Hoofdstuk IV: goniometrische functies

www.karelappeltans.be

August 10, 2021

1 De goniometrische cirkel

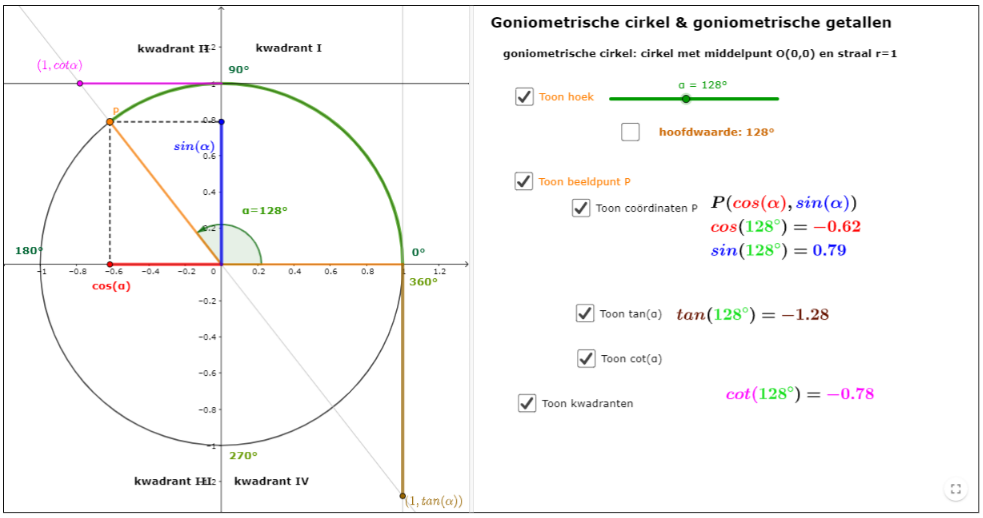

Figure 1: https://www.geogebra.org/m/FrxeHcwA

2 De goniometrische getallen

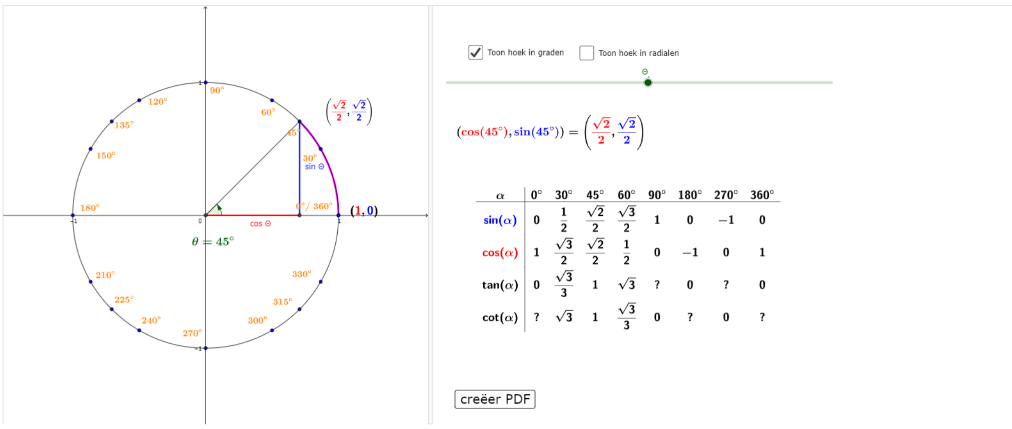

Figure 2: https://www.geogebra.org/m/FrxeHcwA

3 De Radiaal

3.1 Definitie

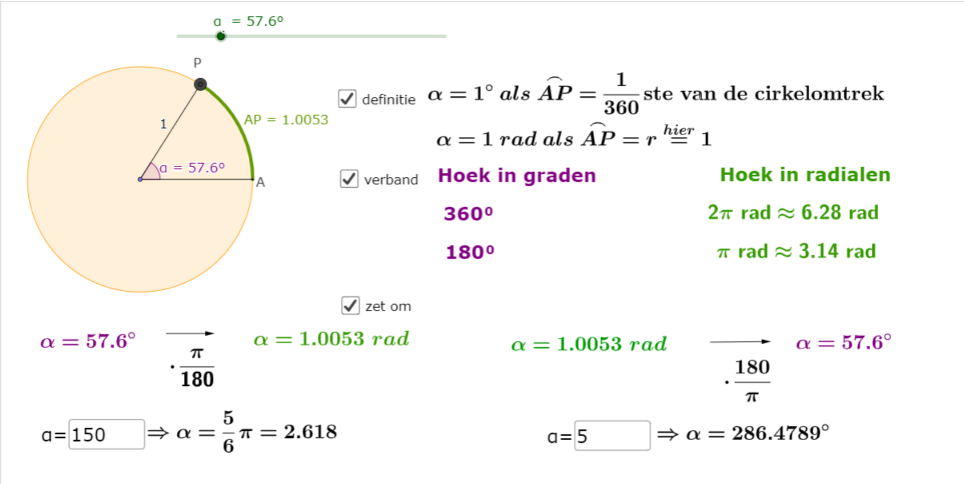

Figure 3: https://www.geogebra.org/m/QjEWqdp3

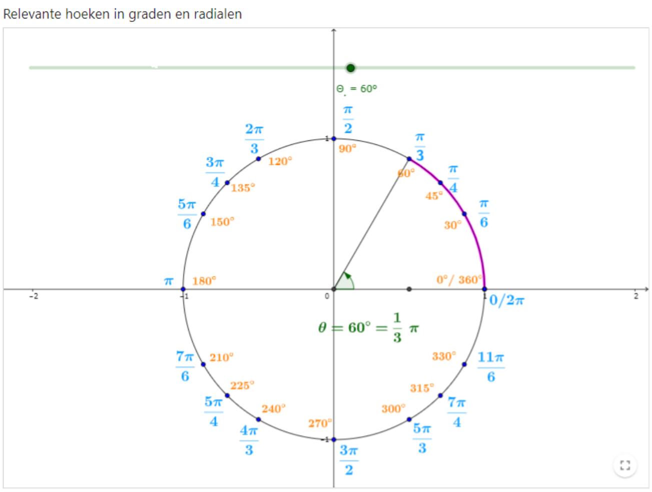

Figure 4: https://www.geogebra.org/m/QjEWqdp3

3.2 booglengte

