Fostering Number Sense Through Digits & Dice

Lynda R. Wiest shares a versatile game to develop place value concepts and number sense.

Having a good "feel" for numbers is vitally important at all age levels. The foundation for developing a flexible working knowledge of numbers and their relationships begins before children enter the classroom. Schooling should formally foster these fundamental aspects of mathematics, particularly in the early grades. This set of knowledge and skills falls within the National Council of Teachers of Mathematics' (NCTM) (2000) Number and Operations standard, where the NCTM states, "All the mathematics proposed for pre-kindergarten through grade 12 is strongly grounded in number" (p. 32).

Van de Walle (2004) notes, "A rich understanding of number, a relational understanding, involves many different ideas, relationships, and skills" (p. 115). Some concepts and skills that fall under the general notion of number sense are counting, number representation, number relationships, and place value.

Games are among the many types of activities that can be used to help promote number sense. Chosen and used wisely, games encourage learning and motivate many students to grasp concepts more quickly and fully. This article describes a popular and easy-to-implement number sense game — here called Digits and Dice — most commonly used in conjunction with teaching place value. Variations of the game are described, in addition to ways in which the game may be extended to other number concepts that span the elementary and middle grades.

Exploring place value with "Digits and Dice"

This simple number sense game requires only dice (or number cubes), paper, and pencil. The teacher rolls a die or number cube and announces the result, continuing to do so in turn as students place digits one at a time on blank lines to try to achieve a known goal. For example, students might draw three short lines, as in Figure 1a, and try to make the smallest number possible on three rolls. If the three numbers rolled are 1, 5, and 3, the smallest number possible is 135. Students must commit a number to a line after each toss of the dice. (Experience shows that this requires some careful monitoring.) After three rolls, the teacher asks who thinks they have gotten the smallest possible number and why. Students share strategies they used to make decisions about placing numbers. A sample response is, "I put the 1 in the hundreds' place because that will make a smaller number than putting a larger digit there." Discussion involves number sense — more specifically, place value — as well as probabilistic thinking in terms of where to place numbers, how likely certain numbers are to appear on the dice tosses, and the ultimately unpredictable nature of chance events despite well-reasoned decisions.

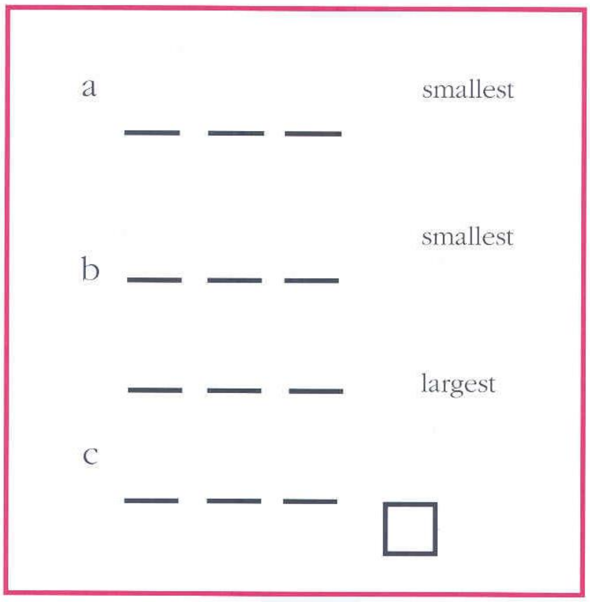


Figure 1. Game format for creating smallest and largest three-digit whole numbers, with a discard-number option

Besides also finding the largest number for this activity, students might use a set-up such as that shown in Figure 1b to find the largest and smallest number possible in a total of three or six rolls (in the first case placing one number in each set of three lines for each roll). Students might also compete against each other in pairs to find the largest or smallest number, with one member of each pair designated to find one or the other. Groups of three might include a student seeking a middle number (i.e., not largest or smallest), in which case students might discuss whether any of the three players has an advantage over the others (more "middle numbers" are possible).

The game's difficulty level can be adjusted —to make the game developmentally appropriate for students — by varying number length and by using different types of multi-sided dice. In the latter case, a six-sided die numbered 1-6 requires different thinking than a ten-sided die numbered 0-9 (or 1-10, where 10 is announced as 0).

A variation of this game is to allow one discard or "throw-away" number. In other words, at any time during the game, students may place one number in a discard box where only the digits placed on lines count in the game (see Figure 1c). Like numbers placed on lines, once a number is entered in the discard box it cannot be relocated. Another important option, especially for more difficult versions of this game, is to allow some type of single replacement move during the game. One method that works well is to use a game format as in Figure 1c but allow one "bump" during the game. The discard-number box is now used for a banked number available for one-time use during the game. The examples in Figure 2 show how this might occur in a game seeking the largest three-digit whole number using a ten-sided die numbered 0-9. The first example is a situation where the player has already placed a 7 and a 6. She then rolled a 9, which she used to replace the 7 that in turn became the banked number. The 7 may not replace the 6 because only one existing number may be replaced during the game. In the second situation, a rolled 8 replaces a 7, which then moves to the tens' place, and in the third case a fourth and final roll of 1 is used to replace the banked number 5, which then moves to the ones' place. In essence, for any situation involving the replacement option, a "bumped" number is crossed out and immediately moved to an open space either on a line or in the bank box, and only one such number may be replaced (thus crossed out) in the game. Because this number transfer must occur before the next die roll, more time should be allowed between rolls for this game version.

