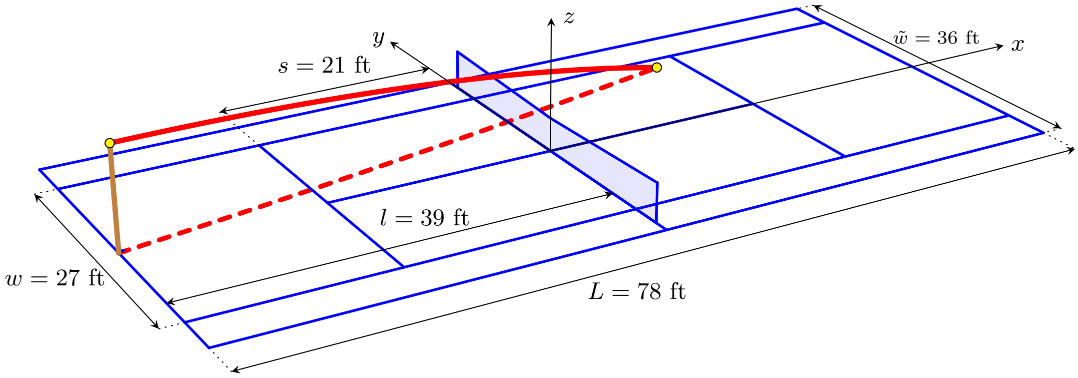
Mathematical Modeling of the Tennis Serve: Adaptive Tasks

The serve is considered one of the most difficult strokes in tennis. In this task, the aim is to find out what challenges a player faces when he opens a rally with the serve.
The dimensions of a tennis court are given in the following figure. In singles play, the net posts are 3 ft outside the sideline of the singles court. The height of the net is 3 ft in the center and 3.5 ft at the net posts.



Choose one of the following models dealing with the tennis serve. To compare the results with a numerical solution see our GeoGebra-App:

Model A

Assume that the ball leaves the serving person's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball), flies in a straight line, and lands in the correct part of the service box. For simplicity, assume that the net is the same height everywhere.
First consider the situation where the ball flies just over the net.

a) What does the path of the ball look like in the xy-plane? Make a sketch.

b) Consider the plane which is perpendicular to the xy-plane and in which the rectilinear path of the ball lies. What is the situation in this plane? Make a sketch. Also draw the net.

c) Determine the smallest possible distance of the landing point of the ball from the net. On which parameters does this distance depend?

d) From what height h must the ball be hit at least so that there is a chance that it will land in the service box?

e) What is the geometric shape of the possible impact surface for a straight flight and correct landing of the ball in the service box?

Model B

Assume that the ball leaves the serving person's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball), flies in a straight line, and lands in the correct part of the service box. Consider the variable net height.
First consider the situation where the ball flies just over the net.

a) What is the path of the ball in the xy-plane? Create a sketch.

b) Consider the plane which is perpendicular to the xy-plane and in which the rectilinear path of the ball lies. What is the situation in this plane? Draw a sketch. Also draw the net.

c) Determine the smallest possible distance of the landing point of the ball from the net. On which parameters does this distance depend?

d) What is the minimum height h from which the ball must be hit to have a chance of landing in the service box?

e) What is the geometric shape of the possible impact surface for a straight flight and correct landing of the ball in the service box? Examine the situation for different heights h. From what other parameters does the shape of the impact surface depend on?

