PROJECTILE MOTION *Tilted surface*

We seek the intersection of a trajectory and a tilted surface. This could be a case of firing down a hill (or like a ski jump), or firing at a target up a hill from the launch point. In either case the surface is modeled as a line starting at the launch point, so that

$$y_H(x) := \tan(\phi)\ x + y_0$$

where ϕ is the angle, positive counterclockwise, made with the local horizontal. A negative angle is thus downhill. The projectile's trajectory equation is as usual

$$y_P(x) := \left[\frac{-g}{2\left(v_0 \cos(\theta)\right)^2}\ x^2 + \tan(\theta)\ x + y_0\right]$$

where θ is the elevation (launch) angle, positive counterclockwise from the local horizontal. Equating these, at the point of intersection we have

$$\frac{-g}{2\left(v_0 \cos(\theta)\right)^2}\ x_I^2 + \tan(\theta)\ x_I + y_0 = \tan(\phi)\ x_I + y_0$$

from which

$$x_I := \frac{2\left(v_0 \cos(\theta)\right)^2}{g}\left(\tan(\theta) - \tan(\phi)\right) \qquad (1)$$

Note that if ϕ is zero, so that the surface is horizontal, we have the usual range expression. If we need the vertical position of the intersection it is just (from the equation of the line)

$$y_I := \tan(\phi)\ x_I + y_0$$

The distance *along the slope* can be found using this x and y, and Pythagoras, or it can be seen directly from the geometry that this distance L is

$$L := \frac{x_I}{\cos(\phi)} \qquad (2)$$

Next we find theTOF. The flight ends at the intersection rather than at y = 0. Thus, at the TOF,

$$y(T) = y_I \qquad y(T) = y_0 + v_0 \sin(\theta)\ T - \frac{1}{2}\ g\ T^2 = \tan(\phi)\ x_I + y_0$$

$$y_0 + v_0 \sin(\theta)\ T - \frac{1}{2}\ g\ T^2 = 2\tan(\phi)\ {v_0}^2\ \frac{\cos(\theta)^2}{g}\left(\tan(\theta) - \tan(\phi)\right) + y_0$$

$$T(\phi,\theta) := \frac{v_0}{g}\left[\sin(\theta) + \sqrt{\sin(\theta)^2 - 4\tan(\phi)\left[\cos(\theta)\left(\sin(\theta) - \tan(\phi)\cos(\theta)\right)\right]}\right] \qquad (3)$$

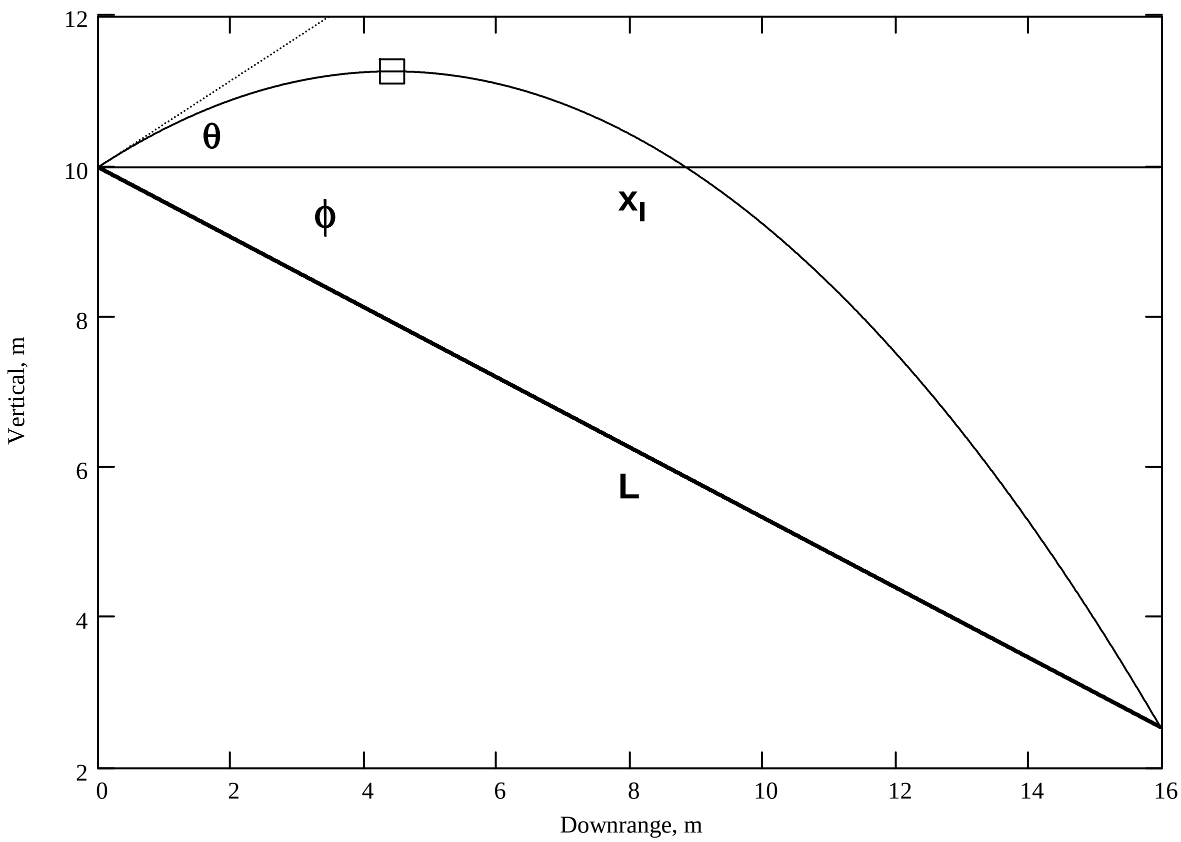
When ϕ is zero, the terrain is horizontal and we have the previous solution for TOF

