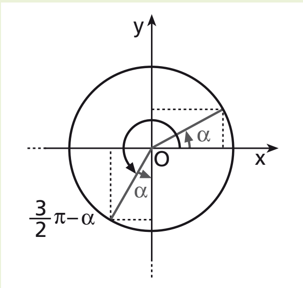
$\text{sen}\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$

$\cos\left(\frac{\pi}{2}+\alpha\right) = -\text{sen}\,\alpha$

$\text{tg}\left(\frac{\pi}{2}+\alpha\right) = -\text{cotg}\,\alpha$

$\text{cotg}\left(\frac{\pi}{2}+\alpha\right) = -\text{tg}\,\alpha$

α e $\frac{3}{2}\pi-\alpha$



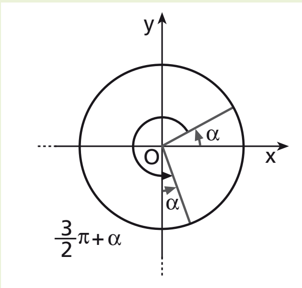
$\text{sen}\left(\frac{3}{2}\pi-\alpha\right) = -\cos\alpha$

$\cos\left(\frac{3}{2}\pi-\alpha\right) = -\text{sen}\,\alpha$

$\text{tg}\left(\frac{3}{2}\pi-\alpha\right) = \text{cotg}\,\alpha$

$\text{cotg}\left(\frac{3}{2}\pi-\alpha\right) = \text{tg}\,\alpha$

α e $\frac{3}{2}\pi+\alpha$



$\text{sen}\left(\frac{3}{2}\pi+\alpha\right) = -\cos\alpha$

$\cos\left(\frac{3}{2}\pi+\alpha\right) = \text{sen}\,\alpha$

$\text{tg}\left(\frac{3}{2}\pi+\alpha\right) = -\text{cotg}\,\alpha$

$\text{cotg}\left(\frac{3}{2}\pi+\alpha\right) = -\text{tg}\,\alpha$

Le formule goniometriche

Le formule di addizione

$\operatorname{sen}(\alpha+\beta)=\operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta$

$\operatorname{tg}(\alpha+\beta)=\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$

con $\alpha, \beta, \alpha+\beta \neq \dfrac{\pi}{2}+k\pi$

Le formule di sottrazione

$\operatorname{sen}(\alpha-\beta)=\operatorname{sen}\alpha\cos\beta-\cos\alpha\operatorname{sen}\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta$

$\operatorname{tg}(\alpha-\beta)=\dfrac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$

con $\alpha, \beta, \alpha-\beta \neq \dfrac{\pi}{2}+k\pi$

Le formule di duplicazione

$\operatorname{sen}2\alpha=2\operatorname{sen}\alpha\cos\alpha$

$\cos 2\alpha=\cos^2\alpha-\operatorname{sen}^2\alpha$

$\operatorname{tg}2\alpha=\dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$

Le formule di bisezione

$\operatorname{sen}\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$\operatorname{tg}\dfrac{\alpha}{2}=\pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

Le formule parametriche

$\operatorname{sen}\alpha=\dfrac{2\operatorname{tg}\dfrac{\alpha}{2}}{1+\operatorname{tg}^2\dfrac{\alpha}{2}}$

$\cos\alpha=\dfrac{1-\operatorname{tg}^2\dfrac{\alpha}{2}}{1+\operatorname{tg}^2\dfrac{\alpha}{2}}$, con $\alpha\neq\pi+k2\pi$

Le formule di prostaferesi

$\operatorname{sen}p+\operatorname{sen}q=2\operatorname{sen}\dfrac{p+q}{2}\cdot\cos\dfrac{p-q}{2}$

$\operatorname{sen}p-\operatorname{sen}q=2\cos\dfrac{p+q}{2}\cdot\operatorname{sen}\dfrac{p-q}{2}$

$\cos p+\cos q=2\cos\dfrac{p+q}{2}\cdot\cos\dfrac{p-q}{2}$

$\cos p-\cos q=-2\operatorname{sen}\dfrac{p+q}{2}\cdot\operatorname{sen}\dfrac{p-q}{2}$

Le formule di Werner

$\operatorname{sen}\alpha\operatorname{sen}\beta=\dfrac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$

