

# Ruimtemeetkunde

www.karelappeltans.be

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## 1 herhaling 2D

### 1.1 vectoren

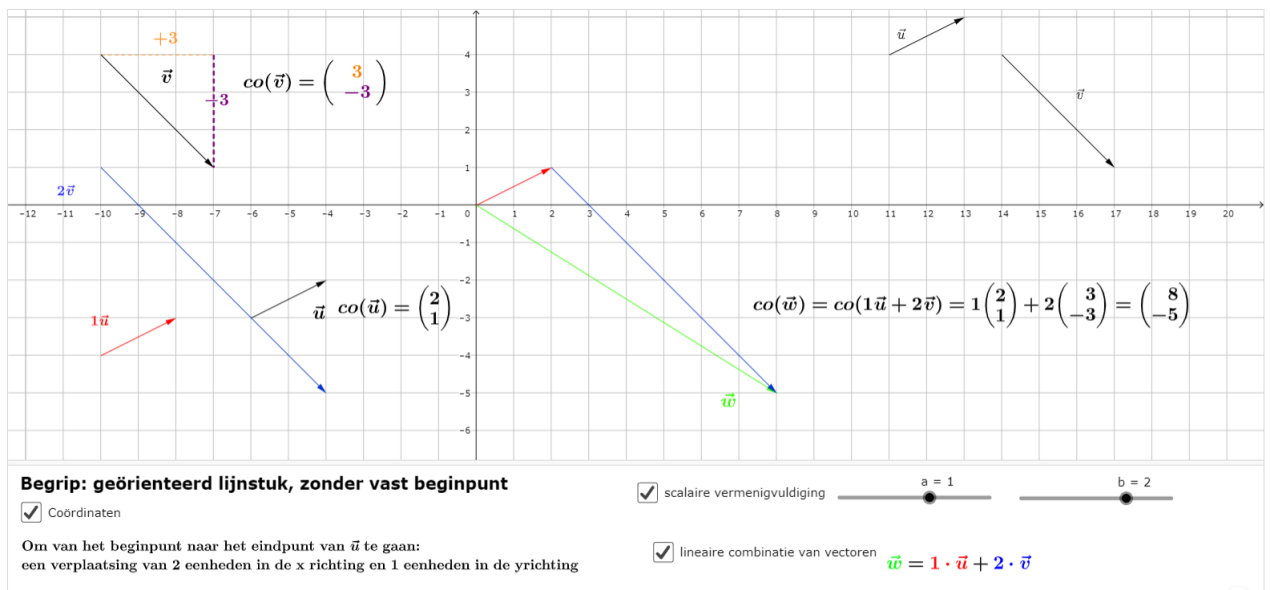


Figure 1: <https://www.geogebra.org/m/cRmm4Ez4>

### 1.2 vergelijking rechte

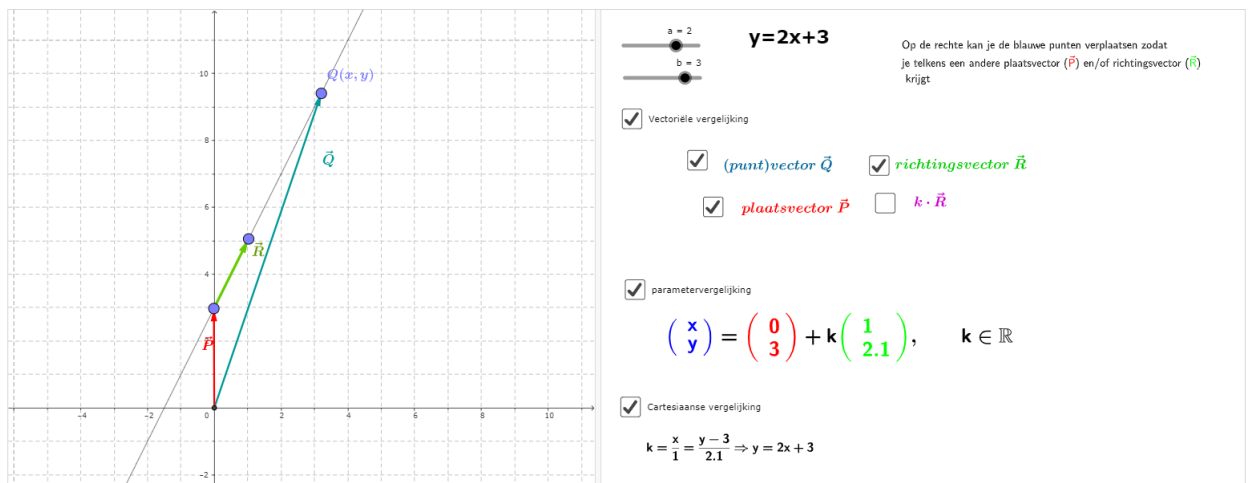


Figure 2: <https://www.geogebra.org/m/vzmgnsny>

## 2 punt

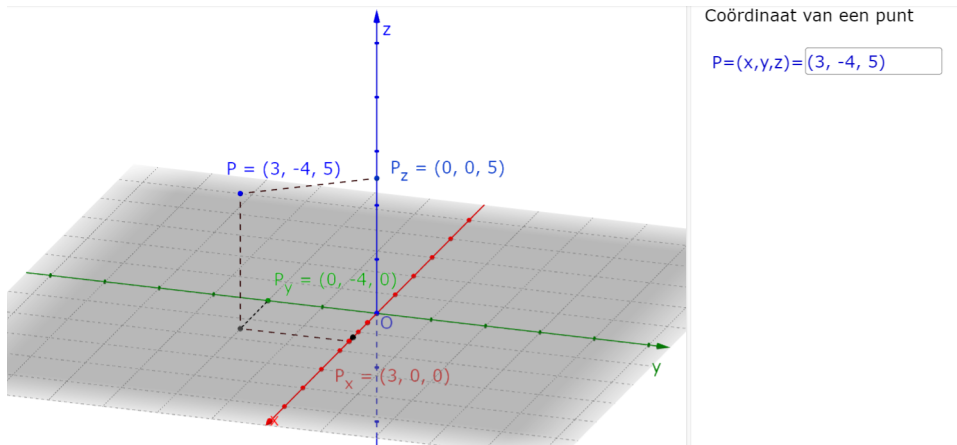


Figure 3: <https://www.geogebra.org/m/Up7HKEmg>

## 3 Vector

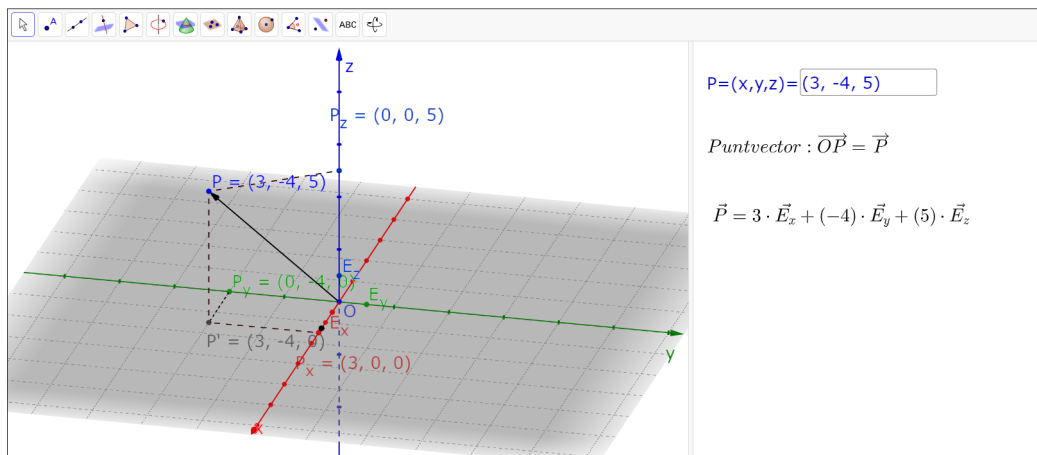


Figure 4: <https://www.geogebra.org/m/Up7HKEmg>

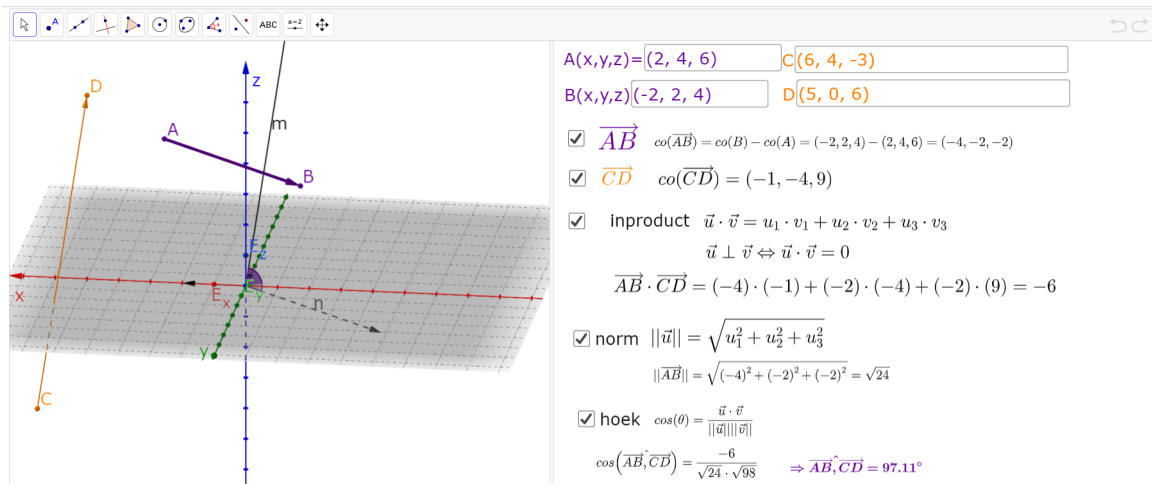


Figure 5: <https://www.geogebra.org/m/Up7HKEmg>