$$T = \frac{2\ v_0}{g}\ \sin(\theta)$$

Note that there must be some sensible relation between the initial height, θ, and ϕ or else no solution is possible. There are many possible arrangements; some will make physical sense, others won't. The positive initial elevation angle must be greater than any positive terrain angle (else the projectile goes into the ground). Also a negative terrain angle must be such that the intercept occurs above ground.

All these conditions can be coded into algorithms so that plots will have meaningful ranges. For *any* situation, the x of the intersection *must be positive* (real-valued solutions will exist for negative x!). Also, for any situation the y of the intersection must also be positive- a negative value can happen if the hill is too steep downward, so that the y = 0 level is hit before the hill is.

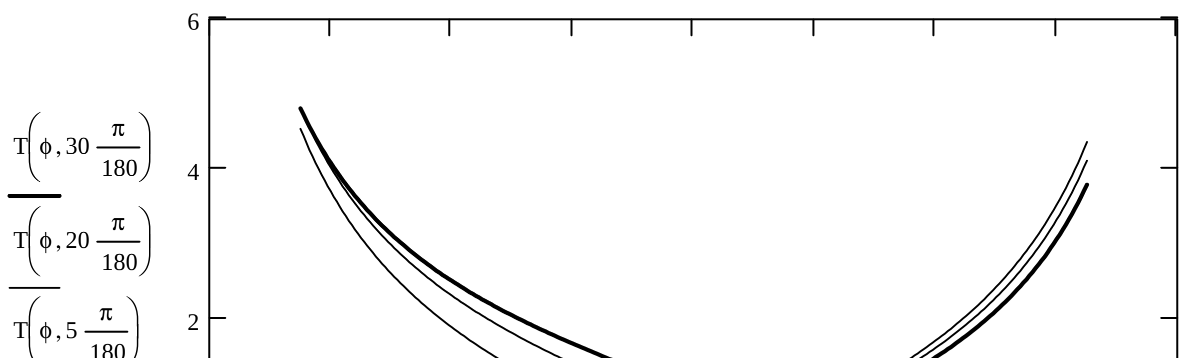
We will not make any plots of the TOF, downslope distances, etc. since the coding necessary is more trouble than it is worth at this time.