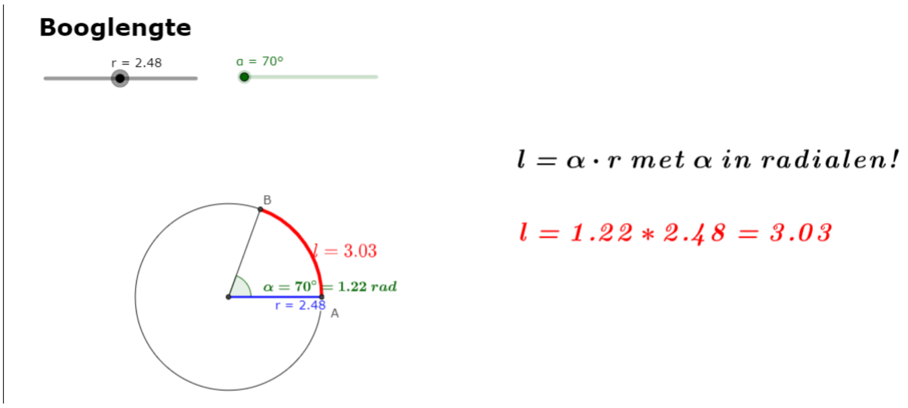

Figure 5: https://www.geogebra.org/m/QjEWqdp3

4 Verwante hoeken

4.1 supplementaire hoeken

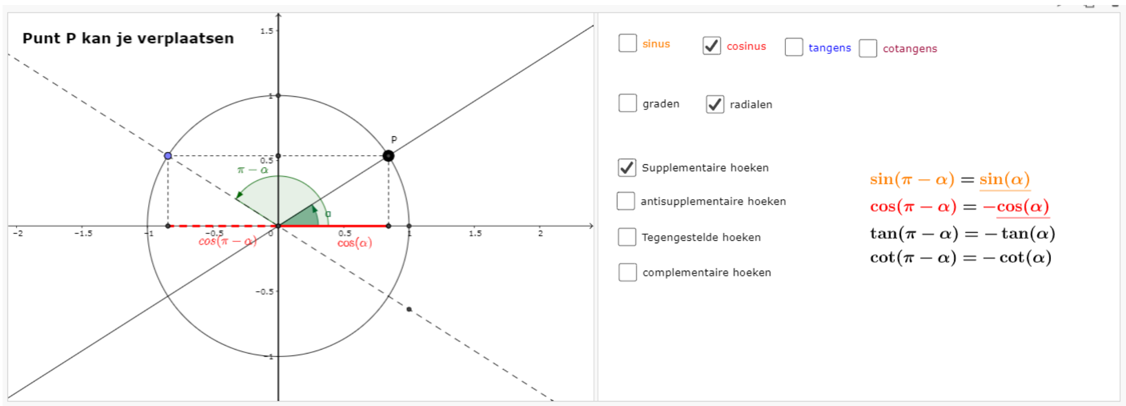

Figure 6: https://www.geogebra.org/m/q27XXAeF

4.2 antisupplementaire hoeken

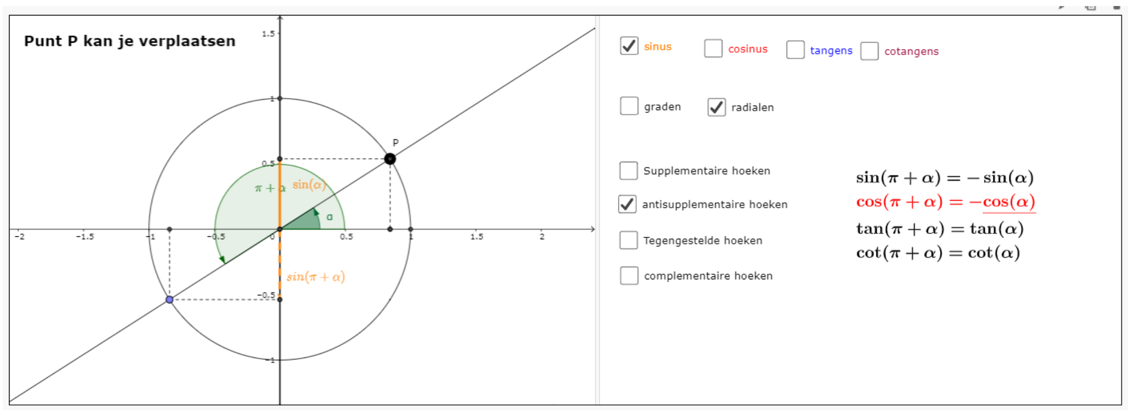

Figure 7: https://www.geogebra.org/m/q27XXAeF

4.3 tegengestelde hoeken

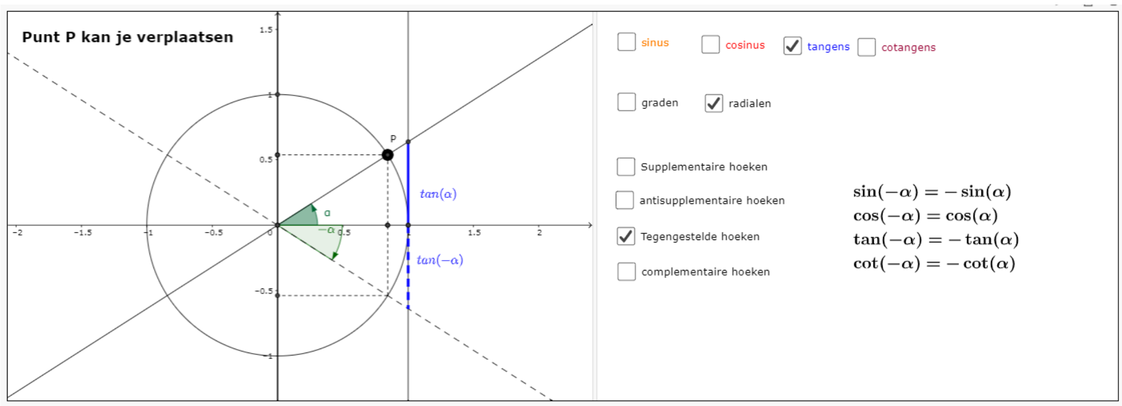

Figure 8: https://www.geogebra.org/m/q27XXAeF

4.4 complementaire hoeken

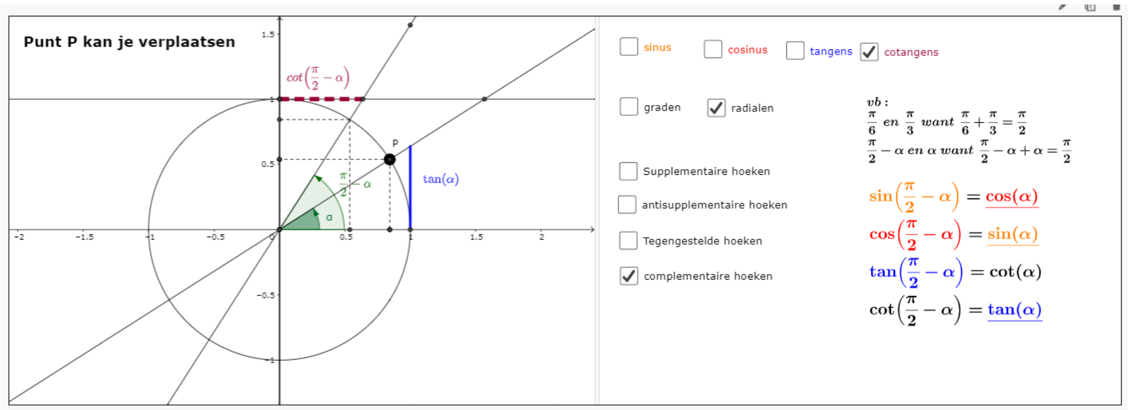

Figure 9: https://www.geogebra.org/m/q27XXAeF