## 4 vectorruimte

## 5 rechte

### 5.1 begripvorming

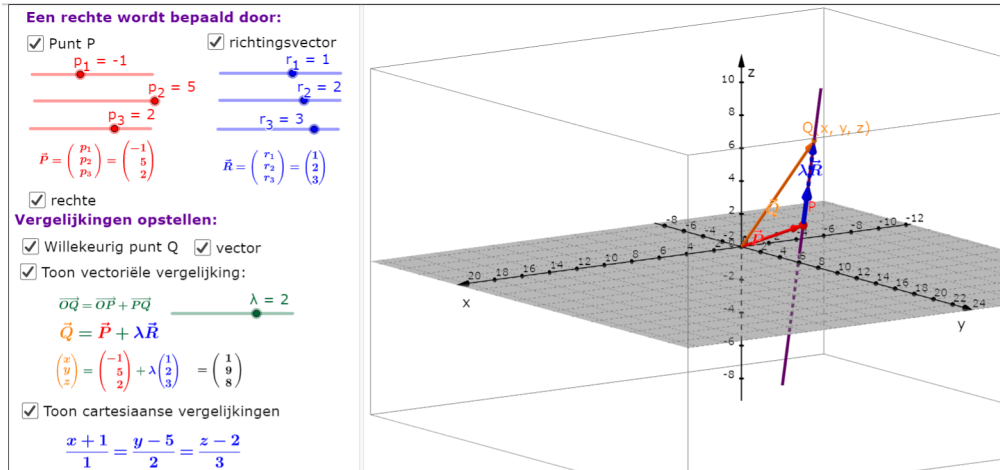


Figure 6: <https://www.geogebra.org/m/h2NSnEu4>

### 5.2 onderlinge stand twee rechten

#### 5.2.1 mogelijke gevallen

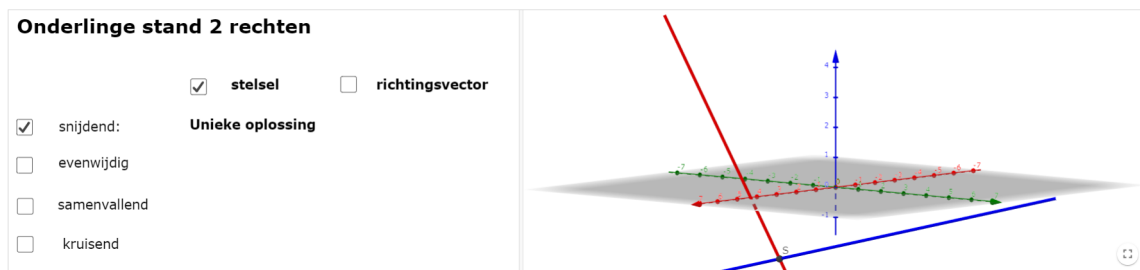


Figure 7: <https://www.geogebra.org/m/tmzxrbbh>

#### 5.2.2 snijdende rechten

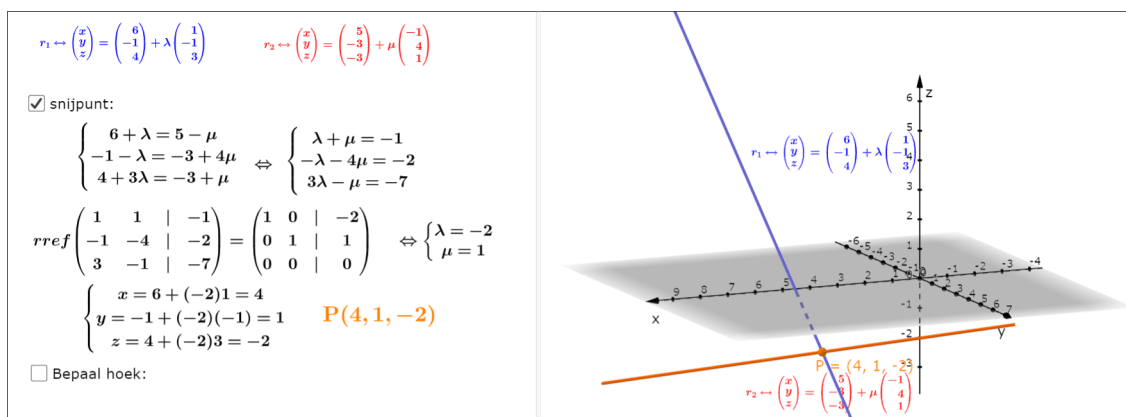


Figure 8: <https://www.geogebra.org/m/tmzxrbbh>

## 6 vlak

### 6.1 begripsvorming

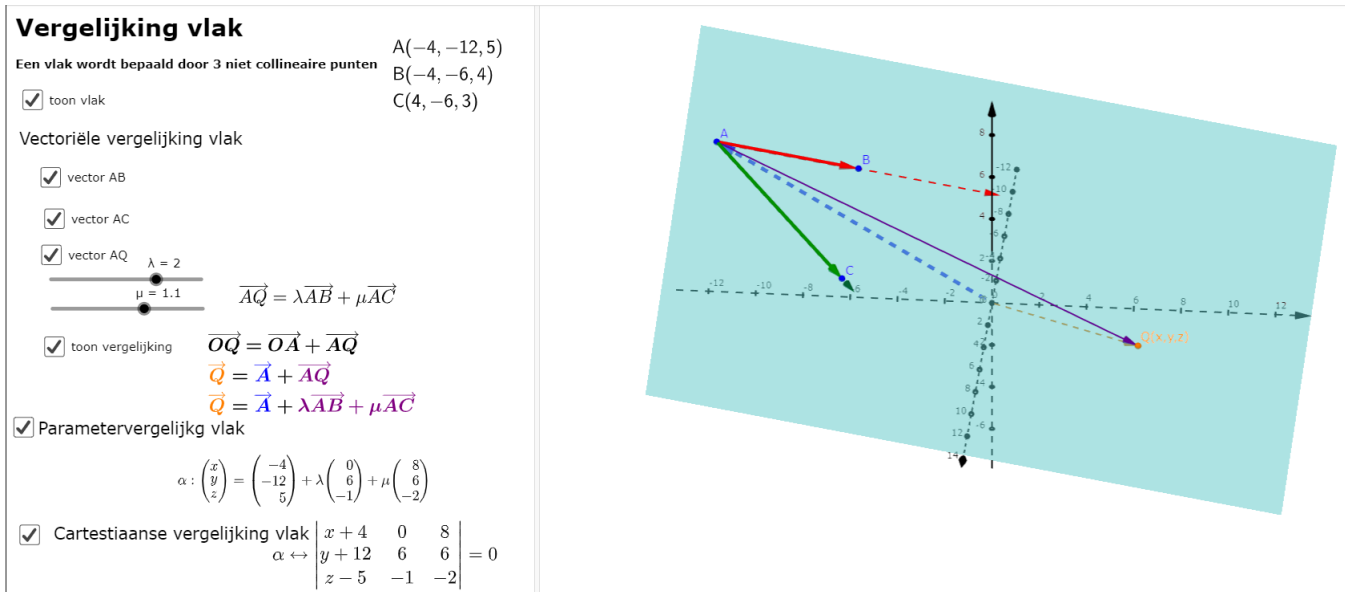


Figure 9: <https://www.geogebra.org/m/tyxzar6k>

### 6.2 onderlinge stand twee vlakken

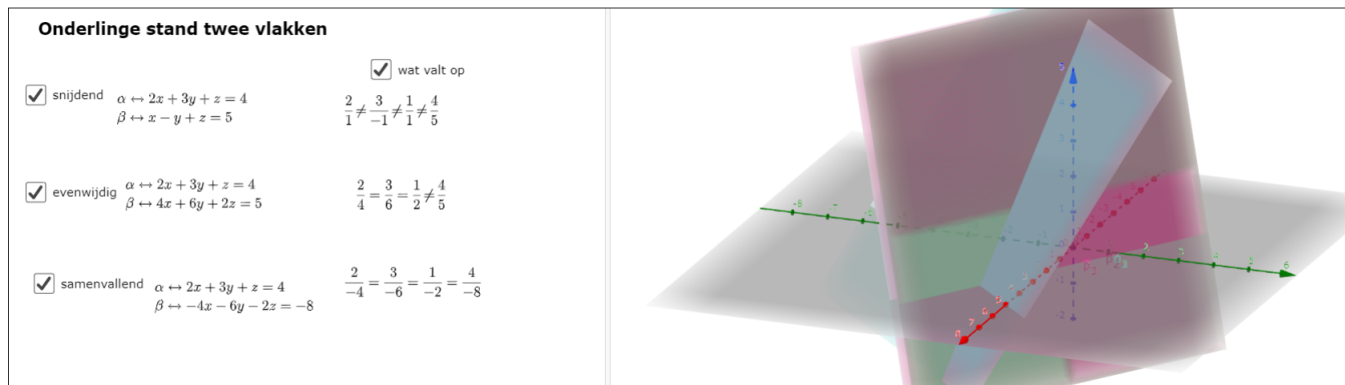


