GeoGebra Tips and Tricks

GeoGebra is an extremely versatile free dynamic mathematics software package which combines elements of geometry software, computer algebra systems and spreadsheets. In this presentation we will explore some tips and tricks for working with GeoGebra including dynamic text, color systems, working with images, spreadsheets, fine tuning animations, the sequence command, the locus command, exporting to GeoGebraTube and more. After attending this session, participants will be better able to use GeoGebra in their classes.

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Introduction

GeoGebra was created by Markus Hohenwarter who began working on the project part of his Master's thesis in Computer Science and Mathematics Education at the University of Salzburg (Austria) in 2002.   He designed the software to combine features of dynamic geometry software with features of computer algebra systems.   Dr. Hohenwarter continued his project at Florida Atlantic University (2006–2008), Florida State University (2008–2009), and now at the University of Linz together with the help of open-source developers and translators all over the world.

Tip 1 – Use the input bar

Many tasks in GeoGebra can be accomplished using either the toolbars or the input bar.

Task	Tool	Input bar
Create a point		A = (1,1)
Create a circle		x^2 + y^2 = 1
Create a text field	ABC	"Hello world"

The Input bar can be used to add objects and perform calculations which are beyond the capabilities of the toolbars.

Task	Input bar
Create a line with sliders	y = m x + b
Graph a function	y = sin(x)
Graph the derivative of the function	Derivative[f]

Tip 2 – Create dynamic text

Dynamic geometry is awesome, but dynamic text can also be helpful in some demonstrations.

Example: Invariance of the trigonometric ratios. For this demonstration, we need a right triangle with an adjustable acute angle which can be easily resized. Since none of the polygon tools provide this, we construct our own triangle using the following directions.

1. Create to points to form the base of the triangle using the following commands. Point A is attached to the origin and point B is attached to the *x*-axis. Pull point B off the origin.
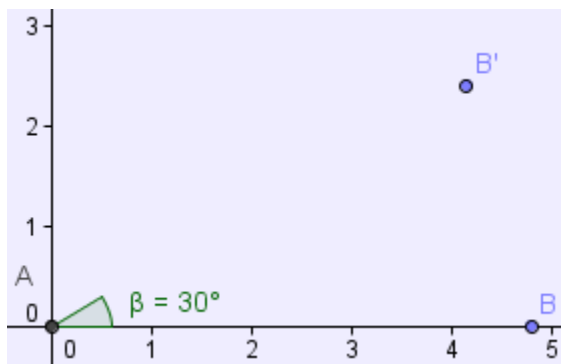
```
A = Intersect[xAxis, yAxis]
B = Point[xAxis]
```

2. Create a slider α for the acute angle with a range from 0° to 90° with an increment of 1°.

3. Use the Angle with Given Size tool to create the angle BAB' equal to α. To use this tool, select the leg point B, the vertex A and enter the angle α.
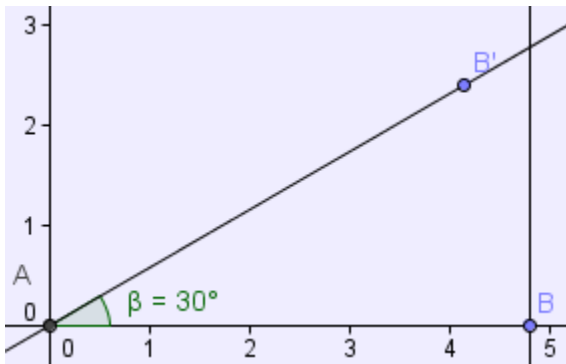
4. Use the line tool to create a line passing through points A and B'.