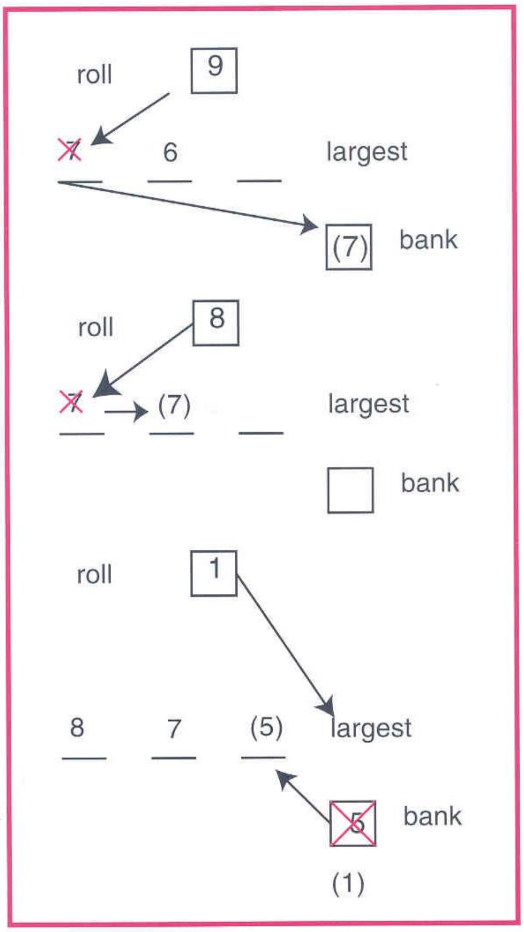


Figure 2. Examples involving the replacement option in separate games seeking the largest three-digit whole number using a ten-sided die numbered 0-9

Extensions to other concepts

Digits and Dice may be used in a variety of other ways to suit classroom goals and student abilities. The game may be tailored to numerous concepts, such as those listed below.

Whole-Number Operations

Playing Digits and Dice with computation problems can enhance conceptual understanding of whole-number operations. For example, students might be asked to find the smallest or largest positive sum, difference, product, or quotient of two numbers by plugging digits into boxes in computation problems. (See sample formats in Figure 3.)

Even-Odd Numbers and Divisibility

Younger students can practice the concept of even-odd simply by trying to form an even or odd number of a given number of digits. This idea, and that of divisibility, might be explored with older students as well, as in fine-tuning the original game idea (Figure 1) by having students find, for example, the smallest even number or largest number divisible by five.

Fractions and Decimals

Students might be asked to find the smallest or largest fraction (see Figure 4a) or a fraction that is as close as possible to one-half using set-ups such as those shown in Figure 4a. They might also try to find a set of two or three fractions that are each as close as possible to one-half. In one such example, one member of a pair attained 2/4, 3/4, and 3/5 in a game with one replacement permitted in seeking to find three fractions as close as possible to one-half. The partner obtained 1/4, 2/4, and 3/5. The pair decided that the game was a tie, with two fractions being equal and each player's third being the same distance from one-half. Students might also order several fractions from largest

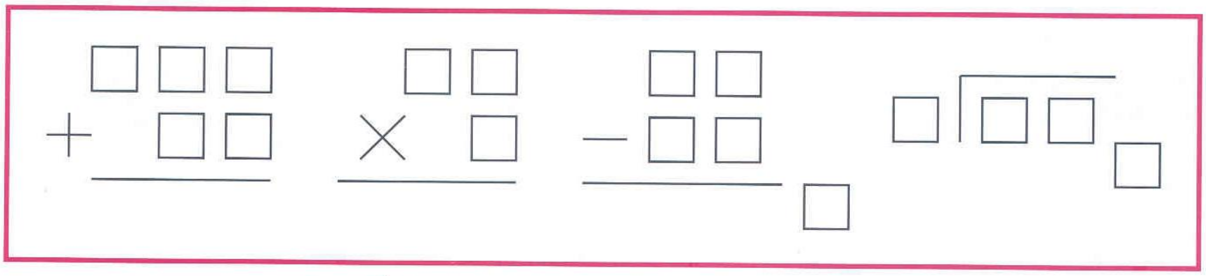
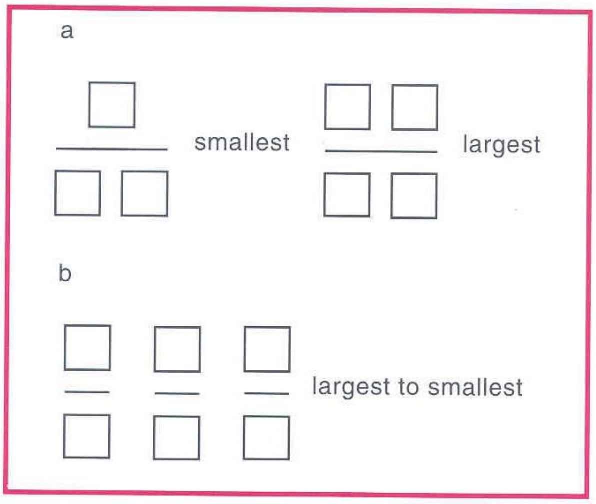


