1 The scalar product Exercise 1

1.Find the scalar product $\mathbf{a} \cdot \mathbf{b}$ for each of the following:

a) $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j}$, $\mathbf{b} = \mathbf{i} + 8\mathbf{j}$, b) $\mathbf{a} = 3\mathbf{i} - 7\mathbf{j}$, $\mathbf{b} = 5\mathbf{i} - \mathbf{j}$

c) $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j}$, $\mathbf{b} = -3\mathbf{i} + 8\mathbf{j}$, d) $\mathbf{a} = -\mathbf{i} + 8\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$,

e) $\mathbf{a} = \mathbf{i} - \mathbf{j}$, $\mathbf{b} = -\mathbf{i} + \mathbf{j}$, f) $\mathbf{a} = 5\mathbf{i} + 8\mathbf{j}$, $\mathbf{b} = 7\mathbf{i} - 5\mathbf{j}$,

2. Find $\mathbf{p} \cdot \mathbf{q}$ and the cosine of the angle between $\mathbf{p}$ and $\mathbf{q}$ if

a) $\mathbf{p} = 2\mathbf{i} + 6\mathbf{j}$, $\mathbf{q} = -\mathbf{i} + 2\mathbf{j}$, b) $\mathbf{p} = -3\mathbf{i} + 2\mathbf{j}$, $\mathbf{q} = 2\mathbf{i} + \mathbf{j}$

3. The angle between two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\arccos \frac{5}{13}$.

$\mathbf{v}_1 = 5\mathbf{i} + 12\mathbf{j}$ and $\mathbf{v}_2 = \mu\mathbf{i} - 5\mathbf{j}$. Find the positive value of μ.

4. In a triangle ABC, $\overrightarrow{AB} = 5\mathbf{i} + 6\mathbf{j}$ and $\overrightarrow{BC} = 6\mathbf{i} - 5\mathbf{j}$. Find the cosine of angle ABC and hence the angle ABC in radians.

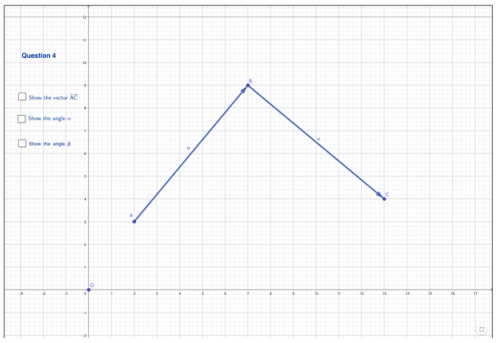


Figure 1: The diagram shows a triangle ABC and 2 given vectors.

Find the vector $\overrightarrow{AC}$ and use it to calculate the angle BAC in radians.

5. The vector $\mathbf{a} = 5\mathbf{i} + 12\mathbf{j}$.

The magnitude of this vector is given by $|\mathbf{a}|=\sqrt{5^2 + 12^2}=\sqrt{25 + 144} = 13$.

The unit vector in the direction of $\mathbf{a}$ is $\hat{\mathbf{a}}$ where $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{13}\mathbf{a}$

Find the unit vector for each of the following:

a) $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$.

b) $\mathbf{b} = \mathbf{i} + \mathbf{j}$.

c) $\mathbf{c} = -\mathbf{i} + \sqrt{3}\mathbf{j}$.

d) $\mathbf{d} = 4\sqrt{5}\mathbf{i} + 8\mathbf{j}$.

Answers:

1) a) 20, b) 22, c) -22, d) -35 e) -2, f) -5

2) a) $\mathbf{p} \cdot \mathbf{q} = 10$

2) b) $cos\theta = \frac{-4}{\sqrt{65}}$

3) $\mu = \frac{119}{24}$

4) a) $\theta = \frac{\pi}{2}$ b) $\theta = \pi$

5) a) $\hat{\mathbf{a}} = \frac{1}{5}(\mathbf{i} + 4\mathbf{j})$

b) $\hat{\mathbf{b}} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$

b) $\hat{\mathbf{c}} = \frac{1}{2}(-\mathbf{i} + \sqrt{3}\mathbf{j})$

b) $\hat{\mathbf{d}} = \frac{\sqrt{5}}{3}\mathbf{i} + \frac{2}{3}\mathbf{j}$