4.5 willekeurige verwantschap

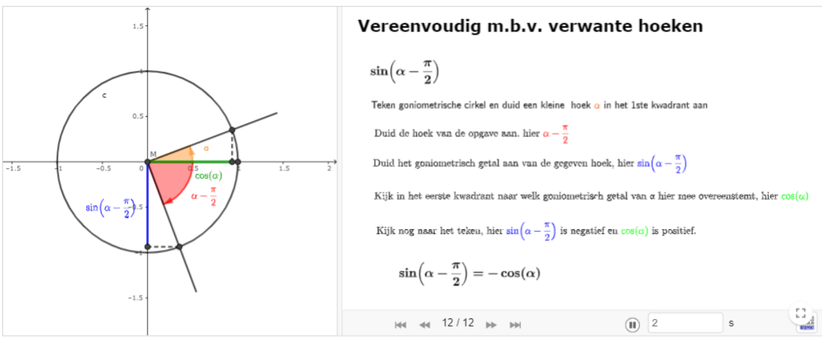

Figure 10: https://www.geogebra.org/m/q27XXAeF

5 Goniometrische functies

5.1 periodieke functies

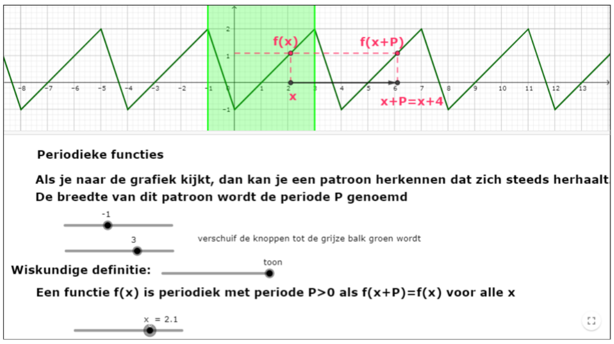

Figure 11: https://www.geogebra.org/m/dqmct5ur

5.2 f(x)=sin(x)

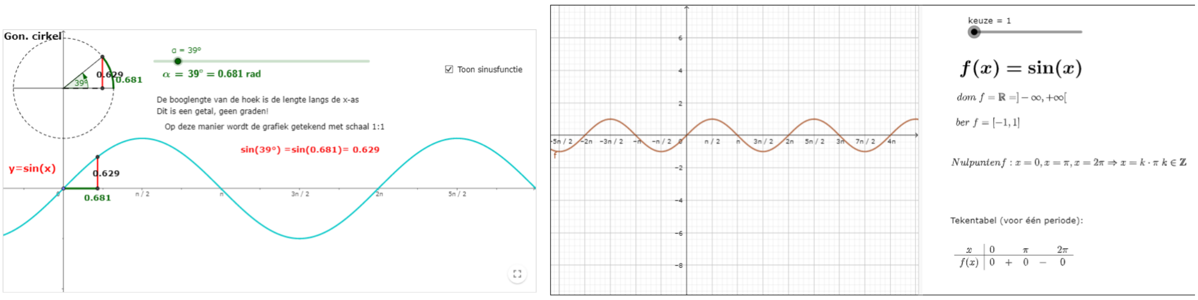

Figure 12: https://www.geogebra.org/m/eeEyfQce]https://www.geogebra.org/m/eeEyfQce

5.3 f(x)=cos(x)

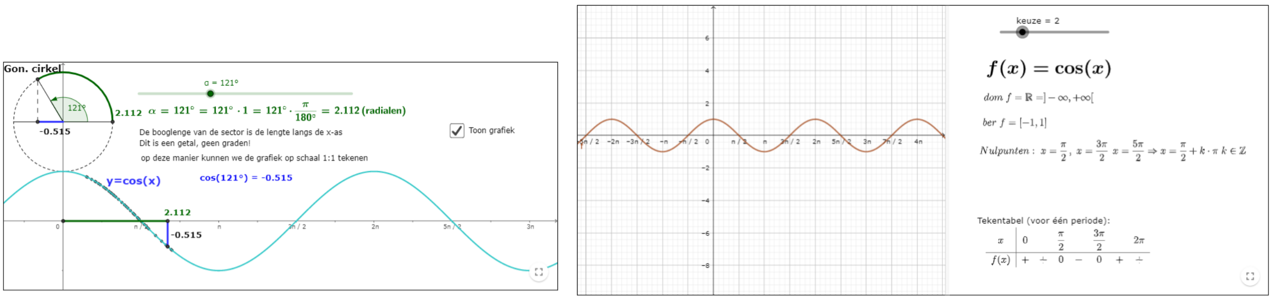

Figure 13: https://www.geogebra.org/m/eeEyfQce]https://www.geogebra.org/m/eeEyfQce

