Construction of Points of Concurrency using GeoGebra

Overview:
Students will use the free GeoGebra dynamic geometry software to construct two of the points of concurrency of a triangle; the Circumcenter and the Incenter.

Connections to the Common Core:

G-CO.12 Make formal geometric construction with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Standards for Mathematical Practice:
1. Reason abstractly and quantitatively
2. Model with mathematics
3. Use appropriate tools strategically
4. Look for and make use of structure

NCTM Geometry Standards:
1. Analyze characteristics and properties of two- and three- geometric shapes and develop mathematical arguments about geometric relationships:
   Expectation: Analyze properties and determine attributes of two- and three-dimensional objects
2. Use Visualization, spatial reasoning, and geometric modeling to solve problems
   Expectation: draw and construct representations of two- and three-dimensional geometric objects using a variety of tools

Objectives:
1. The students will be able to construct the perpendicular bisector of a line segment and the angle bisector of an angle.
2. The students will be able to construct two points of concurrence; the Circumcenter and the Incenter.
3. The students will be able to construct the inscribed and circumscribed circles of a triangle.
4. Students will be able to conjecture about the Circumcenter and the Incenter as well as the circles and the triangle involved with these points of concurrence.

Prior Knowledge:
1. Students should be able to construct a perpendicular bisector and an angle bisector using a straight edge and compass.
2. Students should know that each triangle has three perpendicular bisectors (one per side) and three angle bisectors (one per vertex).
3. Knowledge of using GeoGebra is not necessary.

Materials:
1. GeoGebra is free software and can be downloaded at www.geogebra.org.
2. The class will need use of a computer lab to complete the assignment.
3. Every student (or group of students if you decide to group them) will need a copy of the worksheet provided at the end of this lesson.

Lesson:
1. Have each student work through the worksheet independently or if preferred, they can work in small groups (this may be necessary if the number of computers is limited).
2. There is a small problem set after each of the constructions. The questions are limited and there is room to add more.
3. Have the students submit both the answers to the questions and their completed GeoGebra file as an assessment.

Construction of the Circumcenter

The Circumcenter is the point of concurrence of the three perpendicular bisectors of a triangle. It is the center of a circle that circumscribes the triangle (goes through all three vertices of the triangle).

1. Open up a new GeoGebra file
2. Go to view→axis, click to turn off the axis
3. Select the polygon tool (5th button from the left)
    a. Notice the directions that are given to the right of the last button; *"Select all vertices, then click first vertex again"*
    b. Create a triangle by clicking three times to add points A, B and C and then finish by clicking on the original point A
    c. You can move any of these three points by selecting the move tool (arrow)

Notice that each of these three points is independent (colored blue)

4. We will first start by constructing the perpendicular bisector for side AB
    a. Click on the down arrow for the circle tool (6th button from the left), a drop-down menu will appear
    b. Select the Compass tool
        i. Notice the directions; *"Select segment or two points for radius, then center point"*
    c. Click on segment AB (the compass is now set to this length) and then click on point A
        i. A circle of radius AB, centered at A appears
    d. Click on segment AB again and then click on point B
    e. You should now have two circles of the same radius; one centered at A and one at B
    f. We need to determine the points of intersection between these two circles
    g. Click on the down arrow for the point tool (2nd button from the left)
    h. Select the Intersect Two Objects tool
        i. Notice the directions; *"Select two objects or click directly on intersections"*
    i. Click on one of the circles and then the other
        i. Two points of intersection should appear
    j. The perpendicular bisector of side AB is the line through these two point of intersection
    k. Select the Line tool (3rd button from the left)
        i. Notice the directions; *"Select two points"*
    l. Click on the two points of intersection
        i. A line through these two points should appear

Notice that these 2 points are dependent (colored black) and cannot be moved independently (if you click on ofe of the points and try to drag it, it doesn't move)

    m. We now have one of the three perpendicular bisectors
    n. Our drawing has many things cluttering up the workspace; we are going to hide some of them
        i. In the menu to the left, each object has a bubble next to it
        ii. If the bubble is filled in, the object is showing
        iii. Clicking on a filled in bubble will hide the item and change the bubble to unfilled
        iv. We need to hide the two circles we created and their two points of intersection
    o. After you have finished hiding those four items, create the perpendicular bisectors for sides BC and AC
    p. Make sure to hide the circles and points of intersection after you have finished
    q. You should now have the original triangle and three perpendicular bisectors
    r. Select the Intersect Two Objects tool
    s. Click on two of your perpendicular bisectors
    t. Hide the three perpendicular bisectors
    u. Select the Circle with Center and Point tool
        i. Notice the directions; *"Select center point, then select point on the circle"*
    v. Click the point of intersection of the perpendicular bisectors and then vertex A