5. Use the perpendicular line tool to create a line passing through point B and perpendicular to the *x*-axis.

6. Use the intersect tool to find the intersection of the previous two lines and thus complete the triangle.

7. Hide the lines and point B'. Then use the line segment tool to create line segments between points A and B, points B and C and points C and A. Then hide the points A and C and turn off the axes.
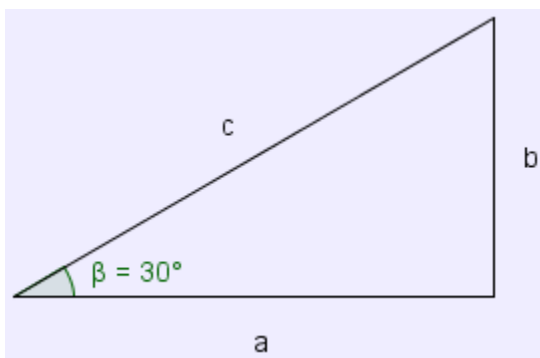
8. Now we are ready to add the dynamic text. Use Latex commands to create the mathematical notation and use the objects drop-down menu to select the sides of the triangle.
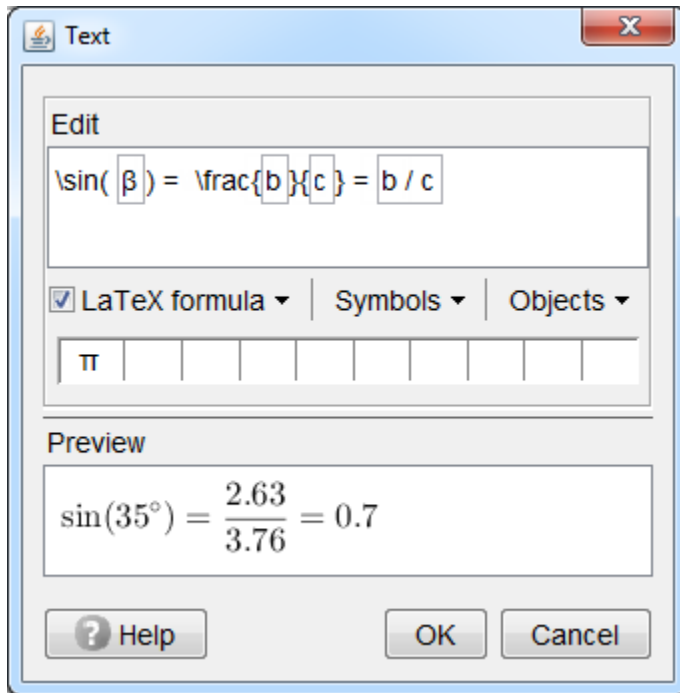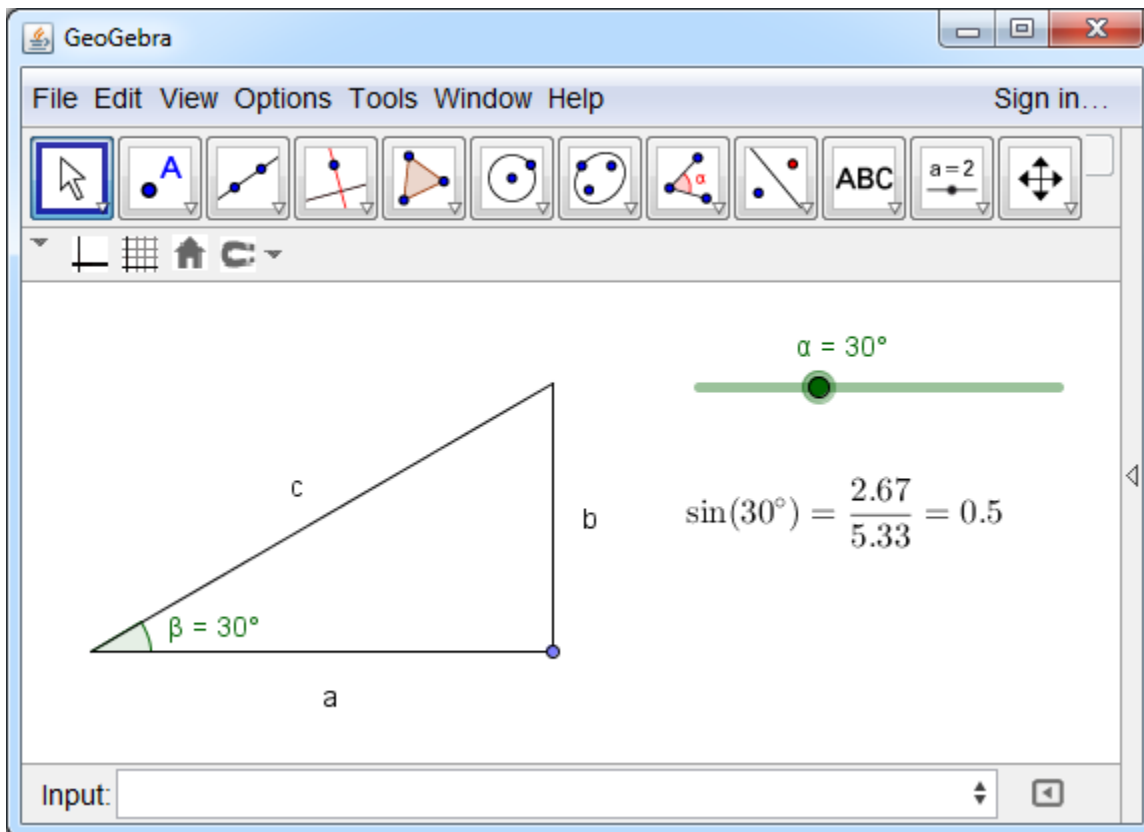
Figure 1: GeoGebra applet for demonstrating the invariance of the trigonometric ratios. Use point B to change the size of the triangle and the slider to adjust the angle.

Tip 3 – Use color

GeoGebra supports three different color systems.

- **RGB color model.** The RGB color model is an additive color model in which red, green and blue light are added together to produce a wide range of colors. This model is used for displaying colors in television and computer screens. Colors in this model are represented using ordered triples of the form (R, G, B).

- **HSV color model.** Hue refers to the gradation of color within the visible spectrum of light and is determined by the dominant wavelength. Saturation indicates the degrees to which the hue differs from a neutral gray. Its values range from gray at 0% to intense color at 100%. Value is the brightness of the color where 0% means black and 100% means full brightness. Graphic designers prefer the HSV color model since it is more intuitive than the RGB model.

