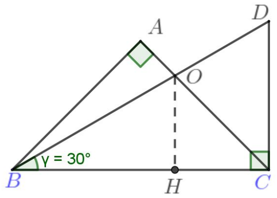
2023学年度第一学期期末九年级普陀区数学一模参考答案

一、选择题：(本大题共 6 题，每题 4 分，满分 24 分)

1、$(\mathbf{C})$　2、$(\mathbf{A})$　3、$(\mathbf{B})$　4、$(\mathbf{C})$　5、$(\mathbf{D})$　6、$(\mathbf{D})$

6、详解：假设 $CD=1$，则 $BD=2, BC=\sqrt{3}, CA=\dfrac{\sqrt{2}}{2}BC=\dfrac{\sqrt{6}}{2}$

作 $OH\perp BC$,交 BC于点 H，设 $OH=x$, 则 $HC=x$ (等腰直角三角形OCH)



$OH//CD \implies OH:CD=BH:HC, x:1=(\sqrt{3}-x):\sqrt{3}$

$x=\dfrac{\sqrt{3}}{1+\sqrt{3}}, OC=\dfrac{x}{\sin 45^\circ}=\dfrac{\sqrt{6}}{1+\sqrt{3}}$

$OC:CA=\dfrac{\sqrt{6}}{1+\sqrt{3}}:\dfrac{\sqrt{6}}{2}=\dfrac{2}{1+\sqrt{3}}=\sqrt{3}-1$

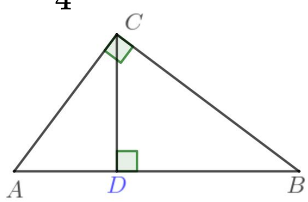
答：$(\mathbf{D})$

二、填空题：(本大题共 12 题，每题 4分，满分 48 分)

7、$\dfrac{7}{2}$　8、$y=kx(k>0)$

9、$\vec{a}+2\vec{b}$　10、$m>2$　11、$\dfrac{10}{3}$

12、$\dfrac{3}{4}$



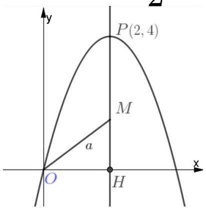
解：勾三股四弦五，$CD=4$，$AC=5$，则$AD=3$，相似比为 CD：$DB=\tan B=\tan\angle ACD=3/4=\dfrac{3}{4}$

13、$A'(3,6)$

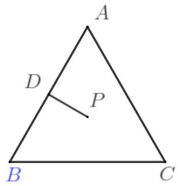
14、$M(2,\dfrac{3}{2})$

解：$P(2,4)$,设对称轴与 x 轴交于一点H，设$OM=PM=a$, 则在$Rt\triangle OMH$ 中，$a^2=2^2+(4-a)^2, a=\dfrac{5}{2}$

$M(2,4-\frac{5}{2}),M(2,\frac{3}{2})$



15、$PD=\frac{\sqrt{3}}{3}$

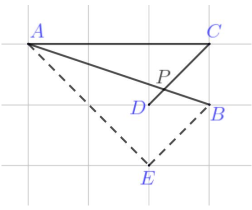


16、$\frac{\sqrt{5}}{5}$

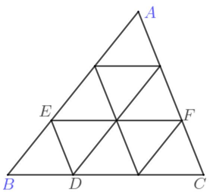
解：$CD//EB,\therefore\angle APD=\angle ABE=\angle BPC,$

在三角形 ABE 中，$BE\perp AE(\angle BED=\angle DEA=45^{\circ}),$

$\therefore\cos ABE=BE/AB=\sqrt{2}/\sqrt{10}=\frac{\sqrt{5}}{5}$



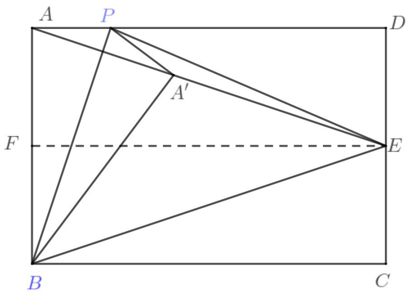
17、$EF=4$



18、$2<AP<2\sqrt{10}-2$

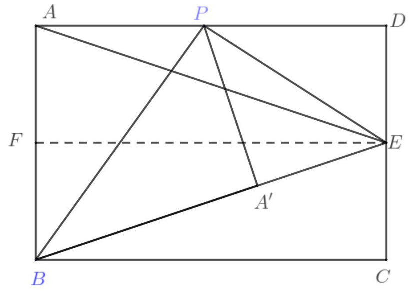
18题详解：(1) A' 落在AE上， $Rt\triangle ABP\sim Rt\triangle FEA,AP:AF=AB:EF,AP=3\times 6\div 9=2$ 也可以由 $Rt\triangle ABP\sim Rt\triangle DAE,$ 还可以由角度的余弦值得到，