Figure 10: <https://www.geogebra.org/m/r3tjnhga>

### 6.3 vlakkenwaaier

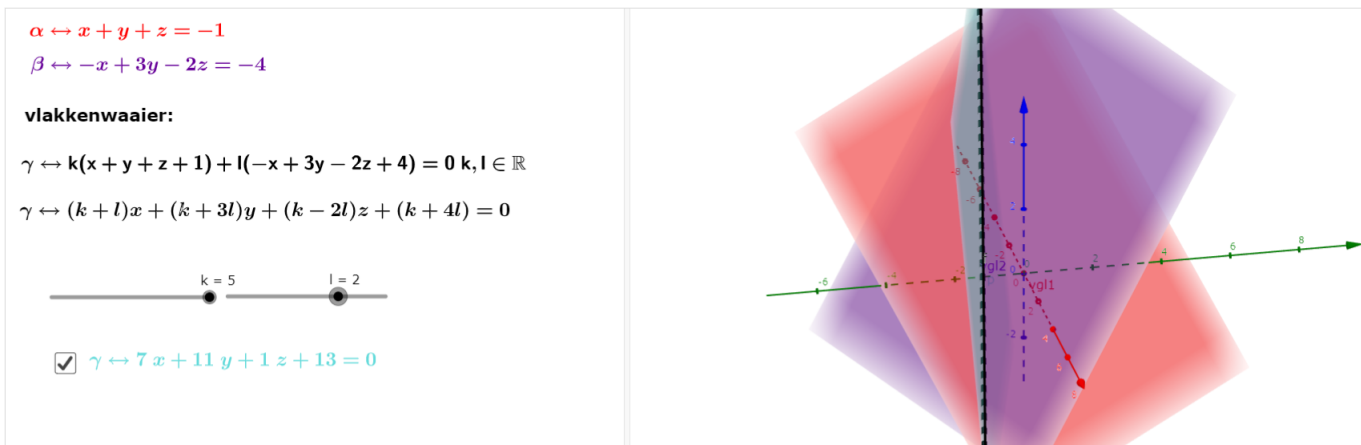


Figure 11: <https://www.geogebra.org/m/zkqecuwg>

### 6.4 onderlinge stand rechte en vlak

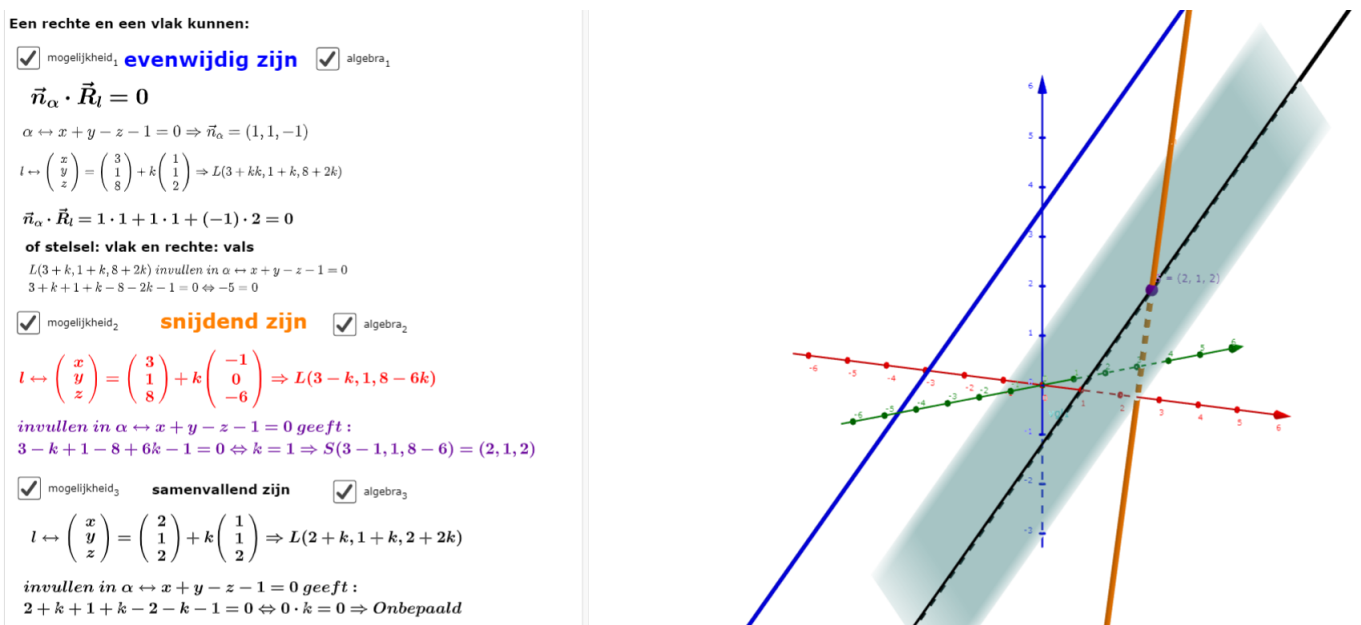


Figure 12: <https://www.geogebra.org/m/qpjszrty>

## 7 loodrechte stand

### 7.1 norm&scalair product van twee vectoren

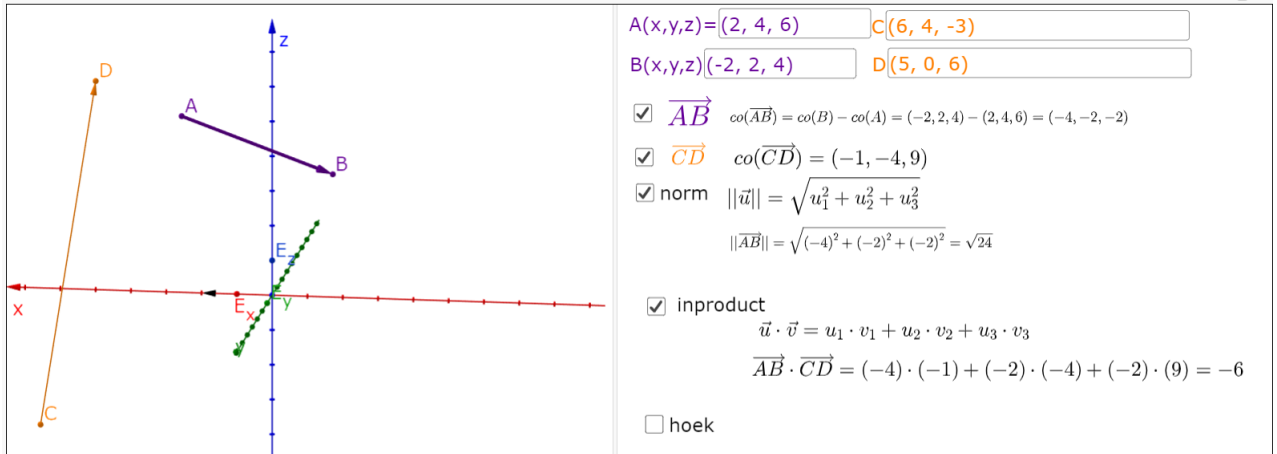


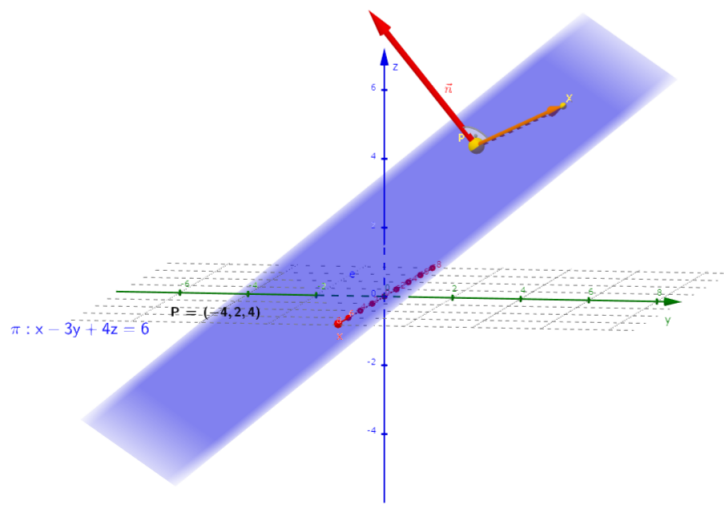
Figure 13: <https://www.geogebra.org/m/Up7HKEmg>

### 7.2 loodrechte stand van twee rechten



Figure 14: <https://www.geogebra.org/m/furru9tk>