- **HSL color model.** Hue is the same as in the HSV model. Saturation is the purity or intensity of a color and ranges from gray at 0% to intense color at 100%. The transition from black to a hue to white is symmetric and controlled by the luminance parameter. Luminance ranges from black at 0%, to full color at 50% to white at 100%. This is different from the value parameter in the HSV model which reaches full brightness at 100%.
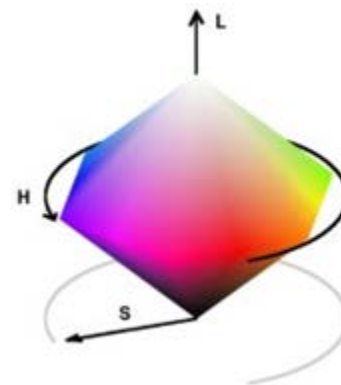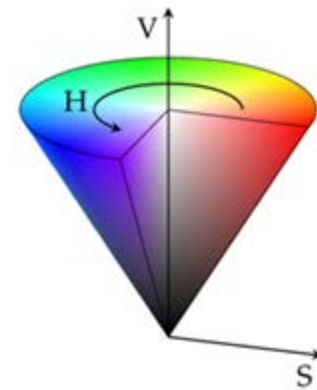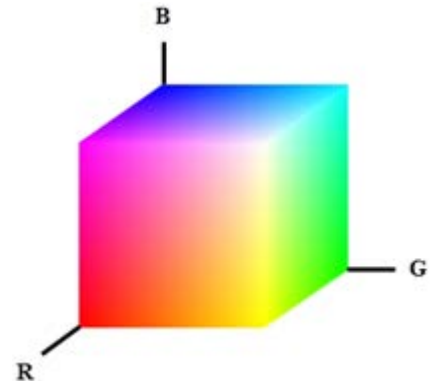
Example: A hypotrochoid is traced by a point P attached to a circle of radius b rolling around the inside of a circle of radius a, where P is a distance h from the center of the interior circle. The parametric equations for the hypotrochoid are

$$\begin{cases} x = (a-b)\cos(t) + h\cos\left(\dfrac{a-b}{b}t\right) \\ y = (a-b)\sin(t) - h\sin\left(\dfrac{a-b}{b}t\right) \end{cases}$$