$AP/AB=\tan\angle ABP=\tan\angle DAE=3/9=1/3$

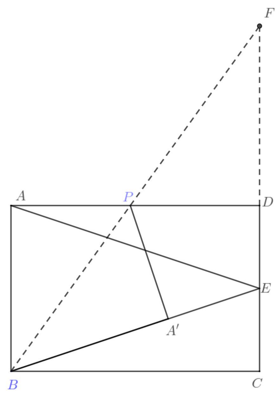


(2) A' 落在BE上，设 $AP=x=A'P,PE^2=PD^2+DE^2=PA'^2+A'E^2$

$\therefore(9-x)^2+3^2=x^2+(3\sqrt{10}-6)^2,x=2\sqrt{10}-2$ 答：$2<AP<2\sqrt{10}-2$

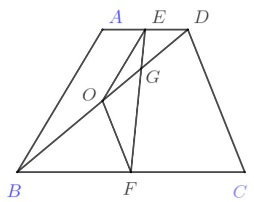


(3) A' 落在BE上时，还可以延长BP，延长CD，交点为F,设 $AP=x$, 则$PD=9-x$。
$AB//CF, \angle F=\angle ABP=\angle A'BP \therefore BE=EF=3\sqrt{10}$
$Rt\triangle ABP \sim Rt\triangle DFP \implies AP:PD=AB:DF$ $x:(9-x)=6:(3\sqrt{10}-3)=2:(\sqrt{10}-1) \implies x:9=2:(\sqrt{10}+1)$ $x=2\sqrt{10}-2$



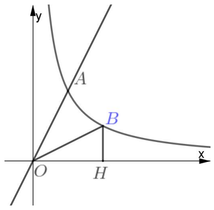
三、解答题：(本大题共 7 题，满分 78 分)

19、$-\dfrac{1}{2}$　20、(1) 解：$OF//CD, BF=FC, \therefore BO=OD$
又 $\because AE=ED \therefore OE$ 是三角形 ABD 的中位线，$OE//AB, OE=\dfrac{1}{2}AB, \dfrac{OE}{AB}=\dfrac{1}{2}$



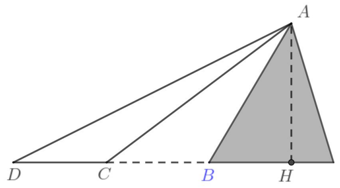
(2)$FO//CD, BF=FC \therefore \overrightarrow{FO}=-\dfrac{1}{2}\vec{b}$
同理 $\overrightarrow{OE}=\dfrac{1}{2}\vec{a}, \therefore \overrightarrow{FE}=\dfrac{1}{2}(\vec{a}-\vec{b})$ $EG:GF=DE:BF=1:3, \therefore EG=\dfrac{1}{4}EF \therefore$
$\overrightarrow{EG}=-\dfrac{1}{4}\times\dfrac{1}{2}(\vec{a}-\vec{b})=\dfrac{1}{8}(\vec{b}-\vec{a})$

21、（1）解：$x=1$ 代入 $y=2x$, 得到 $y=2, A(1,2)$, 将A点坐标代入反比例函数，得到 $k=xy=2$. 这个反比例函数的解析式为 $y=\dfrac{2}{x}$



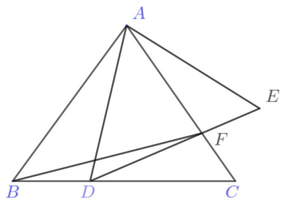
(2) $\tan\angle AOH=2\therefore\tan\angle OBH=OH:BH=2$, 不妨设点B的坐标为 $(a,\dfrac{2}{a})$, $\therefore a:\dfrac{2}{a}=2, a=2\ B(2,1)$

22、解：过点A 作$AH\perp BC$ 垂足为点H，设 $BH=a, AH=h$ 则 $h=\tan 37^{\circ}(33+a)=\tan 26.6^{\circ}(63+a)$



$\dfrac{33+a}{63+a}=\dfrac{0.5}{0.75}=\dfrac{2}{3},\dfrac{33+a}{30}=\dfrac{2}{1}, a=60-33=27\ h=0.5(63+27)=45$, 坡度就是 $h:a=45:27=5:3$

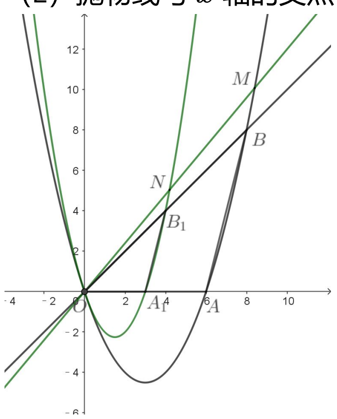
23、(1) 证明：$\because\angle EAF=\angle FDC,\angle AFE=\angle DFC\therefore\triangle AEF\sim\triangle DCF\therefore\angle E=\angle C$, 又 $\angle B=\angle ADE\therefore\triangle ABC\sim\triangle ADE\therefore AB:AD=AC:AE\implies\dfrac{AB}{AC}=\dfrac{AD}{AE}$