Model C

Assume that the ball leaves the serving person's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball) and flies parabolically under the influence of gravity, landing in the right part of the service box. For simplicity, assume that the impact occurs along the center line. (*Note:* 1 ft = 0.3048 m)
The physical equations for the motion of the tennis ball are as follows:

$$\begin{aligned} \dot{\vec{s}}(t) &= \vec{v}(t) \\ \dot{\vec{v}}(t) &= \frac{1}{m}\vec{F}_G \end{aligned}$$

with

$$\vec{F}_G = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix}$$

and initial conditions

$$\begin{aligned} \vec{s}(0) &= \begin{pmatrix} -l \\ 0 \\ h \end{pmatrix} \\ \vec{v}(0) &= \begin{pmatrix} v_0 \cdot \cos(\varphi) \\ 0 \\ -v_0 \cdot \sin(\varphi) \end{pmatrix}. \end{aligned}$$

a) Describe the path of the center of the ball in the xz-system by function terms $x(t)$ and $z(t)$. Determine the corresponding function f for $z = f(x)$.

b) What serve speed v_0 is required to hit the ball from a height h such that the angle φ between $\vec{v}$ and the horizontal is 0° and the ball (center point) passes the net at a height of 3 ft above the ground?

c) At what distance from the net does the ball land on the ground at the speed calculated in b)?

d) The largest serve speed recorded so far in a tournament was 263.4 km/h. Investigate which angles φ between $\vec{v}$ and the horizontal lead to a valid valid serve.

Model D

Assume that the ball leaves the serving player's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball) and flies under the influence of gravity and air resistance and lands in the right part of the service box. For simplicity, assume that the impact occurs along the centerline. (*Note:* 1 ft = 0.3048 m, mass of tennis ball: $m = 0.0577$ kg, density of air $\rho = 1.167$ kg/m^3, radius of tennis ball $r = 0.0335$ m).
The physical equations for the motion of the tennis ball are as follows:

$$\begin{aligned} \dot{\vec{s}}(t) &= \vec{v}(t) \\ \dot{\vec{v}}(t) &= \frac{1}{m}\left(\vec{F}_G + \vec{F}_D\right) \end{aligned}$$

with

$$\vec{F}_G = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \quad \text{und} \quad \vec{F}_D = -\frac{1}{2} \cdot \rho \cdot A \cdot C_D \cdot \vec{v} \cdot |\vec{v}|$$

and initial conditions

$$\begin{aligned}\vec{s}(0) &= \begin{pmatrix}-l\\0\\h\end{pmatrix}\\ \vec{v}(0) &= \begin{pmatrix}v_0\cdot\cos(\varphi)\\0\\-v_0\cdot\sin(\varphi)\end{pmatrix}.\end{aligned}$$

$A = r^2\pi$ is the cross-sectional area of the ball, $C_D = 0.6204$ is the drag coefficient.

a) Solve the equations of motion numerically.

b) For the following serve speed velocities v_0 determine those serve angles φ between $\vec{v}$ and the horizontal line that cause the ball (center point) to fly over the net in each case: 45 m/s, 50 m/s, ..., 75 m/s.

c) For the velocities from b), determine the departure angles φ so that the landing point of the ball is on the service line.

Model E

When serving, good tennis players often impart a spin to the ball. In doing so, the ball experiences a force that is perpendicular to the velocity vector $\vec{v}$ and the rotation vector $\vec{\omega}$, the so-called Magnus force (lift-force).
Assume that the ball leaves the serving player's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball) and flies under the influence of gravity, air resistance, and magnus force and lands in the correct part of the service box. For simplicity, assume that the impact occurs along the center line.
(*Note:* 1 ft = 0.3048 m, mass of tennis ball: $m = 0.0577$ kg, density of air $\rho = 1.167$ kg/m^3, radius of tennis ball $r = 0.0335$ m).
The physical equations for the motion of the tennis ball are as follows:

$$\begin{aligned}\dot{\vec{s}}(t) &= \vec{v}(t)\\ \dot{\vec{v}}(t) &= \frac{1}{m}\left(\vec{F}_G + \vec{F}_D + \vec{F}_L\right)\end{aligned}$$