To create a hypotrochoid in GeoGebra first create sliders for the parameters a, b, h and t. I chose the interval [0, 5] with an increment of 0.10 for parameters a, b and h and the interval [0, 24π] with an increment of π/200 for the variable t. Other choices are possible. Then enter the parametric equations in the input bar.

```
f = (a - b) cos(t) + h cos((a - b) t / b)
g = (a - b) sin(t) - h sin((a - b) t / b)
```
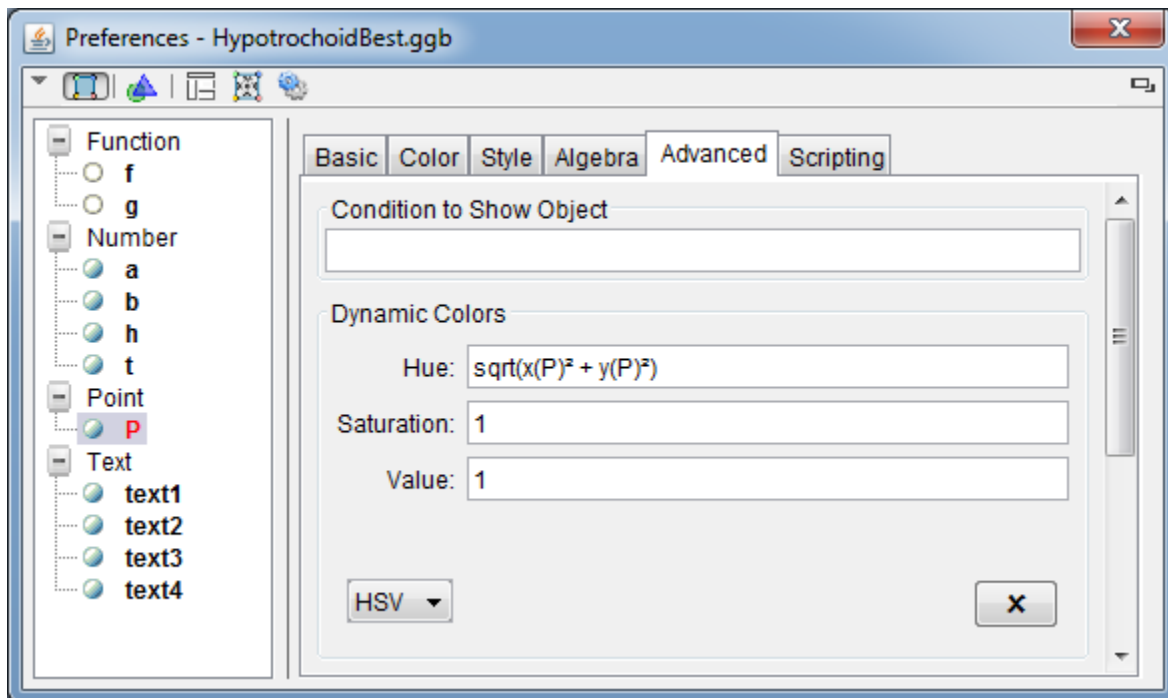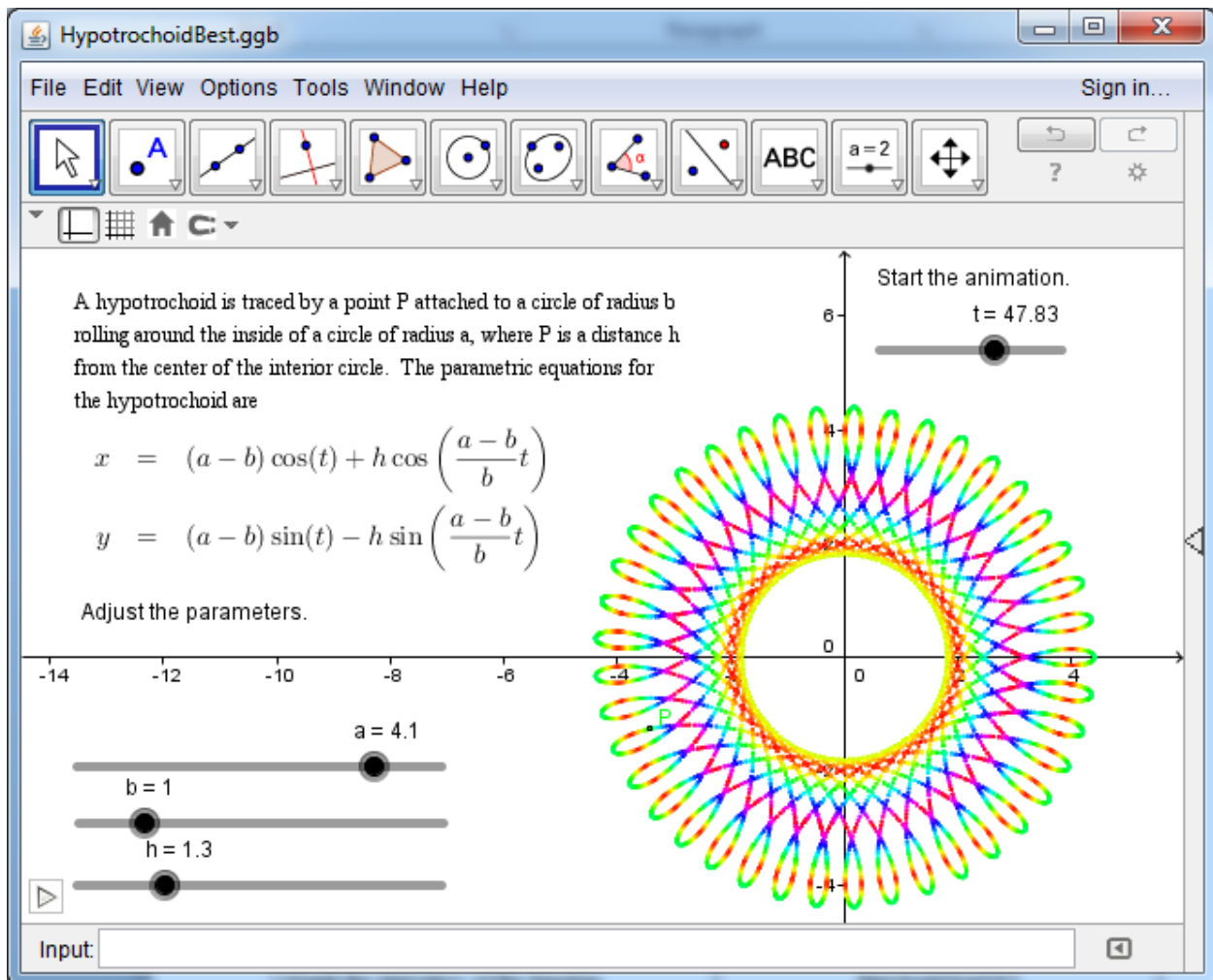
Then enter the point P.

```
P = (f(t), g(t))
```

Right-click on the point P, select Object Properties, click on the Advanced tab, use the drop-down menu to select the HSV color model, enter a formula for the Hue and numbers for Saturation and Value. I chose to determine the hue based on the distance the distance between the point P and the origin. Again, other choices are possible,

```
sqrt(x(P)2 + y(P)2)
```

File Edit View Options Tools Window Help   Sign in...

ABC   a=2

Start the animation.

t = 47.83

A hypotrochoid is traced by a point P attached to a circle of radius b
rolling around the inside of a circle of radius a, where P is a distance h
from the center of the interior circle. The parametric equations for
the hypotrochoid are

$$x = (a-b)\cos(t) + h\cos\left(\frac{a-b}{b}t\right)$$

$$y = (a-b)\sin(t) - h\sin\left(\frac{a-b}{b}t\right)$$

Adjust the parameters.

-14   -12   -10   -8   -6

a = 4.1

b = 1

h = 1.3

Input:

Tip 4 – Using images

Images from the internet can be used in GeoGebra to demonstrate scientific principles.

Example: Flight of a basketball.

1. Insert the photograph into GeoGebra using the menu Edit/Insert Image from/File or by using the Image tool

2. The position of an image may be absolute on screen or relative to the coordinate system. By default, GeoGebra uses the relative to coordinate system option and provides two corner points on the bottom of the image to control the image.

   In this example, it will be convenient to anchor the lower left corner to the origin and the lower right corner to the *x*-axis by redefining the points A and B as follows.