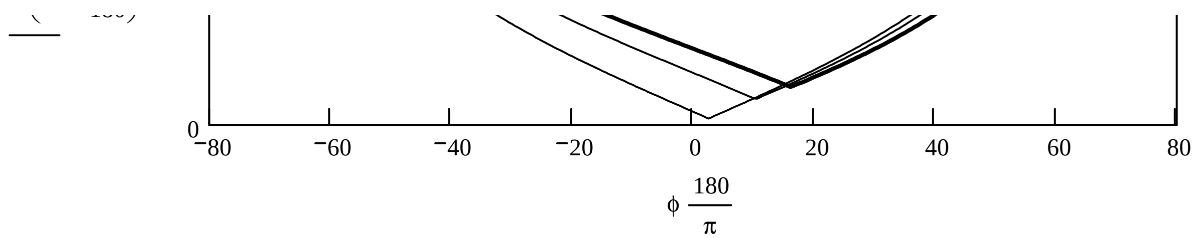


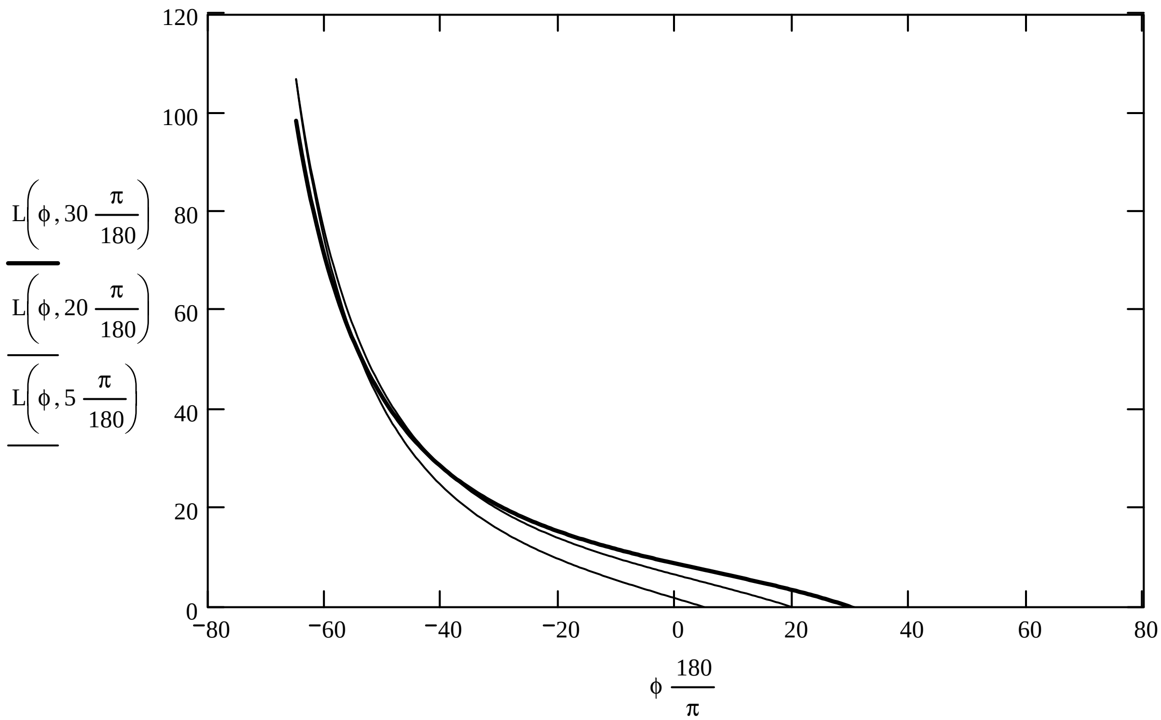
Note that the vertex coordinates are the same as previously developed, since the sloping terrain has no effect on the trajectory itself, other than at the intersection point. The vertex is indicated on the plot.

For θ = 30 degrees, ϕ = - 25 degrees, v_0 = 10 m/s: $x_I = 15.974$ $y_I = 2.551$ $L = 17.626$