with

$$\begin{aligned}\vec{F}_G &= \begin{pmatrix}0\\0\\-mg\end{pmatrix}, \quad \vec{F}_D = -\frac{1}{2}\cdot\rho\cdot A\cdot C_D\cdot\vec{v}\cdot|\vec{v}| \quad \text{und}\\ \vec{F}_L &= \frac{1}{2}\cdot C_L\cdot\left(1-\exp\left(-\frac{|\vec{\omega}|}{\omega_0}\right)\right)\cdot A\cdot\rho\cdot\frac{|\vec{v}|}{|\vec{\omega}|}\cdot\vec{\omega}\times\vec{v}\end{aligned}$$

and initial conditions

$$\begin{aligned}\vec{s}(0) &= \begin{pmatrix}-l\\0\\h\end{pmatrix}\\ \vec{v}(0) &= \begin{pmatrix}v_0\cdot\cos(\varphi)\\0\\-v_0\cdot\sin(\varphi)\end{pmatrix}.\end{aligned}$$

$A = r^2\pi$ is the cross-sectional area of the ball, $C_D = 0.6204$ is the drag coefficient, $\omega_0 = 402.768$ rad/s and $C_L = 0.319$.

a) Write down the equations of motion component by component.

b) Solve the equations of motion for pure topspin numerically, i.e. for

$$\vec{\omega} = \begin{pmatrix}0\\\omega\\0\end{pmatrix}.$$

c) For the following takeoff velocities v_0, determine those serve-angles φ between $\vec{v}$ and the horizontal that will cause the ball (center point) to fly over the net in each case: 45 m/s, 50 m/s, ..., 75 m/s. Calculate with $\omega = 300$ rad/s.

d) For the velocities from c) determine the departure angles φ so that the landing point of the ball is on the service line.

Model F

When serving, good tennis players often impart a spin to the ball. In doing so, the ball experiences a force that is perpendicular to the velocity vector $\vec{v}$ and the rotation vector $\vec{\omega}$, the so-called Magnus force (lift-force).
Assume that the ball leaves the serving player's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball) and flies under the influence of gravity, air resistance, and magnus force and lands in the correct part of the service box. For simplicity, assume that the impact occurs along the center line. Also consider any wind speed $\vec{W}$.
(*Note:* 1 ft $= 0.3048$ m, mass of tennis ball: $m = 0.0577$ kg, density of air $\rho = 1.167$ kg/m^3, radius of tennis ball $r = 0.0335$ m).
The physical equations for the motion of the tennis ball are as follows:

$$\begin{aligned} \dot{\vec{s}}(t) &= \vec{v}(t) \\ \dot{\vec{v}}(t) &= \frac{1}{m}\left(\vec{F}_G + \vec{F}_D + \vec{F}_L\right) \end{aligned}$$

with

$$\begin{aligned} \vec{F}_G &= \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix}, \quad \vec{F}_D = -\frac{1}{2}\cdot\rho\cdot A\cdot C_D\cdot\left(\vec{v}-\vec{W}\right)\cdot\left|\vec{v}-\vec{W}\right| \quad \text{und} \\ \vec{F}_L &= \frac{1}{2}\cdot C_L\cdot\left(1-\exp\left(-\frac{|\vec{\omega}|}{\omega_0}\right)\right)\cdot A\cdot\rho\cdot\frac{\left|\vec{v}-\vec{W}\right|}{|\vec{\omega}|}\cdot\vec{\omega}\times\left(\vec{v}-\vec{W}\right) \end{aligned}$$

and initial conditions

$$\begin{aligned} \vec{s}(0) &= \begin{pmatrix} -l \\ 0 \\ h \end{pmatrix} \\ \vec{v}(0) &= \begin{pmatrix} v_0\cdot\cos(\varphi) \\ 0 \\ -v_0\cdot\sin(\varphi) \end{pmatrix}. \end{aligned}$$

$A = r^2\pi$ is the cross-sectional area of the ball, $C_D = 0.6204$ is the drag coefficient, $\omega_0 = 402.768$ rad/s and $C_L = 0.319$.

a) Write down the equations of motion component by component.

b) For the following serve speeds v_0, determine the serve-angles φ between $\vec{v}$ and the horizontal that cause the ball (center point) to fly over the net: 45 m/s, 50 m/s, ..., 75 m/s. Calculate with sidespin of $\omega = 300$ rad/s and 2 m/s tail wind, i.e.

$$\vec{\omega} = \begin{pmatrix} 0 \\ 0 \\ 300 \end{pmatrix} \quad \text{and} \quad \vec{W} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.$$

c) For the velocities from b), determine the departure angles φ, so that the landing point of the ball is on the service line.