$\cos\alpha\cos\beta=\dfrac{1}{2}[\cos(\alpha+\beta)+\cos(\alpha-\beta)]$

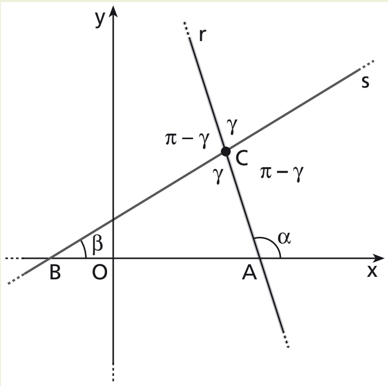
$\operatorname{sen}\alpha\cos\beta=\dfrac{1}{2}[\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)]$

L'angolo fra due rette

$r: y=mx+q,\quad$ con $m=\operatorname{tg}\alpha$

$s: y=m'x+q',\quad$ con $m'=\operatorname{tg}\beta$

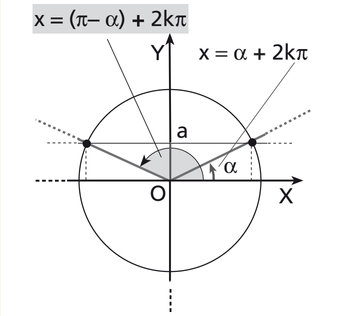
$\mathbf{tg}\,\boldsymbol{\gamma}=\operatorname{tg}(\alpha-\beta)=\dfrac{\boldsymbol{m-m'}}{\boldsymbol{1+mm'}}.$



Equazioni goniometriche elementari

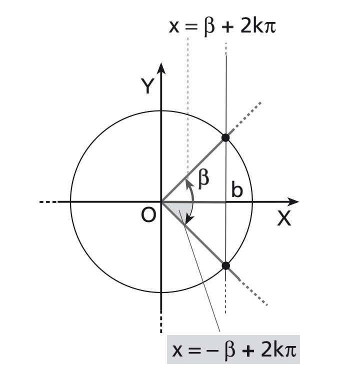
Un'**equazione** si dice **goniometrica** se contiene almeno una funzione goniometrica dell'incognita. Si chiamano **elementari** le equazioni goniometriche del tipo:

$\operatorname{sen} x = a$, $\cos x = b$, $\operatorname{tg} x = c$.

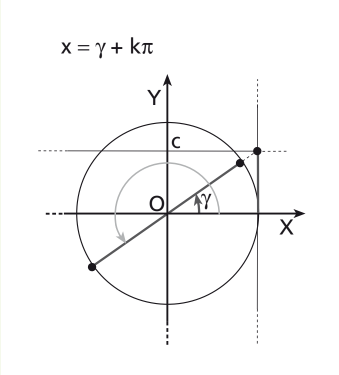


$\operatorname{sen} x = a$ — determinata se $-1 \leq a \leq 1$; impossibile se $a < -1 \vee a > 1$

$\cos x = b$ — determinata se $-1 \leq b \leq 1$; impossibile se $b < -1 \vee b > 1$



$\operatorname{tg} x = c$ determinata $\forall c \in \mathbb{R}$



Ci sono particolari equazioni elementari che si possono risolvere con le proprietà della seguente tabella.

Tipo di equazione	Proprietà
$\operatorname{sen}\alpha = \operatorname{sen}\alpha'$	$\operatorname{sen}\alpha = \operatorname{sen}\alpha' \Leftrightarrow \alpha = \alpha' + 2k\pi \vee \alpha + \alpha' = \pi + 2k\pi$
$\operatorname{sen}\alpha = -\operatorname{sen}\alpha'$	$-\operatorname{sen}\alpha' = \operatorname{sen}(-\alpha')$
$\operatorname{sen}\alpha = \cos\alpha'$	$\cos\alpha' = \operatorname{sen}\left(\frac{\pi}{2} - \alpha'\right)$
$\operatorname{sen}\alpha = -\cos\alpha'$	$-\cos\alpha' = -\operatorname{sen}\left(\frac{\pi}{2} - \alpha'\right) = \operatorname{sen}\left(-\frac{\pi}{2} + \alpha'\right)$
$\cos\alpha = \cos\alpha'$	$\cos\alpha = \cos\alpha' \Leftrightarrow \alpha = \pm\alpha' + 2k\pi$
$\cos\alpha = -\cos\alpha'$	$-\cos\alpha' = \cos(\pi - \alpha')$
$\operatorname{tg}\alpha = \operatorname{tg}\alpha'$	$\operatorname{tg}\alpha = \operatorname{tg}\alpha' \Leftrightarrow \alpha = \alpha' + k\pi$
$\operatorname{tg}\alpha = -\operatorname{tg}\alpha'$	$-\operatorname{tg}\alpha' = \operatorname{tg}(-\alpha')$