(2) $\because AB:AC=AF:AB,\angle BAF=\angle BAC\therefore\triangle ABF\sim\triangle ACB\therefore BF:BC=AB:AC$ 由(1) 得知 $\triangle ABC\sim\triangle ADE\therefore AB:AC=AD:AE$ 综合起来就是 $BF:BC=AD:AE,\implies AD\cdot BC=AE\cdot BF$

24、（1）P, 位似、相似

（2）抛物线与 x 轴的交点 $O(0,0), A(6,0)$



$$\begin{cases}y=\dfrac{1}{2}x^2-3x\\ y=x\end{cases}\implies B(8,8)$$

$\therefore B_1(4,4), A_1=(3,0)$

设过 O,A_1,B_1 的抛物线方程为 $y=ax(x-3)$(两根式)

将 $B_1(4,4)$ 代入上式得到抛物线的解析式为 $y=x(x-3)=x^2-3x$

(3) $\begin{cases} y = \frac{1}{2}x^2 - 3x \\ y = kx \end{cases} \implies M(2(3+k), 2k(3+k))$

$\begin{cases} y = x^2 - 3x \\ y = kx \end{cases} \implies N(3+k, k(3+k))$

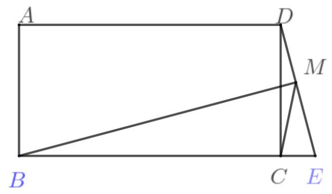
$\therefore ON = MN = \frac{1}{2}OM$

$\because MN, AA_1$ 相交于点 O, 且 $\frac{ON}{OM} = \frac{OA}{OA_1} = \frac{1}{2}, \angle NOA_1 = \angle MOA$

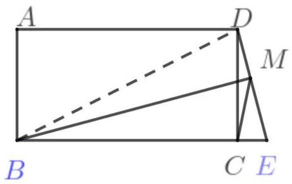
$\therefore \triangle OA_1N \sim \triangle OAM, \triangle OA_1N$ 与 $\triangle OAM$ 是位似三角形。

25、(1) $\because \angle E = \angle E, \angle BME = \angle DCE = 90^\circ \therefore \triangle BME \sim \triangle DCE \quad (\mathbf{AAA})$

$\therefore \angle CDE = \angle MBE$



(2) 连接 B, D,



显然 $\because \angle BMD = \angle BCD, \therefore B, C, M, D$ 四点共圆，直径就是 BD。

$\therefore \angle DBM = \angle DCM$(同弦 DM 所对的圆周角相等)

$\therefore \angle CME = \angle CDM + \angle DCM = \angle MBE + \angle DBM = \angle DBE$

故 $\angle CME$ 为定角, 等于 $\angle DBE$,

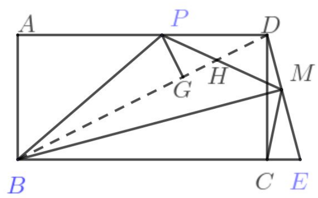
$\therefore \tan \angle CME = \tan \angle DBE = \frac{CD}{BC} = \frac{1}{2}$

(3a) 当 $\angle PBM = \angle CME = \angle DBE$.

$\therefore \angle PBD = \angle MBE \because \Delta BEM \sim DCE$,

$\therefore \angle PBD = \angle MBE = \angle CDE$.

$\therefore \tan \angle PBD = \tan \angle CDE = \frac{a}{2}$,



在 ΔBPD 中:

$BD = 2\sqrt{5}, \tan \angle PDB = \frac{1}{2}$.

解 ΔBPD, 作 $PG \perp BD$, 设 $PG = ak, BG = 2k, \therefore DG = 2ak$.

$\therefore BD=2k+2ak=2\sqrt{5}, \quad k=\dfrac{\sqrt{5}}{a+1}, \quad PD=\sqrt{5}ak=\dfrac{5a}{a+1}$

$\therefore AP=AD-PD=4-\dfrac{5a}{a+1}=\dfrac{4-a}{a+1}$

(3b) 当 $\angle PMB=\angle CME=\angle PDB$ 时，设 PM与BD相交于点 H：

可知 $\Delta PHD\sim\Delta BHM$(斜X型)，

可知：$\Delta PHB\sim\Delta DHM$(斜X型或四点 B,P,D,M 共圆)，

从而得到：$\angle BPD+\angle BMD=180^{\circ},\angle BPD=90^{\circ},P$ 与 A 重合，舍去。