   ```
   A = Intersect[xAxis, yAxis]
   B = Point[xAxis]
   ```

3. Adjust the size of the image such that the height of the rim is equal to 10 feet. To accomplish this, consider adding the line $y = 10$ to the graph by entering the equation in the input bar. Use the zoom tool and then adjust the position of point B.
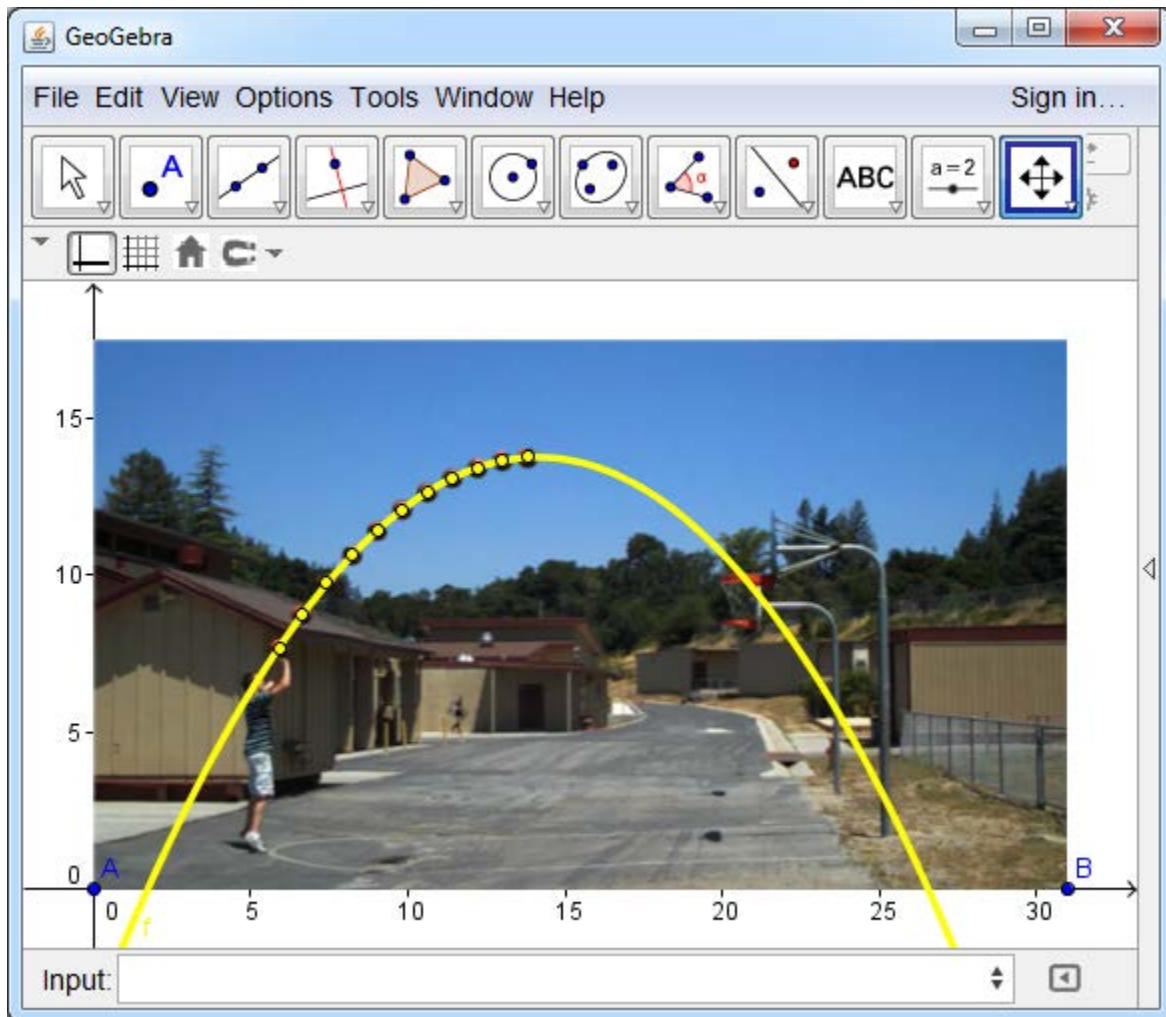

4. Use the point tool to add points to mark the position of the ball in flight.

5. Make a list of the points.

   ```
   points = {C, D, E, F, G, H, I, J, K, L, M}
   ```

6. Use the FitPoly command to find the equation of the best fitting parabola. Enter the following command into the input bar.

```
FitPoly[points, 2]
```

Here is the completed applet showing the flight of the basketball. It appears this shot was good.

Tip 5 – Use the sequence command

The most obvious use of the sequence command is to create numerical sequences.

Example: The triangular numbers can be represented using a triangular grid of points where the first row contains a single dot and each subsequent row contains one additional dot. The nth triangular number is equal to the sum of the first n integers.
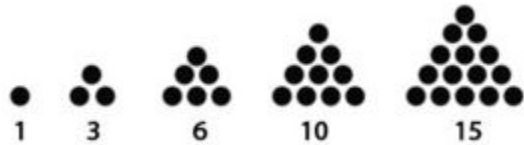
$$T_n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

The first 20 triangular numbers can be computed in GeoGebra by entering the following command in the input bar.

```
Sequence[n (n + 1) / 2, n, 1, 20]
```

In addition to numerical sequences, we can also create sequences of objects including lines, circles, polygons and functions. The one limitation when using the sequence command to generate a list of objects is that all of objects in the list must have the same properties (color, line thickness, opacity,…).

