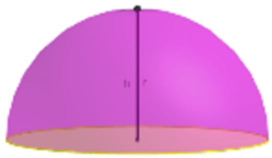
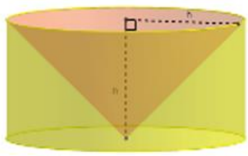
Use *Cavalieri's principle to prove the* volume of cone = $\frac{1}{3}\pi r^2 * h$

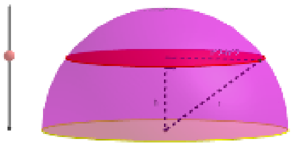
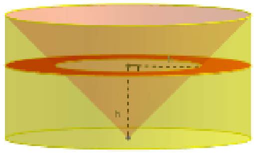
Proof

Let us take a semi sphere of radius *r*. Consider this as object 1.

object 1   object 2

Also, take a cylinder of radius *r, and* height *r.* From the top of the cylinder, immerse a cone of radius *r*¸ height *r,* so that the vertex of the cone coincides at the bottom center of the cylinder. Consider this composition of cylinder and cone as object 2.

Now, we take section of both objects by a plane parallel to the base of the objects and is at height *h* from the base.

h  

In this section, the cross section of object 1 is a circle.

The area of this cross section (circle) is

Area of cross section (circle) = $\pi \times radius^2$

or Area of cross section (circle) = $\pi\left(\sqrt{r^2 - h^2}\right)^2$

or Area of cross section (circle) = $\pi\left(r^2 - h^2\right)$

In this section, the cross section of object 2 is a ring. The area of this cross section (ring) is

Area of cross section (ring) = Area of outer circle-Area of inner circle

or Area of cross section (ring) = $\pi r^2 - \pi h^2$

or Area of cross section (ring) = $\pi\left(r^2 - h^2\right)$

Here, the area of cross sections in both objects are equal. Therefore, by Cavaliries principle [Euclid, SMSG, 22], volume of both objects is equal.

Thus,

Volume of object 1 = Volume of object 2

Volume of semi sphere = Volume of cylinder – Volume of cone

or Volume of cone = Volume of cylinder -Volume of semi sphere

or Volume of cone = $\pi r^2 r - \frac{2}{3}\pi r^3$

or Volume of cone = $\pi r^3 - \frac{2}{3}\pi r^3$

or Volume of cone = $\frac{1}{3}\pi r^3$

or Volume of cone = $\frac{1}{3}\pi r^2 * h$

This completes the proof.