5.4 f(x)=tan(x)

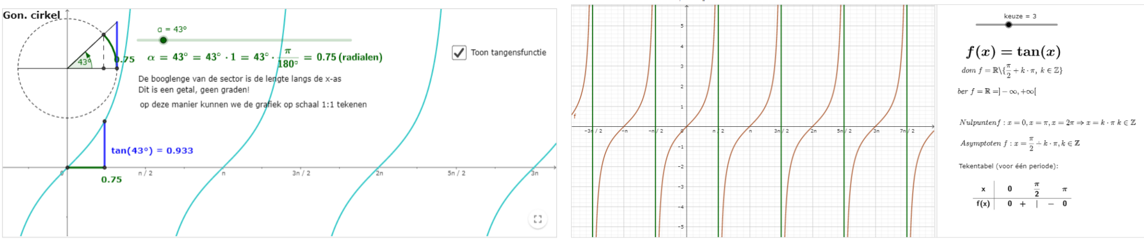

Figure 14: https://www.geogebra.org/m/eeEyfQce|https://www.geogebra.org/m/eeEyfQce

6 algemene sinusfunctie

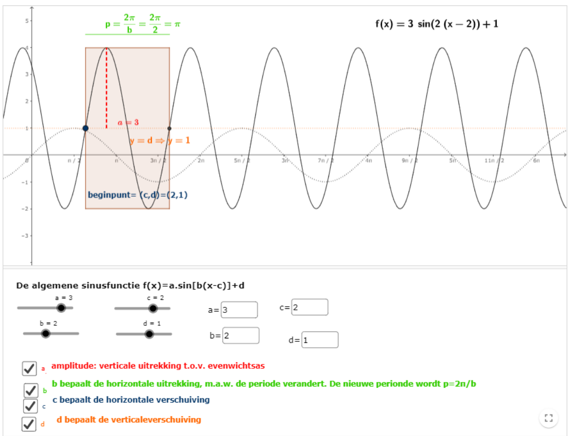

Figure 15: https://www.geogebra.org/m/BFwHmNn4

7 cyclometrische functies

7.1 f(x)=asin(x)

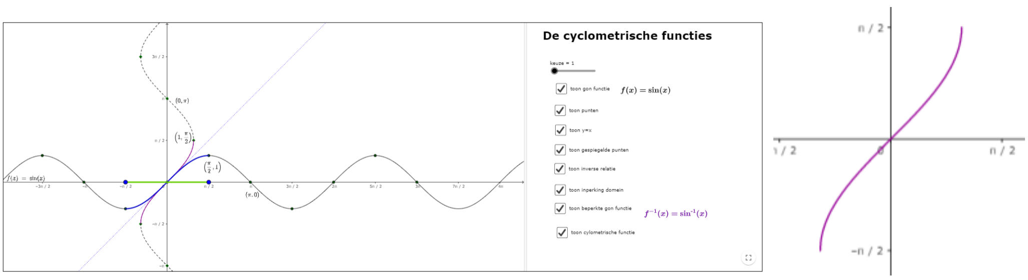

Figure 16: https://www.geogebra.org/m/Vx7MuEsU|https://www.geogebra.org/m/Vx7MuEsU

7.2 f(x)=acos(x)

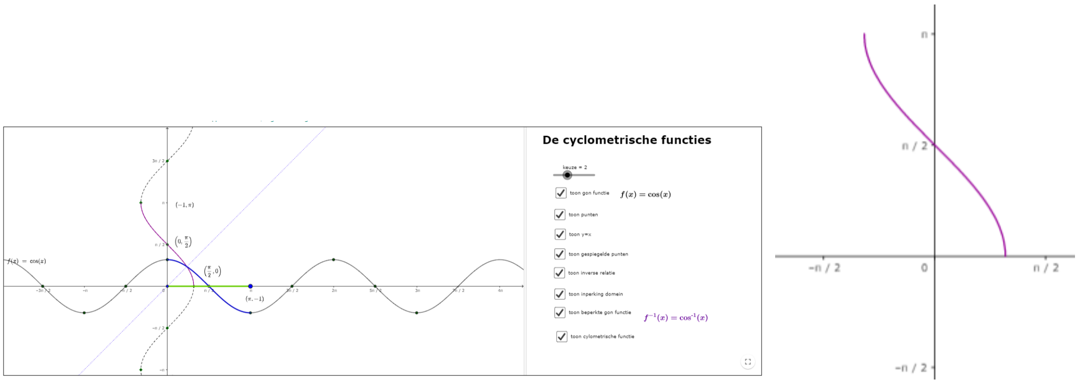

Figure 17: `https://www.geogebra.org/m/Vx7MuEsU`]https://www.geogebra.org/m/Vx7MuEsU

7.3 f(x)=atan(x)

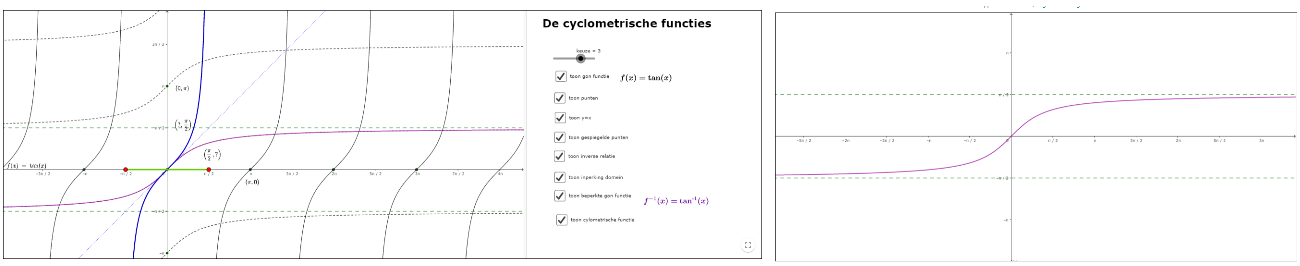

Figure 18: `https://www.geogebra.org/m/Vx7MuEsU`]https://www.geogebra.org/m/Vx7MuEsU

