Setting $R$ to be the radius of our circle $O$ and $r$ to be the radius of the required circle, we get the following equation (Pythagorean theorem).

$$
\begin{aligned}
& r^{2}+(R-h+r)^{2}=(R-r)^{2} \\
& r^{2}+R^{2}-R h+R r-R h+h^{2}-r h+R r-r h+r^{2}=R^{2}-2 R r+r^{2}
\end{aligned}
$$

After we simplify and move some things around, we get

$$
r^{2}+r R r-2 r h=2 R h-h^{2}
$$

This is as far as I could follow the proof solution for. As I progressed to the next step, I did not see how it continued to equal the original equation.