### 7.3 normaalvectoren van vlakken



**Normaalvector van een vlak:**

$$vb : \pi \leftrightarrow 1x - 3y + 4z = 6$$

$$\vec{n}(1, -3, 4)$$

wordt de normaalvector van  $\pi$  genoemd. Dit is de vector die loodrecht op het vlak staat

m.a.w.  $\vec{n}$  staat loodrecht op elke willekeurige *vector* die in het vlak ligt

willekeurige vector in het vlak :  $\vec{PX}$

met  $P(-4, 2, 4) \in \pi$  en  $X(x, y, z)$

$$\begin{aligned} \vec{n} \cdot \vec{PX} &= 0 \\ \vec{n} \cdot (X - P) &= 0 \\ \vec{n} \cdot X &= \vec{n} \cdot P \\ \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} &= \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} \\ (1) \cdot x + (-3) \cdot y + (4) \cdot z &= 6 \quad | : (1) \\ x - 3y + 4z &= 6 \end{aligned}$$

Figure 15: <https://www.geogebra.org/m/tyxzar6k>

### 7.4 loodrechte stand van twee vlakken



**Loodrechte stand van twee vlakken**

$$\alpha \perp \beta \Leftrightarrow \vec{n}_\alpha \perp \vec{n}_\beta$$

verplaats vlakken

Figure 16: <https://www.geogebra.org/m/r3tjnhga>

## 8 afstanden

### 8.1 afstand tussen twee punten

Afstand tussen twee punten

**Afstand tussen twee punten:**

$$A = (2, 3, 1)$$

$$B = (-1, 2, 5)$$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|AB| = \sqrt{((-1) - (2))^2 + ((2) - (3))^2 + ((5) - (1))^2}$$