LE EQUAZIONI LINEARI IN SENO E COSENO

$a \operatorname{sen} x + b \cos x + c = 0 \qquad a \neq 0, b \neq 0$

Metodo algebrico

- $c = 0 \rightarrow$ si divide per $\cos x \rightarrow \operatorname{tg} x = -\dfrac{b}{a}$.
- $c \neq 0 \rightarrow$ si determinano le eventuali soluzioni di tipo $x = \pi + 2k\pi$;
  se $x \neq \pi + 2k\pi$, applicando le formule parametriche si ottiene

$$\begin{cases} t^2(c-b) + 2at + b + c = 0 \\ t = \operatorname{tg}\dfrac{x}{2} \end{cases}$$

Metodo grafico

Si sostituisce $Y = \operatorname{sen} x$ e $X = \cos x$ e si risolve quindi il sistema seguente:

$$\begin{cases} X^2 + Y^2 = 1 \\ aY + bX + c = 0 \end{cases}$$

Metodo dell'angolo aggiunto

Si risolve il sistema seguente:

$$\begin{cases} \operatorname{sen}(x + \alpha) = -\dfrac{c}{r} \\ r = \sqrt{a^2 + b^2} \\ \operatorname{tg}\alpha = \dfrac{b}{a} \end{cases}$$

LE EQUAZIONI OMOGENEE DI SECONDO GRADO IN SENO E COSENO

$a \operatorname{sen}^2 x + b \cos x \operatorname{sen} x + c \cos^2 x = 0$

Primo metodo

- $a = 0 \rightarrow \cos x\,(b \operatorname{sen} x + c \cos x) = 0$
- $a \neq 0 \rightarrow$ si divide per $\cos^2 x \rightarrow a \operatorname{tg}^2 x + b \operatorname{tg} x + c = 0$

Secondo metodo

Sostituendo $\begin{cases} \operatorname{sen} x \cos x = \dfrac{\operatorname{sen} 2x}{2} \\ \operatorname{sen}^2 x = \dfrac{1 - \cos 2x}{2} \\ \cos^2 x = \dfrac{1 + \cos 2x}{2} \end{cases}$ si ottiene un'equazione lineare.

Un'equazione lineare della forma

$$a \operatorname{sen}^2 x + b \operatorname{sen} x \cos x + c \cos^2 x = d \qquad (d \neq 0)$$

è riconducibile a un'equazione omogenea sostituendo $d = d(\cos^2 x + \operatorname{sen}^2 x)$.

Disequazioni goniometriche

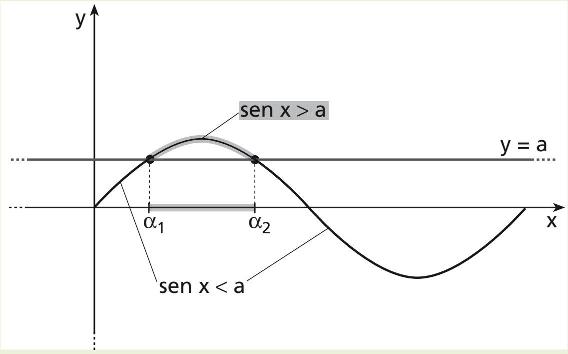
Primo metodo

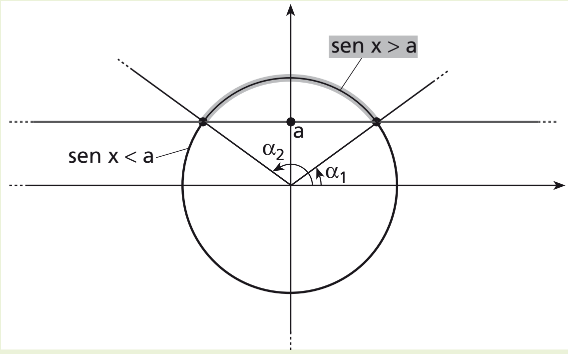
Si studia la posizione reciproca tra il grafico della funzione goniometrica e la retta $y = a$.