8 goniometrische identiteiten

8.1 hoofdformule

Basisformules

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad \sin^2\alpha = 1 - \cos^2\alpha$$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \qquad 1 + \cot^2\alpha = \frac{1}{\sin^2\alpha}$$

$$vb1\ TB: tan^2x(1 + cot^2x) = \frac{1}{1 - sin^2x}$$

$$LL = \frac{sin^2x}{cos^2x}\left(\frac{1}{sin^2x}\right)$$ oef met sinx/cosx en tanx/cotx → tanx/cotx herschrijven en $1 + cot^2x = \frac{1}{sin^2x}$

$$= \frac{sin^2x}{cos^2x} \cdot \frac{1}{sin^2x} = \frac{1}{cos^2x}$$ algebraïsch vereenvoudigen

$$= \frac{1}{1 - sin^2x} = RL$$ kijken naar RL, basis formules gebruiken

$$vb2: \frac{secx + tanx}{secx - tanx} = \frac{1 + 2sinx + sin^2x}{cos^2x}$$

$$LL = \frac{\frac{1}{cosx} + \frac{sinx}{cosx}}{\frac{1}{cosx} - \frac{sinx}{cosx}}$$ oef met sinx/cosx en tanx/cotx → tanx/cotx herschrijven

$$= \frac{\frac{1+sinx}{cosx}}{\frac{1-sinx}{cosx}}$$ algebraïsch vereenvoudigen: T en N als één breuk schrijven

$$= \frac{1 + sinx}{cosx} \cdot \frac{cosx}{1 - sinx} = \frac{1 + sinx}{1 - sinx}$$ algb vereenvoudigen: rekenen met breuk op breuk

$$RL = \frac{(1 + sinx)^2}{1 - sin^2x}$$ algebraïsch vereenvoudigen: merkwaardig product en voor N gekeken naar LL

$$= \frac{(1 + sinx)(1 + sinx)}{(1 - sinx)(1 + sinx)}$$ merkwaardige producten gebruikt:
$(A+B)^2=A^2+2AB+B^2$
$A^2-B^2=(A-B)(A+B)$

$$= \frac{1 + sinx}{1 - sinx}$$

LL=RL

Figure 19: https://www.geogebra.org/m/QfWuM7tH|https://www.geogebra.org/m/QfWuM7tH

8.2 somformules

Som- en verschilformules

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$
$$\tan(\alpha - \beta) = \frac{\tan\alpha - tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

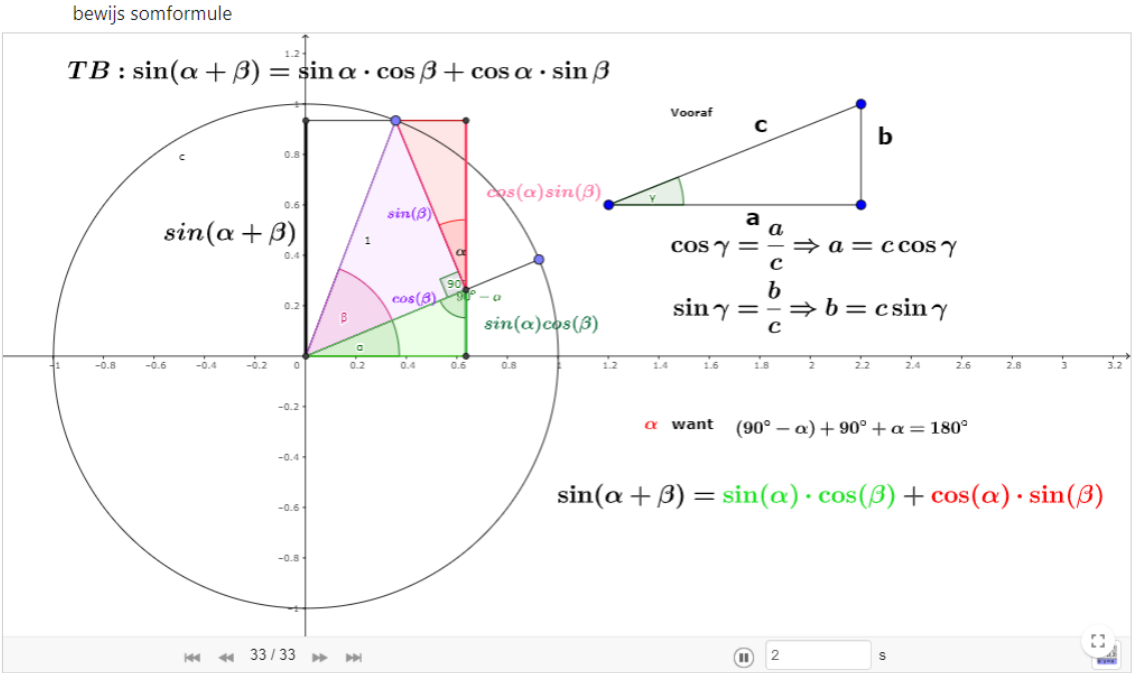

Figure 20: https://www.geogebra.org/m/QfWuM7tH]https://www.geogebra.org/m/QfWuM7tH

8.3 formules dubbele hoek

Verdubbelings- en halveringsformules

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1 \qquad \cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha \qquad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

8.4 Formules van Simpson

Formules van Simpson:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

8.5 overzicht alle formules

Basisformules

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad \sin^2\alpha = 1 - \cos^2\alpha$$

$$\cos^2\alpha = 1 - \sin^2\alpha$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \qquad 1 + \cot^2\alpha = \frac{1}{\sin^2\alpha}$$

Som- en verschilformules

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

Verdubbelings- en halveringsformules

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\cos(2\alpha) = 2\cos^2\alpha - 1 \qquad \cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\cos(2\alpha) = 1 - 2\sin^2\alpha \qquad \sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Formules van Simpson:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Figure 21: https://www.geogebra.org/m/QfWuM7tH|https://www.geogebra.org/m/QfWuM7tH