$$|AB| = \sqrt{(-3)^2 + (-1)^2 + (4)^2} = \sqrt{9 + 1 + 16}$$

$$|AB| = \sqrt{26}$$

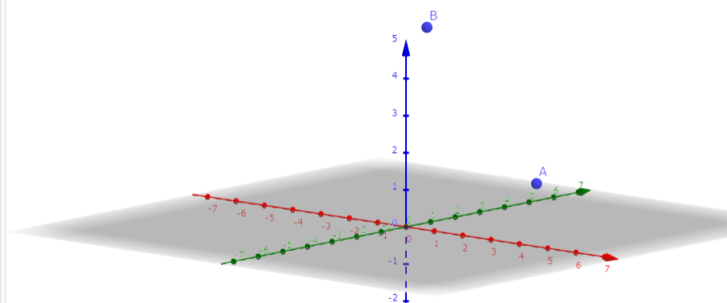


Figure 17: <https://www.geogebra.org/m/a7fx9aqv>

## 8.2 afstand tussen punt en rechte

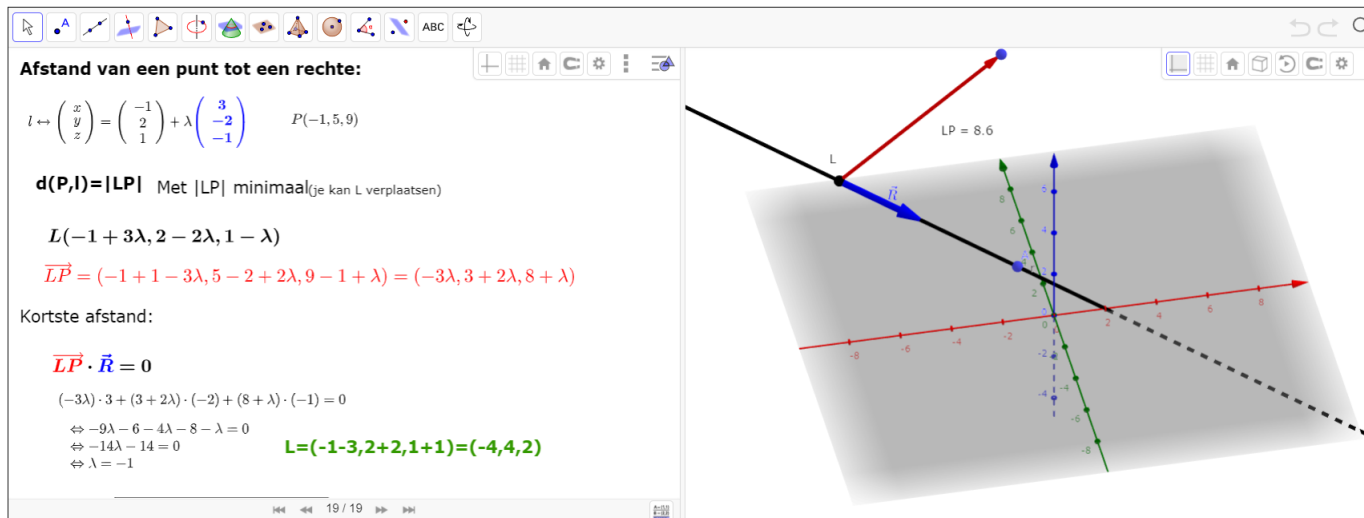


Figure 18: <https://www.geogebra.org/m/a7fx9aqv>

## 8.3 afstand tussen twee kruisende rechten

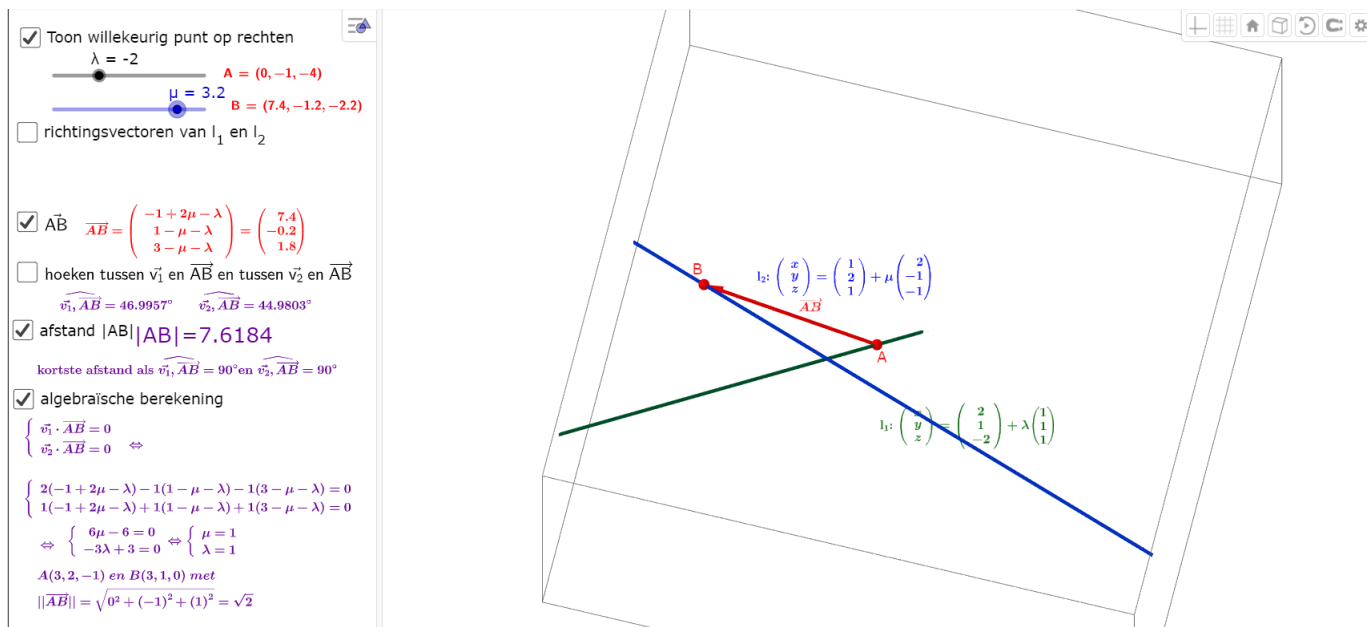