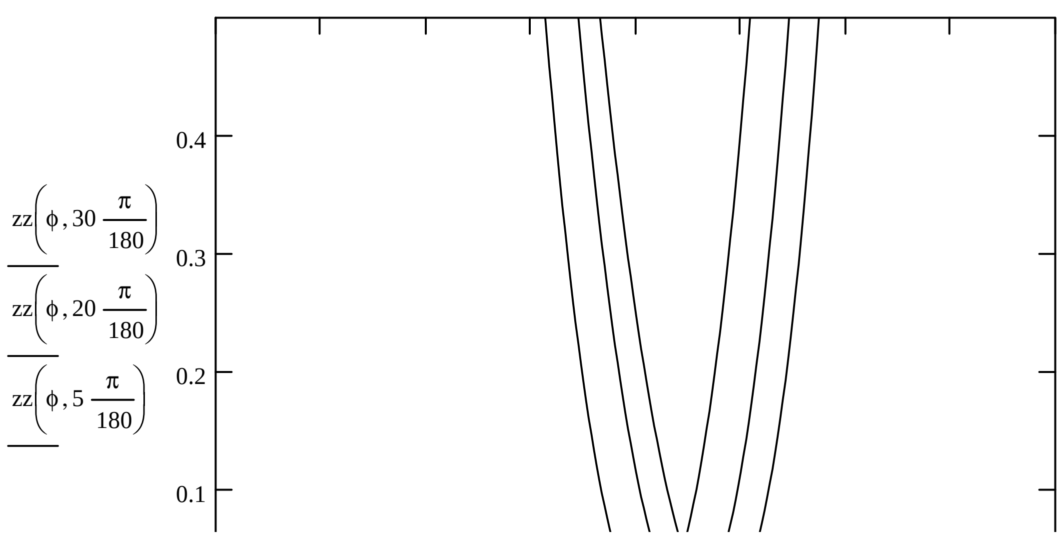
these plots are wrong- they do not check for negative x. some solns are below ground, or behind the initial position (negative y or negative x, respectively)   DO NOT PRINT THESE- MISLEADING

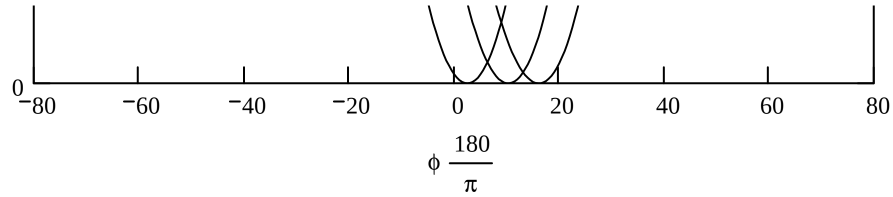




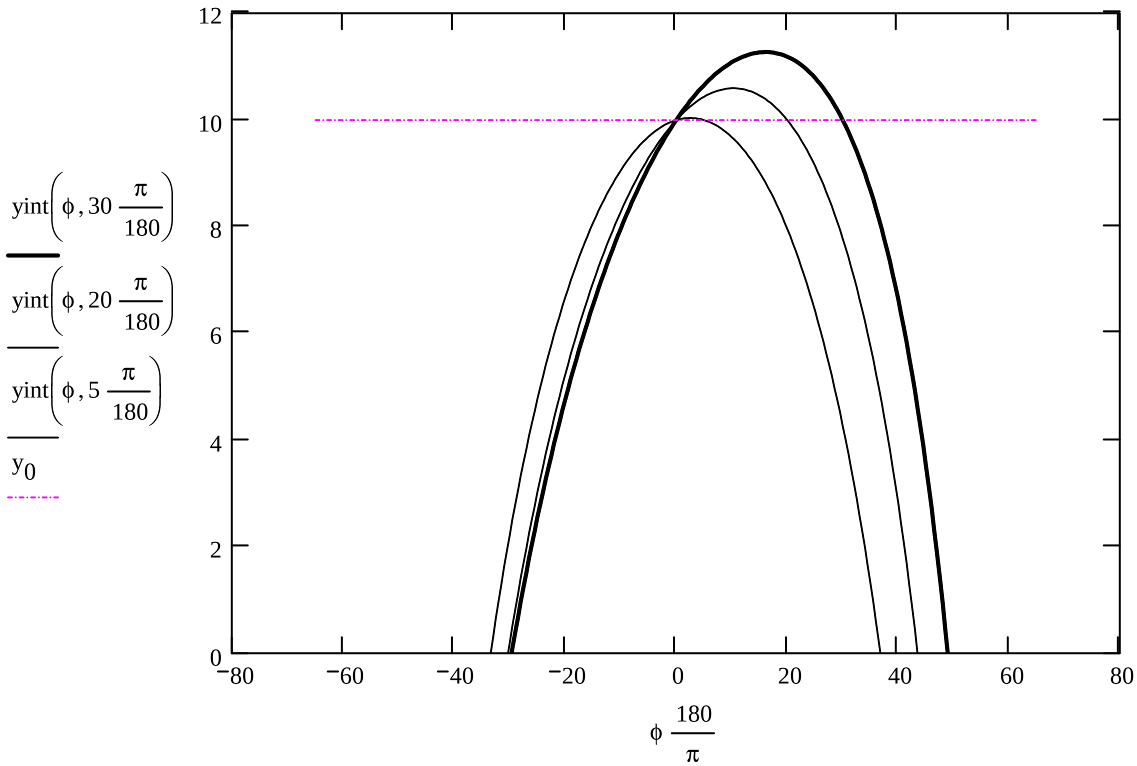


$$zz(\phi,\theta) := \sin(\theta)^2 - 4\,\tan(\phi)\,[\cos(\theta)\,(\sin(\theta) - \tan(\phi)\,\cos(\theta))]$$