Example: Mary Boole introduced string art at the end of the 19th century in an effort to make mathematics more accessible to children (Philosophy and Fun of Algebra (Boole, 1909)). The basic idea was to use a family of straight lines to create the impression of a curve. Mathematically, the equation of the envelop of the family of curves $f(x, y, \alpha) = 0$ can be obtained by eliminating the parameter α from the equations

$$\begin{cases} f(x, y, \alpha) = 0 \\ \dfrac{\partial f(x, y, \alpha)}{\partial \alpha} = 0 \end{cases}$$

See Differential Calculus by S Balachandra Rao for details. We now use this method to find the envelope of the family of straight lines

$$y = mx + a/m$$

where m is the parameter and a is a constant. In this case, the implicit representation of the family and its derivative with respect to m are given by

$$\begin{cases} y - mx - a/m = 0 \\ -x + a/m^2 = 0 \end{cases}$$

To eliminate m, multiply the first equation by m to obtain

$$my - m^2 x - a = 0 \tag{1}$$

and solve the second equation for m^2 to obtain

$$m^2 = \frac{a}{x} \tag{2}$$

Substituting equation (2) into equation (1) yields

$$my - \left(\frac{a}{x}\right)x - a = 0$$
$$my = 2a$$
$$m^2 y^2 = 4a^2 \tag{3}$$

Finally, substitute (3) into (2) to obtain the parabola

$$y^2 = 4ax$$

Hence, the envelope of the family of lines $y = mx + a/m$ is the parabola $y^2 = 4ax$. To improve the appearance of the string art in GeoGebra, discretize the slope using $m = \tan\theta$ where the values of θ are equally spaced. This ensures a consistent spacing of the angles between the lines.

Create sliders for the parameters *a* and *n* by entering the following commands in the input bar. The parameter *a* is used to determine the position of the focus $(a, 0)$ and the parameter *n* is used to control the number of lines drawn.

```
a=Slider[-5, 5, 0.01]
n=Slider[2, 100, 1]
```

Create the lines forming the envelope of the parabola by using the sequence command.

```
lines = Sequence[tan(π k / (2n)) x + a / tan(π k / (2n)), k, -n + 1, n - 1]
```

The parabola may be added to the graph using

```
y^2 = 4a x
```
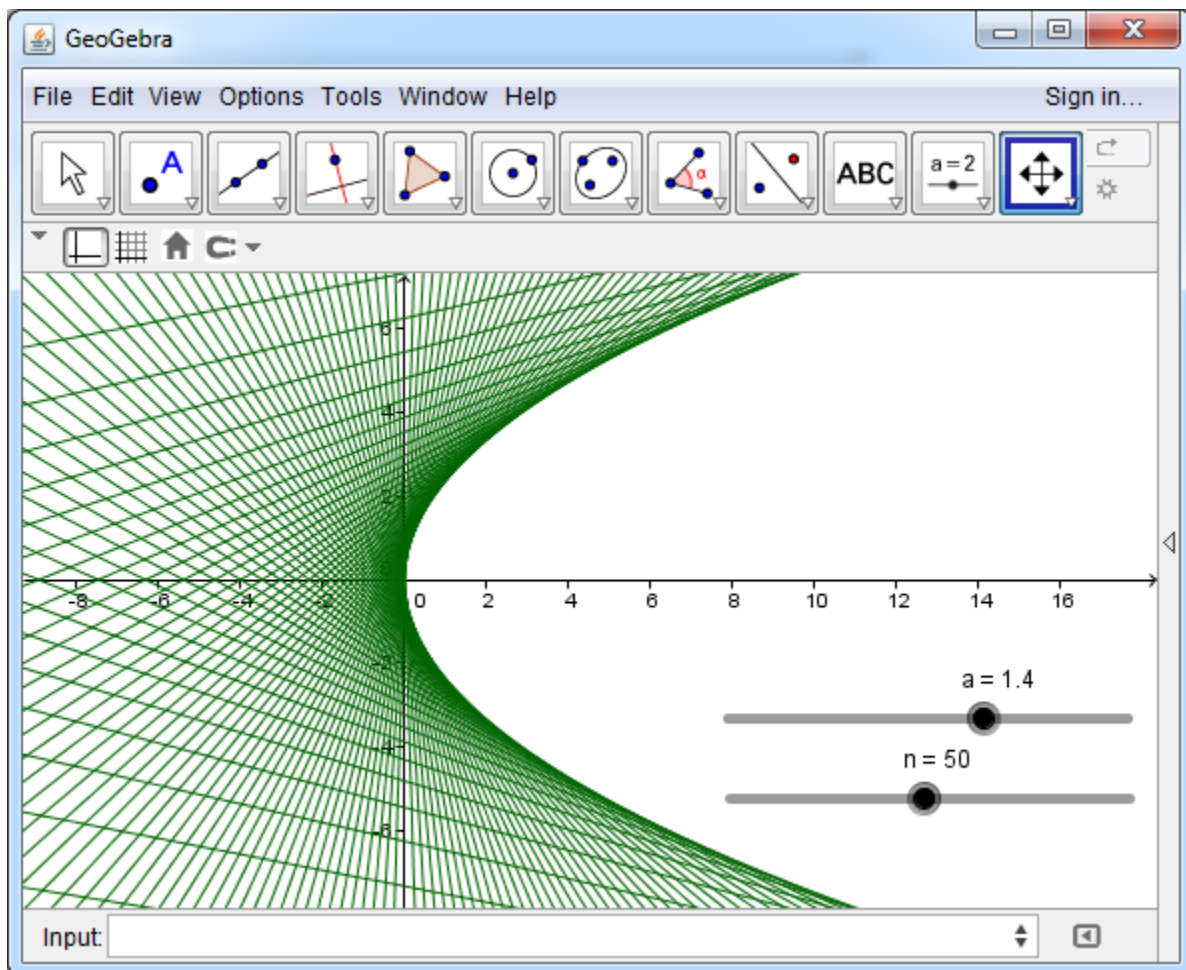
Tip 6 – Use the spreadsheet