Model G

When serving, good tennis players often give the ball a spin. In doing so, the ball experiences a force that is perpendicular to the velocity vector $\vec{v}$ and the rotation vector $\vec{\omega}$, the so-called Magnus force (lift-force). Assume that the ball leaves the serving player's racket at height $h = 9.25$ ft above the baseline (relative to the center of the ball) and flies under the influence of gravity, air resistance, and Magnus-force and lands in the correct part of the service box. For simplicity, assume that the impact occurs along the center line.
(*Note:* 1 ft $= 0.3048$ m, mass of tennis ball: $m = 0.0577$ kg, density of air $\rho = 1.167$ kg/m^3, radius of tennis ball $r = 0.0335$ m).
The physical equations for the motion of the tennis ball are as follows:

$$\begin{aligned} \dot{\vec{s}}(t) &= \vec{v}(t) \\ \dot{\vec{v}}(t) &= \frac{1}{m}\left(\vec{F}_G + \vec{F}_D + \vec{F}_L\right) \end{aligned}$$

with

$$\begin{aligned} \vec{F}_G &= \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix}, \quad \vec{F}_D = -\frac{1}{2}\cdot\rho\cdot A\cdot C_D\cdot\vec{v}\cdot|\vec{v}| \quad \text{und} \\ \vec{F}_L &= \frac{1}{2}\cdot C_L\cdot\left(1-\exp\left(-\frac{|\vec{\omega}|}{\omega_0}\right)\right)\cdot A\cdot\rho\cdot\frac{|\vec{v}|}{|\vec{\omega}|}\cdot\vec{\omega}\times\vec{v} \end{aligned}$$

and initial conditions

$$\begin{aligned} \vec{s}(0) &= \begin{pmatrix} -l \\ 0 \\ h \end{pmatrix} \\ \vec{v}(0) &= \begin{pmatrix} v_0\cdot\cos(\varphi) \\ 0 \\ -v_0\cdot\sin(\varphi) \end{pmatrix}. \end{aligned}$$

$A = r^2\pi$ is the cross-sectional area of the ball, $C_D = 0.6204$ is the drag coefficient, $\omega_0 = 402.768$ rad/s and $C_L = 0.319$.

a) Write down the equations of motion component by component.

b) For the serve speed $v_0 = 160$ km/h, determine the serve-angles φ between $\vec{v}$ and the horizontal that cause the ball (center point) to fly over the net. Calculate with the following spin-vectors:

$$\vec{\omega}_1 = \begin{pmatrix} 0 \\ 600 \\ 0 \end{pmatrix} \quad \vec{\omega}_2 = \begin{pmatrix} 0 \\ 580 \\ 155 \end{pmatrix} \quad \vec{\omega}_3 = \begin{pmatrix} 0 \\ 520 \\ 300 \end{pmatrix} \quad \vec{\omega}_4 = \begin{pmatrix} 0 \\ 424 \\ 424 \end{pmatrix}$$

$$\vec{\omega}_5 = \begin{pmatrix} 0 \\ 300 \\ 520 \end{pmatrix} \quad \vec{\omega}_6 = \begin{pmatrix} 0 \\ 155 \\ 580 \end{pmatrix} \quad \vec{\omega}_7 = \begin{pmatrix} 0 \\ 0 \\ 600 \end{pmatrix}$$

c) For the spin vectors from b), determine the serve-angles φ so that the landing point of the ball is on the service line.