9 goniometrische vergelijkingen

9.1 basisvergelijkingen

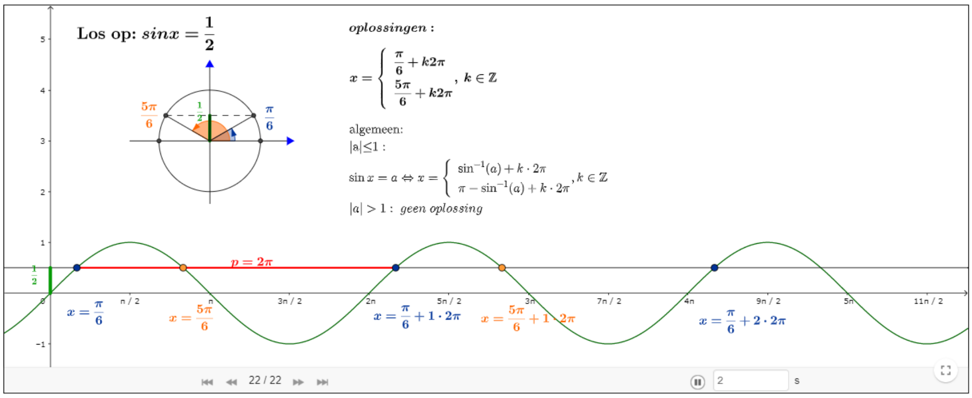

Figure 22: https://www.geogebra.org/m/ej2fhRDY|https://www.geogebra.org/m/ej2fhRDYt

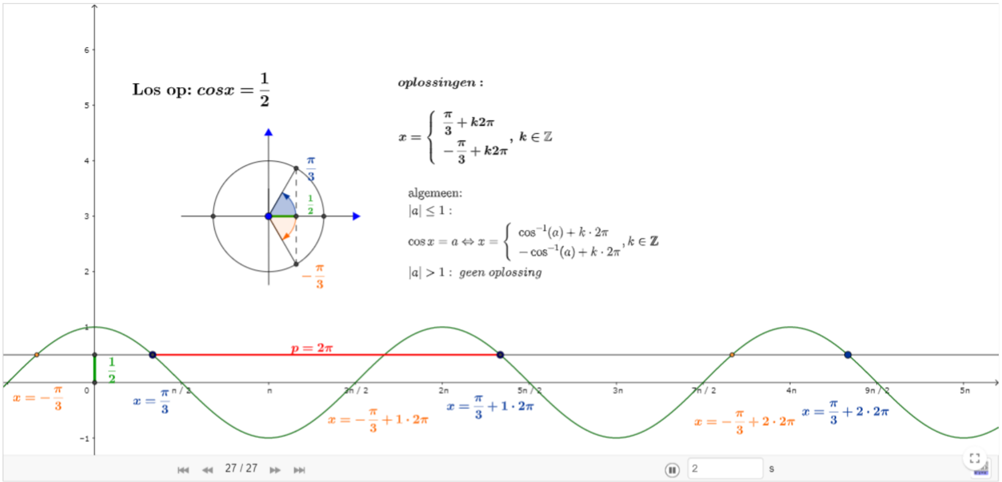

Figure 23: https://www.geogebra.org/m/ej2fhRDYt

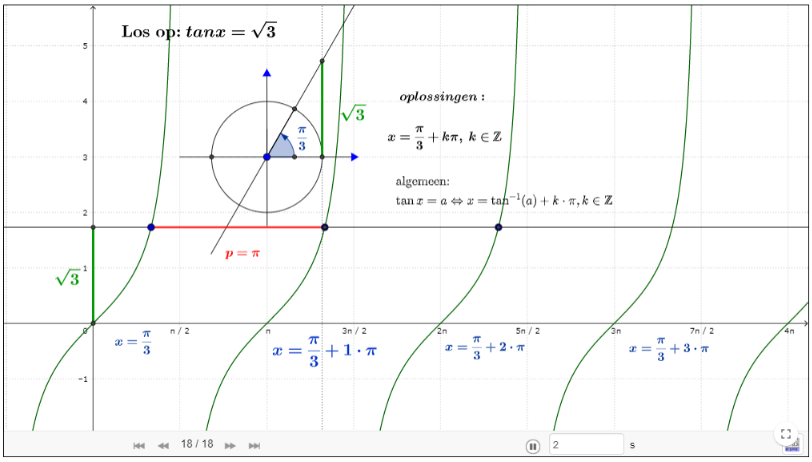

Figure 24: https://www.geogebra.org/m/ej2fhRDYt

9.2 herleidbaar tot basisvergelijking

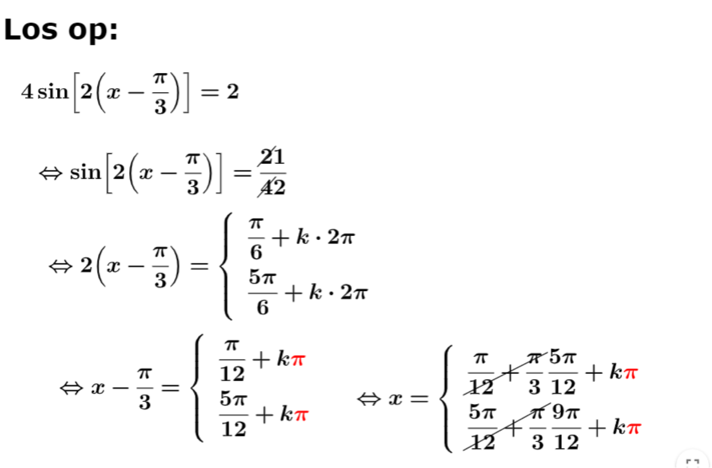

Figure 25: https://www.geogebra.org/m/esxkvnhr

9.3 m.b.v. ontbinding in factoren

Los op:

cos(x)+sin(2x)=0

$$\Leftrightarrow \cos x + 2\sin x \; cosx = 0$$

$$\Leftrightarrow \cos x(1+2\sin x) = 0$$

$$\Leftrightarrow \cos x = 0 \;\vee\; 1+2\sin x = 0$$

$$\Leftrightarrow \cos x = 0 \;\vee\; \sin x = -\frac{1}{2}$$

$$\Leftrightarrow x = \begin{cases} \frac{\pi}{2} + k\cdot 2\pi \\ -\frac{\pi}{2} + k\cdot 2\pi \end{cases} \vee x = \begin{cases} -\frac{\pi}{6} + k\cdot 2\pi \\ \pi - \left(-\frac{\pi}{6}\right) + k\cdot 2\pi \end{cases}$$

$$\Leftrightarrow x = \frac{\pi}{2} + k\cdot\pi \;\vee x = \begin{cases} -\frac{\pi}{6} + k\cdot 2\pi \\ \frac{7\pi}{6} + k\cdot 2\pi \end{cases}$$