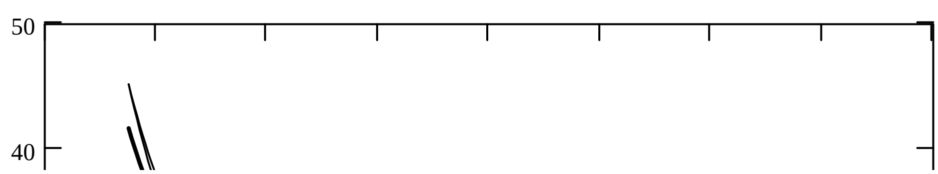


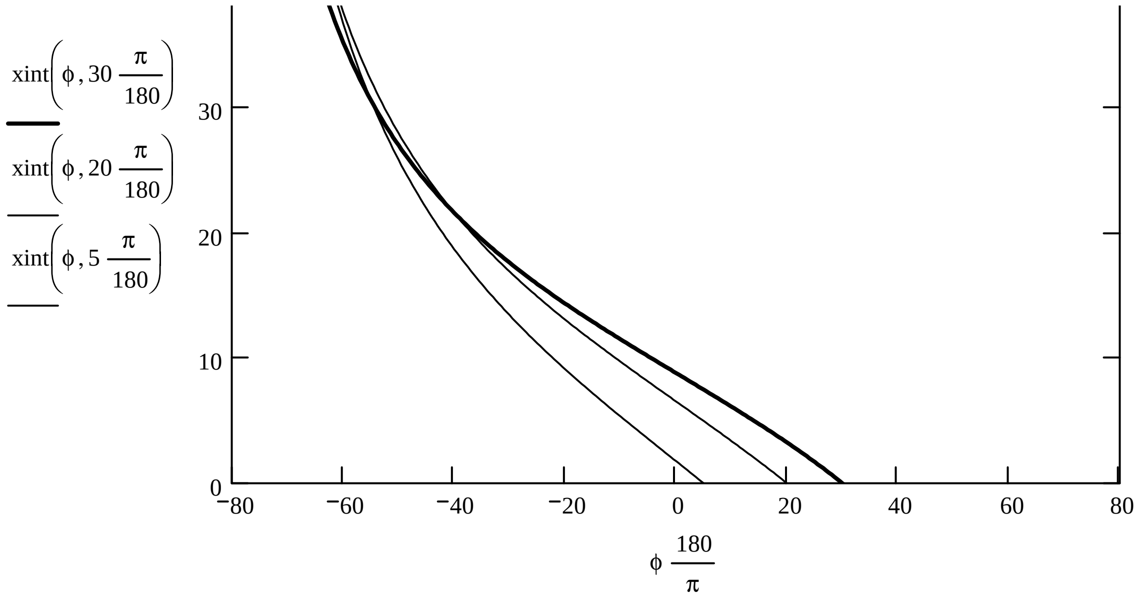
$$\text{yint}(\phi,\theta) := \tan(\phi)\left[\frac{2\left(v_0\cos(\theta)\right)^2}{g}\left(\tan(\theta)-\tan(\phi)\right)\right]+y_0$$



$$\text{xint}(\phi,\theta) := \frac{2\left(v_0\cos(\theta)\right)^2}{g}\left(\tan(\theta)-\tan(\phi)\right)$$

the basic condition is that the x of the intersection cannot be negative; neither can the y, but this is first, since other calcs depend on this





$xint\left(\phi, 30\frac{\pi}{180}\right)$
$xint\left(\phi, 20\frac{\pi}{180}\right)$
$xint\left(\phi, 5\frac{\pi}{180}\right)$
30
20
10
0
−80
−60
−40
−20
0
20
40
60
80
$\phi\frac{180}{\pi}$