Figure 19: <https://www.geogebra.org/m/a7fx9aqv>



## 8.4 afstand tussen punt en vlak

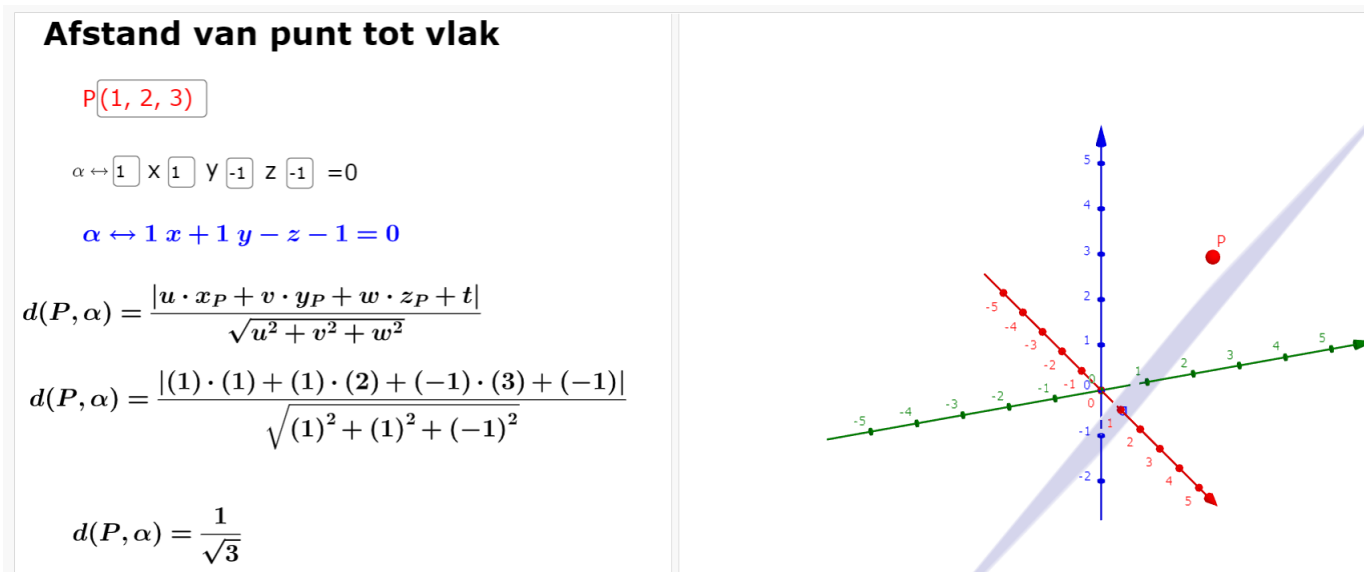


Figure 20: <https://www.geogebra.org/m/a7fx9aqv>

## 9 hoeken

### 9.1 tussen 2 rechten

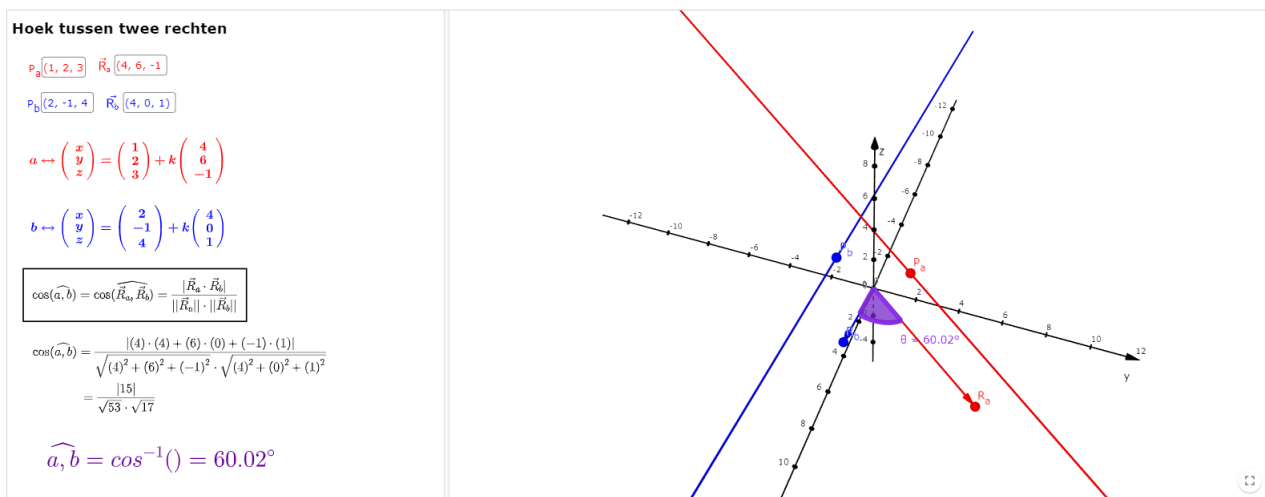


Figure 21: <https://www.geogebra.org/m/furru9tk>

## 9.2 tussen 2 snijdende vlakken

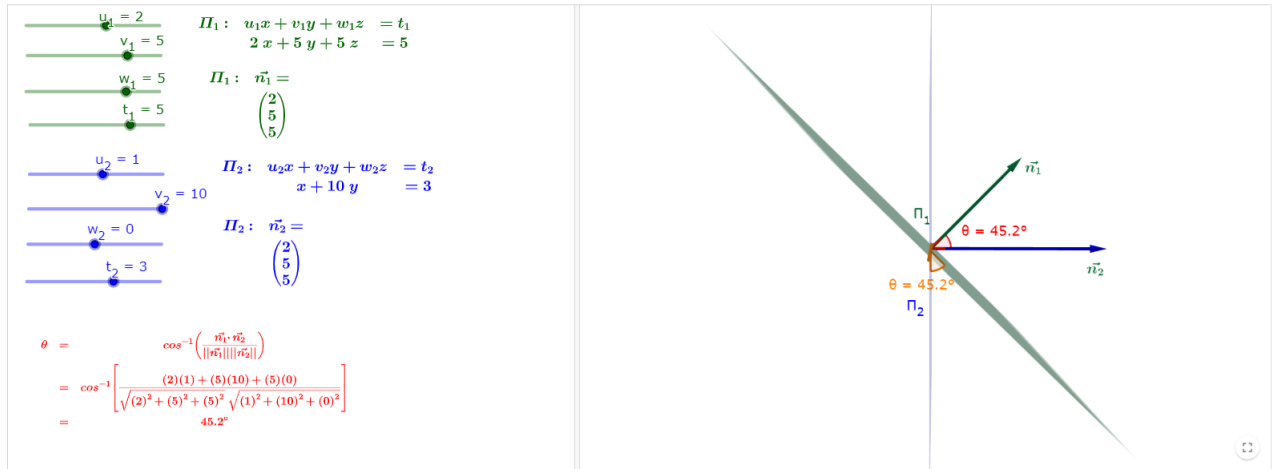


Figure 22: <https://www.geogebra.org/m/r3tjnhga>