Secondo metodo

Si disegna la circonferenza goniometrica, si risolve l'equazione associata, si determinano gli archi in cui è soddisfatta.

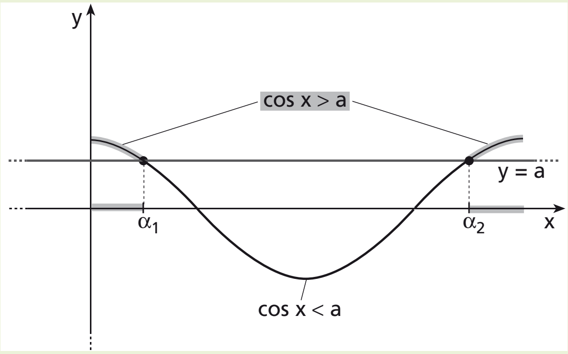
La funzione seno

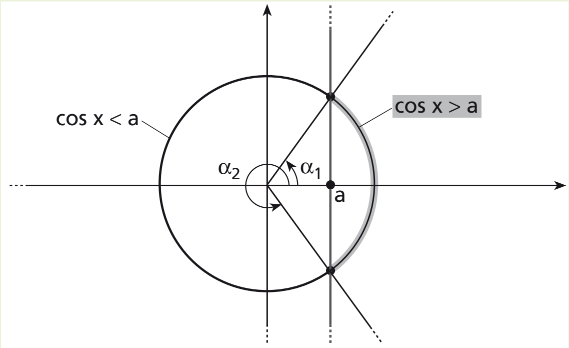




$$\text{sen}\, x > a \rightarrow \alpha_1 + 2k\pi < x < \alpha_2 + 2k\pi; \quad \text{sen}\, x < a \rightarrow 0 + 2k\pi < x < \alpha_1 + 2k\pi \vee \alpha_2 + 2k\pi < x < 2\pi + 2k\pi$$

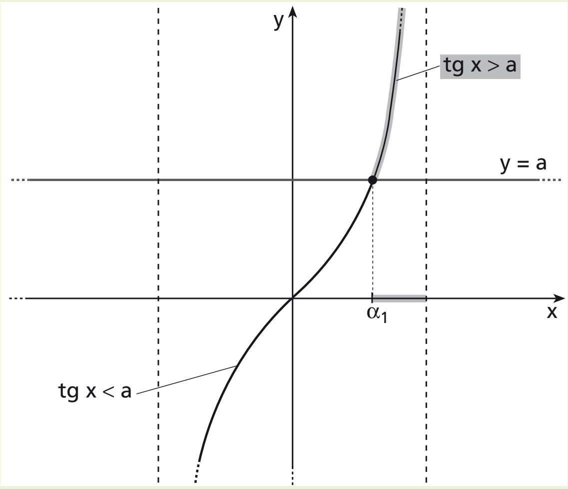
La funzione coseno

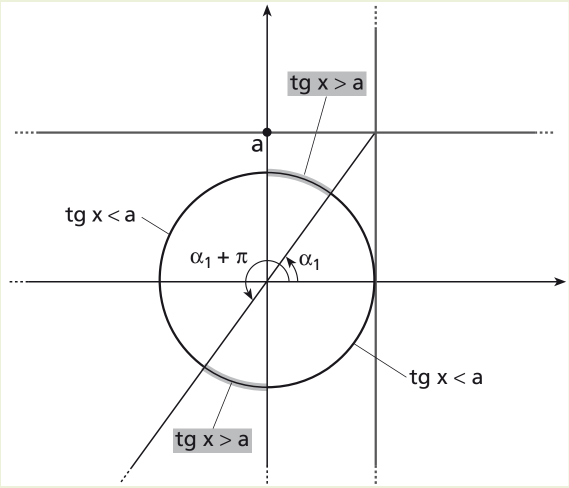




$$\cos x > a \rightarrow 0 + 2k\pi < x < \alpha_1 + 2k\pi \vee \alpha_2 + 2k\pi < x < 2\pi + 2k\pi; \quad \cos x < a \rightarrow \alpha_1 + 2k\pi < x < \alpha_2 + 2k\pi$$

La funzione tangente





$$\text{tg}\, x > a \rightarrow \alpha_1 + k\pi < x < \frac{\pi}{2} + k\pi; \quad \text{tg}\, x < a \rightarrow -\frac{\pi}{2} + k\pi < x < \alpha_1 + k\pi$$