Taak: verloop van veelterm&rationale functies en toepassingen

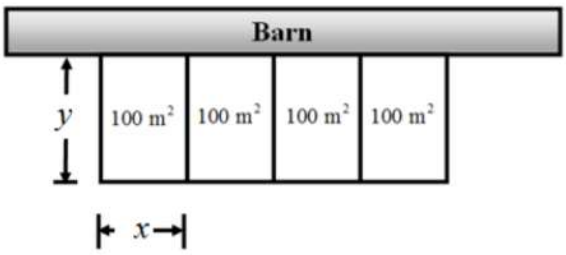
Virga Jessecollege **Toets WISKUNDE**

Hasselt **leerkracht: Karel Appeltans**

Datum: **schooljaar……**

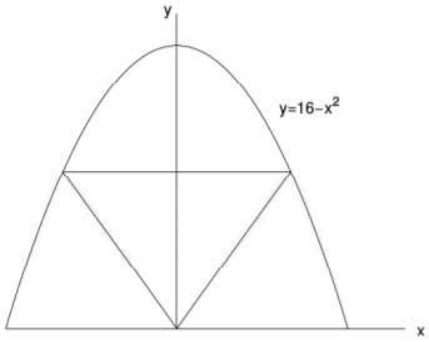
Klas: V **Afdeling:**

Naam:…… **Aantal uren wiskunde: 6**

1	Volledig verloop van $f(x) = \frac{1}{6}x^6 - 5x^4,$
2	3. (30 points) Given $f(x) = \frac{-x^2+x-1}{x-1}$, $f'(x) = \frac{x(2-x)}{(x-1)^2}$, $f''(x) = \frac{-2}{(x-1)^3}$, a.) (1 point) Find the y-intercept of $f(x)$. Note that $f(x)$ does not have any x-intercepts. b.) (5 points) Find any vertical and horizontal asymptotes of $f(x)$. c.) (8 points) State the intervals of increase and decrease of $f(x)$ using interval notation. Find the x and y values of any local extrema. d.) (6 points) On what intervals (using interval notation) is $f(x)$ concave up? Concave down? State the x and y values of any inflection points. e.) (10 points) Sketch $f(x)$
3	EXAMPLE 10: Determine the points on $y = x^2 + 1$ that are closest to $(0, 2)$.
4	11. A rancher plans to make four identical and adjacent rectangular pens against a barn, each with an area of 100 m^2. What are the dimensions of each pen that minimize the amount of fence that must be used. (There is no fence along the barn). (a) $x = 10$ m, $y = 10$ m (b) $x = 5\sqrt{5}$ m, $y = 4\sqrt{5}$ m (c) $x = 2\sqrt{5}$ m, $y = 10\sqrt{5}$ m (d) $x = 4$ m, $y = 25$ m (e) $x = \frac{25}{3}$ m, $y = 12$ m 

5	Let a be a constant with $a > 1$. A function $w(x)$ and its derivative $w'(x)$ are given below. $$w(x) = a + \frac{x}{x^2 + a^2} \quad \text{and} \quad w'(x) = \frac{-(x-a)(x+a)}{(x^2+a^2)^2}.$$ **a.** [5 points] Find and classify the local extrema of $w(x)$. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank, write NONE if appropriate.
6	**14. There is one (and only one) line through the point $(2, 1)$ that is tangent to the graph of $y = \frac{x}{x-1}$ at some point P on the graph. What is the x-coordinate of P?** **A)** $\frac{3}{2}$ **B)** 1 **C)** 2 **D)** -1 **E)** -2 **F)** $\frac{1}{4}$ **G)** $-\frac{2}{3}$ **H)** $\frac{5}{3}$ **I)** 3 **J)** $\frac{3}{4}$
7	5. Find the value of a such that the tangent lines to $y = x^3 - 2x^2 + x + 1$ at $x = a$ and $x = a+1$ are parallel. A) $\frac{1}{2}$ E) $\frac{1}{6}$ B) $\frac{-1}{2}$ F) 2 C) 1 G) 3 D) -2 H) None of these
8	Toon aan dat de functie $f: x \to \frac{x^2-x}{x^2+1}$ een absoluut maximum Men een absoluut minimum m heeft en dat M+m=1
9	An open-top box is to have a square base and a volume of 10 m^3. The cost per square meter of material is \$5 for the bottom and \$2 for the four sides. Let x and y be lengths of the box's width and height respectively. Let C be the total cost of material required to make the box. (a) Express C as a function of x and find its domain. (b) Find the dimensions of the box so that the cost of materials is minimized. What is this minimum cost?
10	A rectangular box with square base is constructed from material costing 1¢ per square centimetre for the sides and base; the top costs 2¢ per square centimetre. If the total cost of the box is to be 900¢, what is the maximum volume possible?
11	Volledig verloop van $f(x) = \frac{8}{5}x^5 - 8x^3$
12	**Oefening 11** In welk punt in het eerste kwadrant van de parabool $y = 4 - x^2$ bepaalt de raaklijn samen met de coördinaatassen een driehoek met minimale oppervlakte.
13	[13 points] Answer (a)–(h) with regard to the function $f(x) = \frac{x^2 - 2x + 4}{x - 2}$. The first and second derivatives of f are given below. $$f'(x) = \frac{x(x-4)}{(x-2)^2} \qquad f''(x) = \frac{8}{(x-2)^3}$$