The discrete logistic map was popularized in a 1976 paper by the biologist Robert May. His discrete-time, demographic model is a simple quadratic equation intended to capture two main effects.

1. Reproduction rates increase at a rate proportional to the population when the population is small.
2. Reproduction rates decrease as the population approaches the carrying capacity (the maximum population an environment can sustain).

The discrete logistic equation is given by

$$x_{n+1} = r\, x_n \left(1 - x_n\right)$$

where x_n is the population of the current generation expressed as a proportion of the carrying capacity, x_{n+1} is the population of the next generation and r is the maximum growth rate. This is the growth rate we would expect to see when the population is small, before the effects of overcrowding become apparent. To implement this model in GeoGebra, first create sliders for the growth parameter r and the initial population x_0. Use an interval of [0, 4] with an increment of 0.01 for r and an interval of [0, 1] with an increment of 0.01 for x_0.

r = 3.4

x0 = 0.47
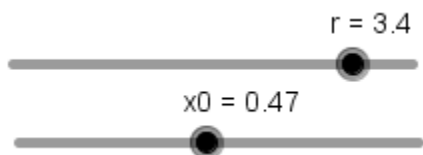
It may be easier to use the spreadsheet view in GeoGebra to handle the recursively defined sequence. To do this enter formulas into the first two rows and then use the fill handle to copy the formulas into a total of 100 rows.

	A	B	C	D
1	0	x0	(A1, B1)	
2	1	r B1 (1 - B1)	(A2, B2)	Segment[C1, C2]

For column A you will be copying the pattern and need to select the first two rows and then use the fill handle.

	A
1	0
2	1

For columns B, C and D you will be copying the formula and only need to select the row containing the formulas before using the fill handle. Since column A has already been completed, simply double-click the fill handle to copy the formulas.

B	C	D
0.47	(0, 0.47)	
0.85	(1, 0.85)	1.07
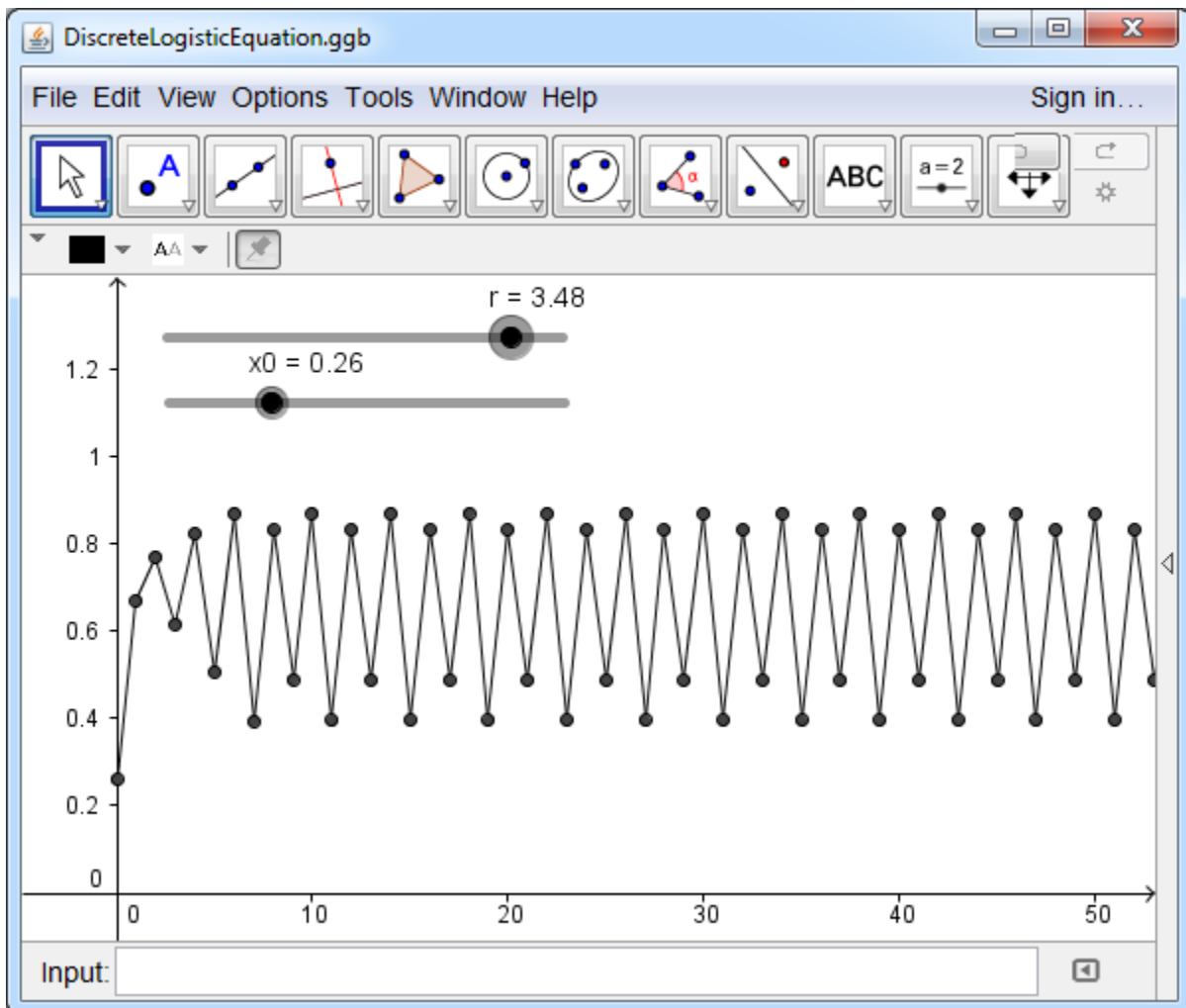
Tip 7 – Create a custom tool

