DIA DAY | MES MONTH | TEMA SUBJECT

Sección 1.4 Metodo de variables separables

5) $2\sqrt{x}\,\frac{dy}{dx} = \sqrt{1-y^2}$

$dy/dx = \frac{\sqrt{1-y^2}}{2\sqrt{x}} \rightarrow dy/dx = \sqrt{1-y^2} \cdot \frac{1}{2\sqrt{x}}$

$dy = \sqrt{1-y^2} \cdot \frac{1}{2\sqrt{x}}\,dx \rightarrow \frac{dy}{\sqrt{1-y^2}} = \frac{1}{2\sqrt{x}}\,dx$

$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{1}{2\sqrt{x}}\,dx \rightarrow \arcsin y = \sqrt{x} + c$

$\operatorname{sen}^{-1} y = \sqrt{x} + c$

$y = \operatorname{sen}(\sqrt{x} + c) \rightarrow$ familia explicita de soluciones

12) $yy' = x(y^2+1)$

$y\frac{dy}{dx} = x(y^2+1) \rightarrow \frac{dy}{dx} = \frac{x(y^2+1)}{y}$

$dy = \frac{x(y^2+1)}{y}\,dx \rightarrow \int \frac{y}{y^2+1}\,dy = \int x\,dx$

$\frac{1}{2}\ln(y^2+1) = \frac{x^2}{2} + c \rightarrow \ln(y^2+1) = x^2 + c_2$

$e^{\ln(y^2+1)} = e^{x^2+c_2}$

$y^2+1 = e^{x^2} \cdot e^{c_2}$

$y^2+1 = Ae^{x^2} \rightarrow$ solución general de forma impicita

$\int \frac{y}{y^2+1}\,dy$ $\quad u = y^2+1$

$du = 2y\,dy$

$\frac{du}{2} = y\,dy$

$\int \frac{1}{y^2+1}\,y\,dy$

$\int \frac{1}{u} \cdot \frac{du}{2}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln u + c$

$\frac{1}{2}\ln(y^2+1) + c$

DIA DAY | MES MONTH | TEMA SUBJECT

Banco de Bogotá
Un Banco hecho entre dos

17) $y' = 1+x+y+xy$

$\frac{dy}{dx} = 1+x+y+xy \rightarrow \frac{dy}{dx} = (1+x)+(y+xy)$

$\frac{dy}{dx} = (1+x)+y(1+x) \rightarrow \frac{dy}{dx}\ (1+x)(1+y)$

$dy = (1+x)\cdot(1+y)\,dx \rightarrow \int \frac{dy}{1+y} = \int (1+x)\,dx$

$\ln(1+y) = x + \frac{x^2}{2} + c$

$e^{\ln(1+y)} = e^{x+\frac{1}{2}x^2+c} \rightarrow e^{\ln(1+y)} = e^{x+\frac{1}{2}x^2}\,e^{c}$

$\ln(1+y) = Ae^{\frac{1}{2}x^2+x}$

$y(x) = Ae^{\frac{1}{2}x^2+x} - 1 \rightarrow$ solución general de forma explicita

18) $x^2y' = 1-x^2+y^2-x^2y^2$

$x^2\frac{dy}{dx} = (1-x^2)+(y^2-x^2y^2)$

$x^2\frac{dy}{dx} = (1-x^2)+y^2(1-x^2)$

$x^2\frac{dy}{dx} = (1-x^2)(1+y^2) \rightarrow \frac{dy}{dx} = \frac{1-x^2}{x^2}(1+y^2)$

$dy = \frac{1-x^2}{x^2}(1+y^2)\,dx$

$\int \frac{dy}{1+y^2} = \int \frac{1-x^2}{x^2}\,dx \rightarrow \int \frac{dy}{1+y^2} = \int \frac{1}{x^2} - 1\,dx$

Scanned with CamScanner

DIA DAY | MES MONTH | TEMA SUBJECT

$\arctan y = -\frac{1}{x} - x + c \rightarrow y = \tan\left(c - x - \frac{1}{x}\right)$

✓ solución de forma explícita

25) $x\frac{dy}{dx} - y = 2x^2y, \quad y(1) = 1$

$x\frac{dy}{dx} = 2x^2y + y \rightarrow x\frac{dy}{dx} = y(2x^2+1)$

$\frac{dy}{dx} = y\frac{(2x^2+1)}{x} \rightarrow dy = y \cdot \frac{(2x^2+1)}{x}dx$

$\int\frac{dy}{y} = \int\frac{2x^2+1}{x}dx \rightarrow \int\frac{dy}{y} = \int 2x + \frac{1}{x}dx$

$\ln y = \frac{2x^2}{2} + \ln x + C$

$e^{\ln y} = e^{x^2+\ln x + C} = e^{x^2}e^{\ln x}e^{C}$

$y = Axe^{x^2}$; como el PVI establece que $y(1) = 1$

$1 = A(1)e^{(1)^2}$

$1 = Ae \rightarrow y(x) = \frac{1}{e}xe^{x^2}$

$1/e = A$

Solución general forma explícita

$y(x) = xe^{x^2-1}$ → Solución general en forma explícita al PVI $y(1) = 1$

DIA DAY | MES MONTH | TEMA SUBJECT

27) $\frac{dy}{dx} = 6e^{2x-y}$, $y(0) = 0$

$\frac{dy}{dx} = 6e^{2x} e^{-y} \rightarrow dy = 6e^{2x} e^{-y} dx$

$\frac{dy}{e^{-y}} = 6^{2x} dx \rightarrow \int e^{y} dy = \int 6e^{2x} dx$

$e^{y} = \frac{6e^{2x}}{2} + C$

$e^{y} = 3e^{2x} + C \rightarrow$ como el PVI establece que $y(0) = 0$

$e^{0} = 3e^{2(0)} + C$

$1 = 3 + C$

$-2 = C$

$e^{y} = 3e^{2x} - 2 \rightarrow \ln e^{y} = \ln(3e^{2x} - 2)$

$y(x) = \ln(3e^{2x} - 2) \rightarrow$ Solucion particular al PVI $y(0) = 0$