PROJECTILE MOTION *Kinematic Equations*

We begin with the basic assumption of a constant acceleration. Using x as a generic displacement variable, we can write

$$\frac{d^2x}{dt^2} = \frac{d}{dt}v = a$$

Since we need the velocity anyway, we can just use the first-order ODE for the velocity. Integrating this,

$$v(t) = v_0 + a\ t$$

For the time-dependent displacement we then integrate again to get

$$x(t) := x_0 + v_0\ t + \frac{1}{2}\ a\ t^2$$

This is the general format. Now we make this specific to the projectile motion situation. As usual we take the horizontal to be x and the vertical is y.

The key is that the x and y motions are independent.

We also need to resolve the initial velocity into its x and y components. With the initial angle θ, measured positive counterclockwise from the local horizontal, we have

$$x(t) = x_0 + v_0\ t\cos(\theta) + \frac{1}{2}\ a_x\ t^2 \qquad y(t) = y_0 + v_0\ t\sin(\theta) + \frac{1}{2}\ a_y\ t^2$$

Usually we define the coordinate system so that the initial x is zero. The x acceleration is also zero since no force acts in that direction. The y acceleration is g, and usually y is positive upward, so the y acceleration is negative. This gives the results for projectile motion:

$$x(t) = v_0\ t\cos(\theta) \qquad y(t) = y_0 + v_0\ t\sin(\theta) - \frac{1}{2}\ g\ t^2$$

$$v_x(t) = v_0\cos(\theta) \qquad v_y(t) := v_0\sin(\theta) - g\ t$$

Note that the x-velocity is constant, and the y-velocity changes linearly with time. Conditions we can use in solving problems are: (1) the y-velocity is zero at the top of the trajectory; (2) the y coordinate is zero at the end of the flight (the coordinate system can always be selected to make this so). It is the case that the analysis is much simpler when the initial y is zero.

This is a set of parametric equations, with time as the parameter. We can of course eliminate t to find y(x), to describe the motion, and this will be the subject of another paper in this series.