## 9.3 tussen een rechte en een vlak

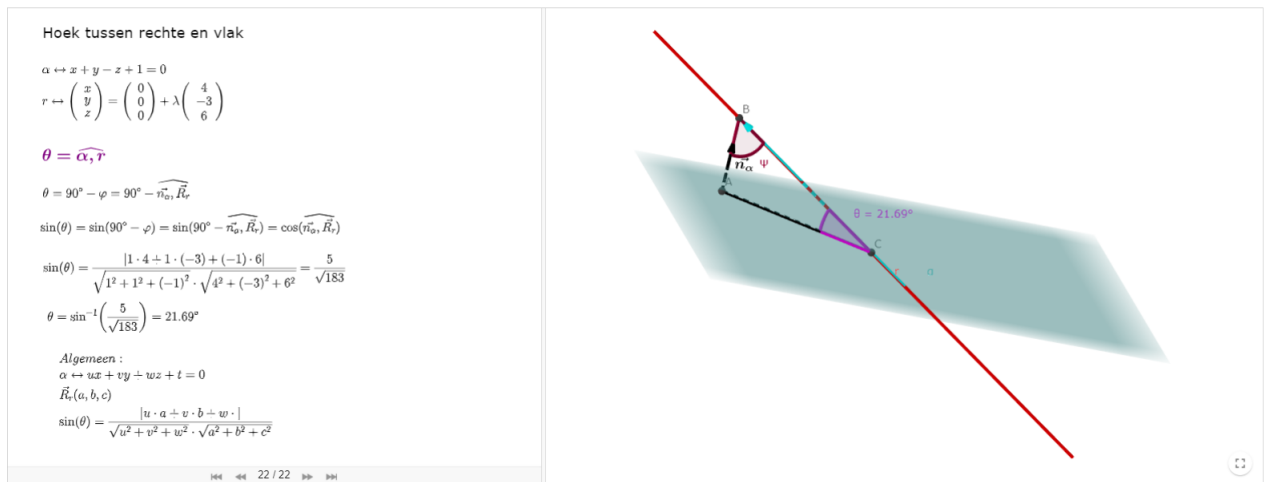


Figure 23: <https://www.geogebra.org/m/qpjszrty>

## 10 meetkundige plaatsen

### 10.1 bollen



Figure 24: <https://www.geogebra.org/m/jsffxvqd>

### 10.2 middelloodvlakken

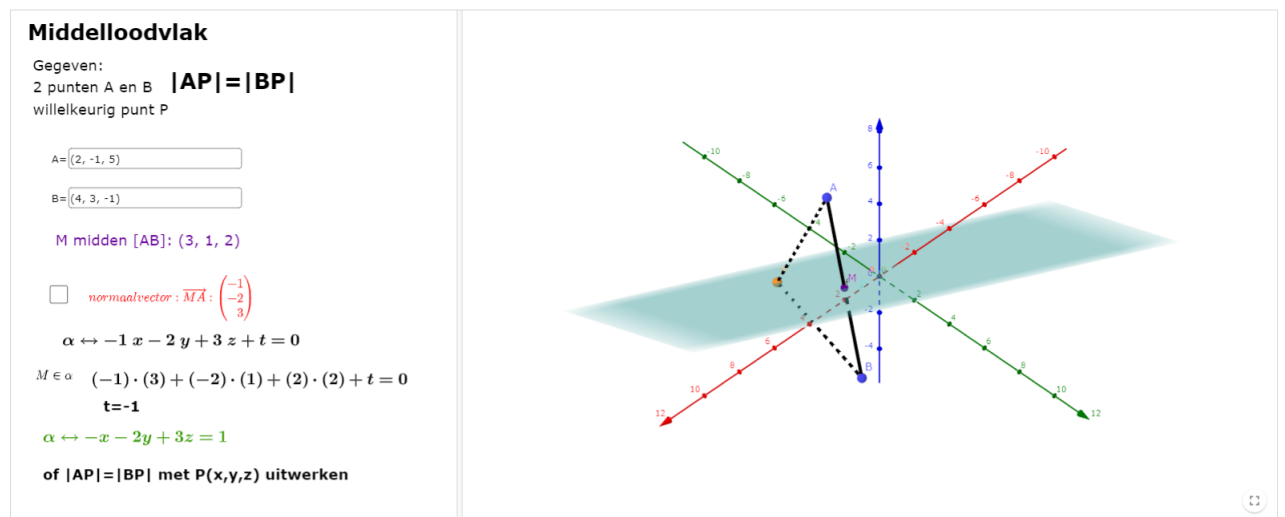


Figure 25: <https://www.geogebra.org/m/jsffxvqd>

### 10.3 bissectorvlakken

## 11 oefeningen

## 12 taken