Setting R to be the radius of our circle O and r to be the radius of the required circle, we get the following equation (Pythagorean theorem).

$$r^{2} + (R - h + r)^{2} = (R - r)^{2}$$
  

$$r^{2} + R^{2} - Rh + Rr - Rh + h^{2} - rh + Rr - rh + r^{2} = R^{2} - 2Rr + r^{2}$$

After we simplify and move some things around, we get

$$r^2 + rRr - 2rh = 2Rh - h^2$$

This is as far as I could follow the proof solution for. As I progressed to the next step, I did not see how it continued to equal the original equation.