Figure 3. Sample game format for whole-number computation, two with a discard number option

to smallest or vice-versa by using a set-up such as that shown in Figure 4b. (Teachers will need to decide whether or not to specify that only proper fractions — or perhaps proper fractions plus improper fractions equivalent to 1 — "count" for some of these activities.) Similar decimal versions of Digits and Dice may be created.

Figure 4: Smple game format for use with fractins: fractin size



(a) and ordering fractions by size (b).

Negative Numbers

Many activities described in this article may include or be applied to negative numbers. For example, whole-number computation may involve finding the largest difference between two numbers, without stipulating that the result must be positive, and negative signs may be placed in front of numbers, such as the three-digit numbers in Figure 1.

General game considerations

With a little creativity, it is likely that Digits and Dice can be applied to other number concepts or extended to other ideas for the topics included here. Other number skills can be practiced in conjunction with this game, such as reading whole numbers, fractions, and decimals correctly and estimating the difference between the two numbers attained in Figure 1b or the answers to computation problems (Figure 3). As mentioned earlier, rich discussions are likely to bring out probabilistic thinking in addition to ideas about numbers and their relationships. Students reason that: larger numbers must be in higher place values to create larger numbers; waiting for a 9 for a particular spot in a three-digit number may not be a wise chance-based decision; two numbers to be subtracted must be as far apart in size as possible to create the greatest difference; and so forth.

Note that throughout these games it is particularly important to include the concept of "smallest" where relevant. Students have more difficulty with the concept of less than with the concept of more (Van de Walle, 2004), so practice finding the smallest number should take place as often as or more often than finding the largest number. Similarly, ordering numbers from largest to smallest should be done as much as the more commonly performed smallest to largest. (Such variation in wording also demands good listening skills and attentiveness to detail.) As noted earlier, the more difficult the concept or goal (e.g., ordering three fractions by size), the more useful it is to include a discard — or especially a replacement — option.

Decisions must be made about certain specifics of these activities, as with any classroom task. Teachers might make these themselves or in conjunction with student input. For this game, teachers will have to decide when answers to be attained must be positive or fractions must be proper. They will also have to decide how to handle "0 cases" for games that include the digit 0. In other words, a fraction with a denominator of 0 is "undefined" in mathematics and might therefore render a student's game invalid when such a case results. A 0 at the front of a number (e.g., 038 for a smallest three-digit number) might be considered to represent exactly what it shows — a 0 in the hundreds' place such that 038 equals 38, unorthodox though the first notation is. At times games might be played as a whole class and at others in small groups. One advantage of whole-class games is the potential for good discussions that centre about all students having played a game using the same numbers, whereas small groups allow more individuals to participate in discussions. Still other minor details to decide are whether students will draw their game formats or have them provided on a worksheet and whether students should write in pencil or a permanent marking device, such as a pen.

Reflection on game decisions that includes adequate discussion of strategies employed is important to realise the potential of these activities to develop significant mathematical ideas. Students might reason about why they placed particular numbers where they did and how often each number is likely to appear, which varies with the type of die used. They might also discuss what types of numbers they choose to exchange if a number exchange is permitted, as discussed at the end of "Exploring place value with 'Digits and Dice'." (These numbers may be ones that students find the most difficult to place, such as numbers that are neither at an extreme nor in the middle of the range of possible numbers.) Students should also discuss what they might do differently when they play the game again. Accordingly, it is important to play the same game numerous times consecutively so that students have an opportunity to improve and solidify thinking. More time will be needed for reflection and discussion when the games are first played. After substantial experience with individual games, it is reasonable to interchange the game variations more readily.

A Final Word

The Digits and Dice game presumes initial conceptual understanding and therefore should not be used until students have meaningful knowledge of the mathematics topics employed. Students develop such understanding by, in particular, working with and reflecting on manipulative materials and story problems. This simple, practical game idea allows students to deepen and broaden conceptual content knowledge and to make strong use of two of the NCTM's (2000) process standards — reasoning and communication. It has the added benefit of providing a motivational context where all students who achieve the optimal solution may "win."

References

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally (5th Ed.)*. Boston: Pearson.

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