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Figure 26: https://www.geogebra.org/m/esxkvnhr

9.4 herleidbaar tot vierkantsvergelijking

Los op:

$$6\sin^2 x + 5\cos x = 7$$

$$\Leftrightarrow 6(1-\cos^2 x) + 5\cos x - 7 = 0$$

$$\Leftrightarrow -6\cos^2 x + 5\cos x - 1 = 0$$

$$D = 5^2 - 4(-6)(-1) = 1$$

$$(\cos x)_{1,2} = \frac{-5 \pm 1}{-12} = \begin{matrix} \nearrow \frac{1}{3} \\ \searrow \frac{1}{2} \end{matrix}$$

$$\Leftrightarrow \cos x = \frac{1}{3} \;\vee\; \cos x = \frac{1}{2}$$

$$\Leftrightarrow x = \begin{cases} cos^{-1}\left(\frac{1}{3}\right) + k\cdot 2\pi \\ -\cos^{-1}\left(\frac{1}{3}\right) + k\cdot 2\pi \end{cases} \vee x = \begin{cases} \frac{\pi}{3} + k\cdot 2\pi \\ -\frac{\pi}{3} + k\cdot 2\pi \end{cases}$$

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Figure 27: https://www.geogebra.org/m/esxkvnhr

9.5 homogeen

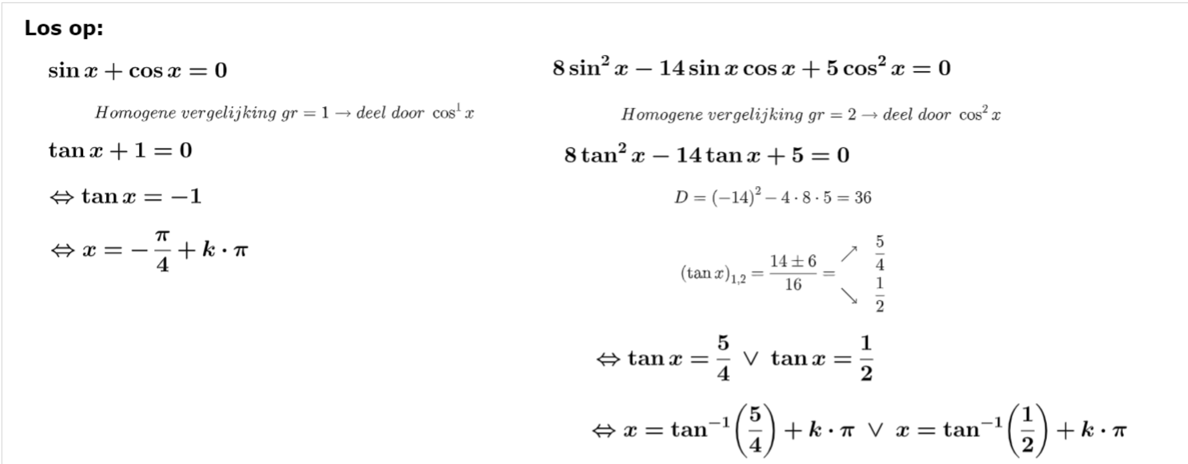


Figure 28: https://www.geogebra.org/m/esxkvnhr

10 goniometrische ongelijkheden

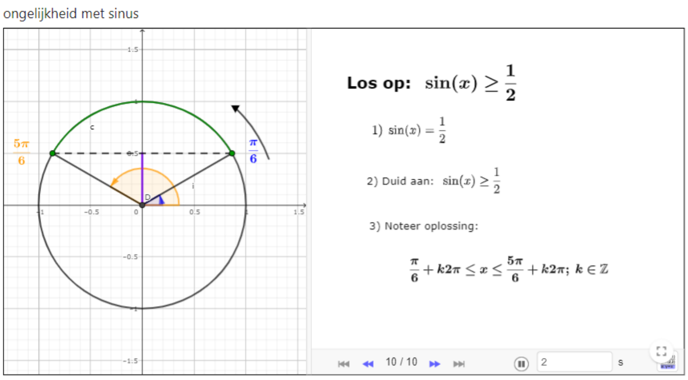


Figure 29: https://www.geogebra.org/m/jr7kkdkt

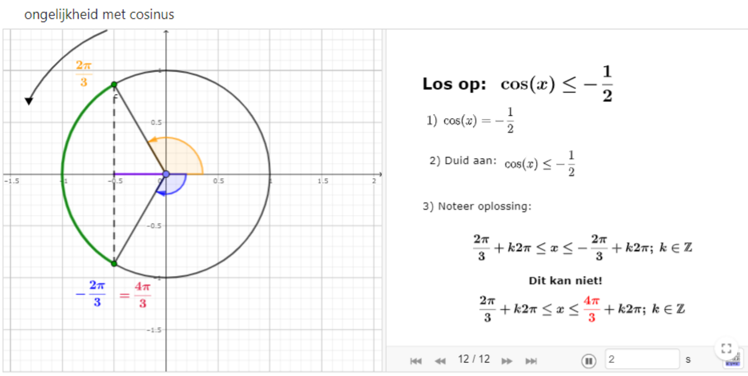


Figure 30: https://www.geogebra.org/m/jr7kkdkt

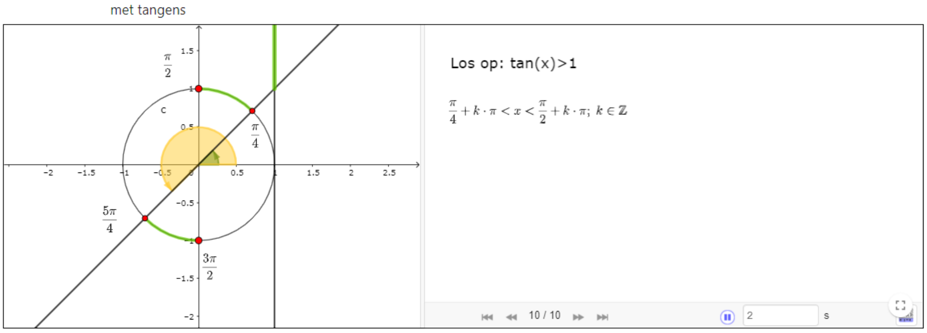


Figure 31: https://www.geogebra.org/m/jr7kkdkt

11 oefeningen

11.1 de radiaal

1. Zet om van graden naar radialen
   (a) 150°
   (b) -900°
   (c) 56°25’
2. zet om van radialen naar graden
   (a) $\frac{11\pi}{6}$
   (b) -5π
   (c) 1.74
3. Over hoeveel radialen draait de minutenwijzer van een klok in 3 uur?
4. Los op:

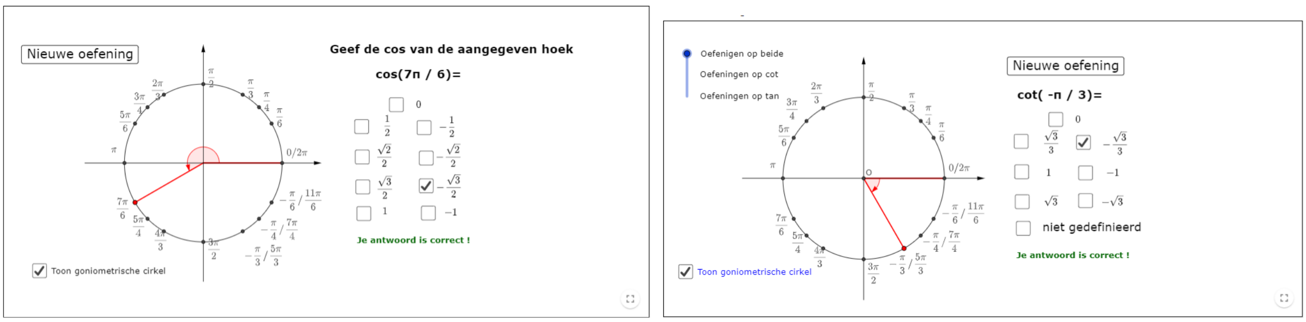
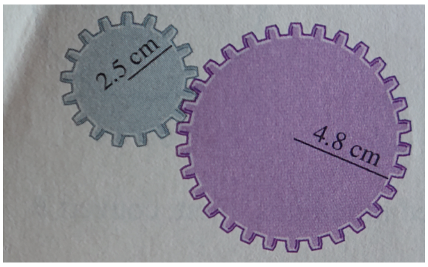
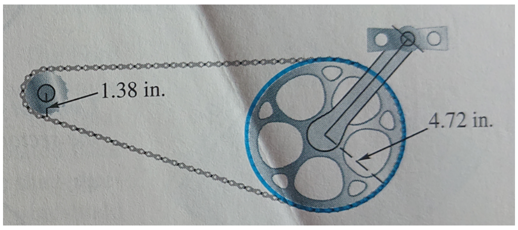


Figure 32: https://www.geogebra.org/m/kqsesuxn]https://www.geogebra.org/m/kqsesuxn

5. Bepaal de afstand tussen de steden Reno en Los Angeles. Deze steden liggen op dezelfde meridiaan. De latitude van Reno is $40N$ en deze van Los Angeles is $34N$. Je mag voor de straal van de aarde 6400 km nemen.
6. Het kleine tandwiel drijft het grote tandwiel aan. Als het kleine tandwiel 225 gedraaid wordt, over hoeveel graden zal dan het grote tandwiel gedraaid zijn? (A.≈ 117)

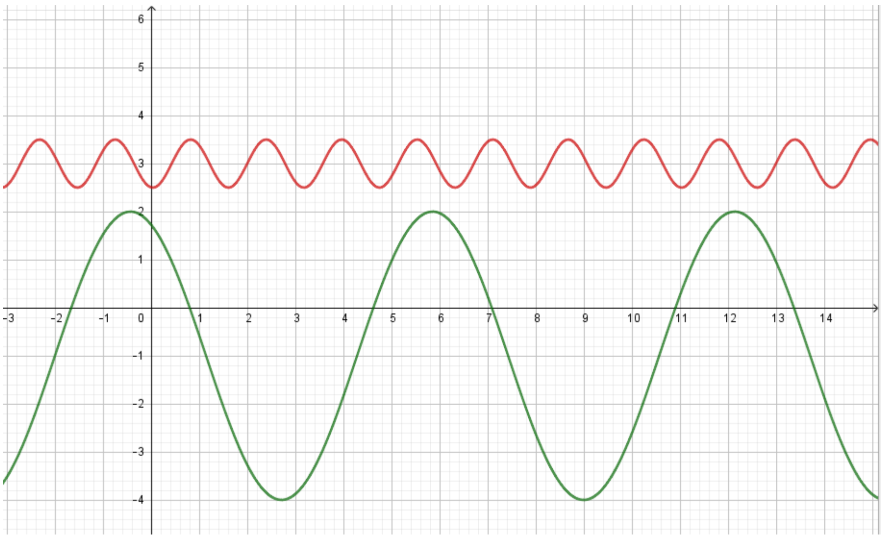


7. De figuur toont de wielaandrijving van een fiets. Hoever zal de fietser zich verplaatsen als de pedalen over een hoek van 180° gedraaid worden? Neem aan dat de straal van het fietswiel 13.6 inches is.



11.2 algemene sinusfunctie

1. Schets de grafiek van volgende algemene sinusfunctie
   (a) $y = 2\sin\left(x + \frac{\pi}{2}\right)$
   (b) $y = \frac{1}{2} + \sin\left[2\left(x + \frac{\pi}{4}\right)\right]$
2. Bepaal het voorschrift van onderstaande grafieken van algemene sinusfuncties



3. De gemiddelde temperatuur (in °F) op een zekere plaats kan bij benadering gemodelleerd worden door de functie $f(x) = 12\sin\left[\frac{\pi}{6}\left(x - 3.9\right)\right] + 72$ met $x = 1$ ingevuld de temperatuur voor januari geeft enz.
   (a) Wat zal de gemiddelde temperatuur in april zijn?
   (b) Wat zijn de grootste en laagste gemiddelde temperatuur?

12 taken

1. Verwante hoeken

2. Goniometrische formules I
3. Goniometrische formules II
4. Goniometrische vergelijkingen
5. Goniometrische ongelijkheden