14	Consider the function $$f(x) = \frac{2+x-x^2}{(x-1)^2}, \quad f'(x) = \frac{x-5}{(x-1)^3}, \quad f''(x) = \frac{2(7-x)}{(x-1)^4}$$ (a) (3 pts) Find any vertical, horizontal, or slant asymptotes of f. Use appropriate limits to justify your answer. (b) (3 pts) On what intervals is f increasing? decreasing? (c) (4 pts) Find all local maximum and minimum values of f. (d) (4 pts) On what intervals is f concave up? concave down? (e) (3 pts) Find all inflection points of f. (f) (3 pts) Using the information from (a)-(e), sketch a graph of f. Clearly label any points of interest, including any asymptotes, local extrema, and inflection points.
15	Volledig verloop van $$f(x) = \frac{(1-2x)(x-2)}{(x-1)^2}, \qquad f'(x) = \frac{-(x+1)}{(x-1)^3}, \qquad f''(x) = \frac{2(x+2)}{(x-1)^4}.$$
16	Volledig verloop van You are given the following information: $$f(x) = \frac{2(x+1)}{3x^2}, \quad f'(x) = \frac{-2(x+2)}{3x^3}, \quad \text{and } f''(x) = \frac{4(x+3)}{3x^4}.$$
17	Volledig verloop van $f(x) = x^4 - 2x^3 + 2x - 1$
18	Welke uitspraken zijn allemaal juist: De functie met voorschrift $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + \frac{1}{3}$ A. heeft P(2,1) als buigpunt B bereikt een maximum voor x=1 C bereikt een minimum voor x=3 D heeft een horizontale raaklijn in P(2,1)
19	Het punt $P(a,b)$ ligt op de grafieken van de functies $f : \mathbb{R} \to \mathbb{R} : x \mapsto f(x) = 4x^2 + 2x - 8$ en $g : \mathbb{R} \to \mathbb{R} : x \mapsto g(x) = x^3 - x + 10$. In dit punt P hebben de grafieken van beide functies dezelfde raaklijn. Bepaal de richtingscoëfficiënt van deze raaklijn. (A) 24 (B) 26 (C) 28 (D) 30
20	**13.** (10 points) Given $f(x) = \frac{x+1}{(x-3)^2}$, $f'(x) = -\frac{(x+5)}{(x-3)^3}$, $f''(x) = \frac{2(x+9)}{(x-3)^4}$. Find (if any): (a) The domain of f. (b) The x and y intercept(s). (c) The vertical and horizontal asymptotes. (d) Intervals on which f is increasing or decreasing. (e) Local (relative) extrema. (f) Intervals of upward or downward concavity. (g) Inflection points(s) (h) On the next page, sketch the graph of f. Label all intercepts, asymptotes, extrema, and points of inflection.
21	A storage container is to be made in the form of a right circular cylinder and have a volume of 28π m^3. Material for the top of the container costs \$5 per square metre and material for the side and base costs \$2 per square metre. What dimensions will minimize the total cost of the container?

22	. (9 pts) An isosceles triangle with its vertex pointing down has its base parallel to and above the x-axis as showm below. The base vertices lie on the parabola $y = 16 - x^2$. What is the largest area the triangle can have? 
23	Volledig verloop van $$y = f(x) = \frac{1}{8}x^8 - \frac{5}{6}x^6 + x^4$$
24	. An open-top box is to have a square base and a volume of 10 m^3. The cost per square meter of material is \$5 for the bottom and \$2 for the four sides. Let x and y be lengths of the box's width and height respectively. Let C be the total cost of material required to make the box. (a) Express C as a function of x and find its domain. (b) Find the dimensions of the box so that the cost of materials is minimized. What is this minimum cost?
25	Volledig verloop van $$f(x) = 7 - 9x - 3x^2 + 3x^3,$$
26	. Find all tangent lines to the graph of $y = x^3$ that pass through the point $(2, 4)$.

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[4pts] 10. (a) Find the equation of the tangent line to the curve $y = (x+1)^{-2}$ at $x = a$ for $a > 0$.

[5pts] (b) Find the value of $a > 0$ such that the area of the triangle formed in the first quadrant by the x-axis, the y-axis and the tangent line to $y = (x+1)^{-2}$ at $x = a$ is a maximum. (You do not have to prove that it is the maximum.)

(a) $\frac{dy}{dx} = -2(x+1)^{-3}$ so $\frac{y-(a+1)^{-2}}{x-a} = -2(a+1)^{-3}$

$$y = -2(a+1)^{-3}(x-a) + (a+1)^{-2}$$

(b) $(y=0)$ $\quad 2(a+1)^{-3}(x-a) = (a+1)^{-2}$

$$2(x-a) = a+1 \qquad x = \frac{a+1}{2} + a = \frac{3a+1}{2}$$

$(x=0)$ $\quad y = -2(a+1)^{-3}(-a) + (a+1)^{-2}$

$$= \frac{2a+(a+1)}{(a+1)^3} = \frac{3a+1}{(a+1)^3}$$

$$\text{area} = A(a) = \frac{1}{2}\left(\frac{3a+1}{2}\right)\left(\frac{3a+1}{(a+1)^3}\right) = \frac{(3a+1)^2}{4(a+1)^3}$$

$$A'(a) = \frac{2(3a+1)(3)4(a+1)^3 - (3a+1)^2(4)(3(a+1)^2)}{16(a+1)^6}$$

$$24(3a+1)(a+1)^3 = 12(3a+1)^2(a+1)^2$$

$$2(a+1) = 3a+1 \qquad 2a+2 = 3a+1 \qquad a = 1$$