        i. A circle passing through the three vertices of the triangle should appear

5. We are now going to create a show hide button for the Circumcenter
    a. Select the Check Box for Show / Hide Objects tool (2nd button from right)
    b. Notice the directions; “Click on the Graphics View to specify position
        i. This is saying to click the location where you want the checkbox to appear (you can move it later if you need)
    c. After you click where you want your checkbox, a menu appears
    d. Under Caption type the name you want for the checkbox; *“Show / Hide Circumcenter”*
    e. There are a couple ways to select the objects you want for your Show / Hide checkbox
        i. You can click the actual object(s) that you want to hide; click on the Circumcenter and on the circle through the three vertices, OR
        ii. You can click on the objects equation in the algebra view menu to the left; click on the Circumcenter and on the circle through the three vertices, OR
        iii. From the drop-down menu, select the Circumcenter and then select the circle through the three vertices (each object will appear in the space below the drop-down menu after selected)
    f. If you accidentally select the wrong object, you can remove it by highlighting it and clicking on the *x*.
        i. Click Apply
    g. You can now show or hide your circle and Circumcenter by clicking on your checkbox
6. Questions:
    a. Do the three perpendicular bisectors always intersect at a single point?
    b. The point of concurrence (the Circumcenter) can be inside the triangle, on the triangle or outside the triangle. Which type of triangle (acute, right or obtuse) produces which result?

You can move any of the three vertices and the circle changes size, but still passes through all three vertices!

Construction of the Incenter

The Incenter is the point of concurrence of the three angle bisectors of a triangle. It is the center of a circle that inscribes the triangle (the three sides of the triangle are tangent to the circle).

1. We are going to continue using the file from above where we constructed a Circumcenter
2. Use your checkbox to hide the Circumcenter and the circle through the three vertices
3. We will start by constructing the angle bisector for point A
   a. Select the Point tool
   b. Create a point on side AB
      i. Lets rename this point
         1. Right-click on the point and select *rename*
         2. Call the point K
   c. Select the Circle with Center through Point tool
   d. Click on point A (the center) and the point K (the point you just created)
      i. A circle with center A and through K should appear
   e. Select the Intersect Two Objects tool
      i. Notice that the directions tell you that you can just click the point of intersection instead of picking both objects.
      ii. Click on the point of intersection between the circle and side AC
         1. A point of intersection should appear
         2. Lets rename this point
            a. Right-click on the point and select *rename*
            b. Call the point L

> Point L and point K are equidistant from point A. The angle bisector for point A is the set of points that is equidistant from L and K.

   f. Hide the circle with center A through K
   g. Select the Circle with Center through Point tool
      i. Create a circle with center K through A
      ii. Create a circle with center L through A
   h. Select the Intersect Two Objects tool
      i. Click on the point of intersection of these two circles
      ii. You should have a point of intersection between these two circles (point A is also a point of intersection with these two circles)
      iii. Name the point of intersection M
   i. Select the Line through Two Points tool
      i. Create the line through point A and point M
   j. This line is the angle bisector for angle A
   k. Hide the circles and the points of intersection (all you should have is the triangle and the angle bisector)
      i. Create the other two angle bisectors
   l. Select the Intersect Two Objects tool
      i. Click on two of your angle bisectors
      ii. The point of concurrence (the Incenter) should appear
   m. Hide your angle bisectors
4. We need to find the distance from this point of concurrence (the Incenter) to one of the sides of the triangle and use this for the radius of the inscribed circle (with the Incenter as its center)
   a. Select the Perpendicular Line tool (4^{th} button from the left)
      i. Notice the directions; “Select point and perpendicular line”
      ii. Click on the Incenter and one of the sides of the triangle

iii. A line perpendicular to this side should appear

b. Select the Intersect Two Objects tool

c. Create the point of intersection between the side and the perpendicular line

5. Select the Circle with Center through Point tool

   a. Click on the Incenter and the point of intersection you just found

   b. A circle tangent to the sides of the triangle should appear

   c. Hide the perpendicular line and the point of intersection

   d. Only the Incenter, the inscribed circle and the triangle should be visible

6. Create a Show / Hide Checkbox for the Incenter

7. Questions:

   a. Can the point of concurrence (the Incenter) be outside the triangle? Can it be on the triangle?

   b. If you show both the Incenter and the Circumcenter, what type triangle is necessary for them to be the same point?

   c. What type of triangle is necessary for the points of concurrence and one of the vertices of the triangle to be collinear?

You can move any of the three vertices and the circle changes size, but is still tangent to all three sides