If you find yourself completing the same construction multiple times, it may be worthwhile to create a custom tool.

Example: Construct a custom tool for creating a Sierpinski gasket. Begin by constructing an equilateral triangle using the regular polygon tool.

Then use the midpoint tool to locate the midpoint of each side of the triangle.
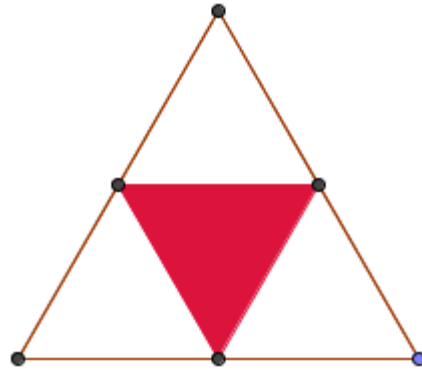
Construct a second equilateral triangle with the regular polygon tool using the midpoints. Right-click on the larger triangle and select object properties and set the opacity to 0. Use the same approach to select a color for the smaller triangle and set its opacity to 100.

Now we are ready to create a custom tool by going to Tools/Create New Tool.
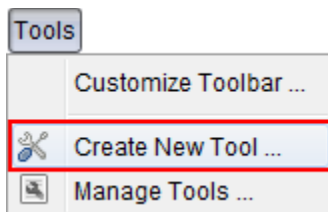
Select the output objects, most everything in this case.

Create New Tool

Output Objects | Input Objects | Name & Icon

Select objects in construction or choose from list

Polygon **poly1**: Polygon[A, B, 3]
Triangle **poly2**: Polygon D, E, F
Segment **a**: Segment [A, B] of Polygon poly1
Segment **b**: Segment [B, C] of Polygon poly1
Segment **c**: Segment [C, A] of Polygon poly1
Segment **f**: Segment [D, E] of Triangle poly2
Segment **d**: Segment [E, F] of Triangle poly2
Segment **e**: Segment [F, D] of Triangle poly2
Point C: Polygon[A, B, 3]
Point D: Midpoint of c
Point E: Midpoint of b
Point F: Midpoint of a

▲ ▼ ✕

< Back | Next > | Cancel

Select the input objects, points A and B.

Create New Tool

Output Objects | Input Objects | Name & Icon

Select objects in construction or choose from list

Point A
Point B

▲ ▼

< Back | Next > | Cancel
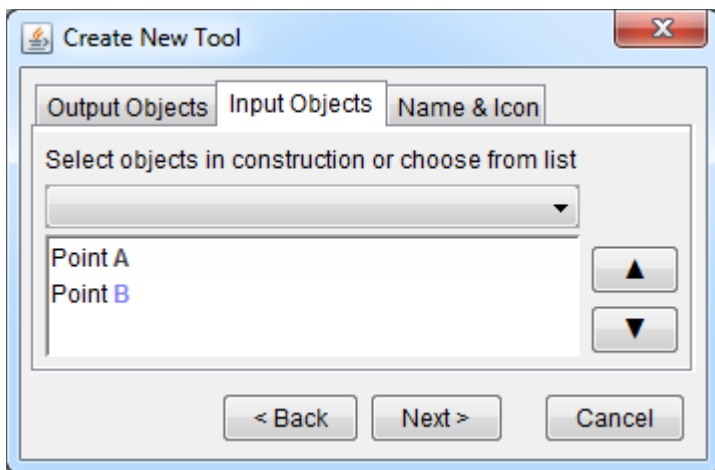
Provide the tool with a name and a hint for how to use the tool.
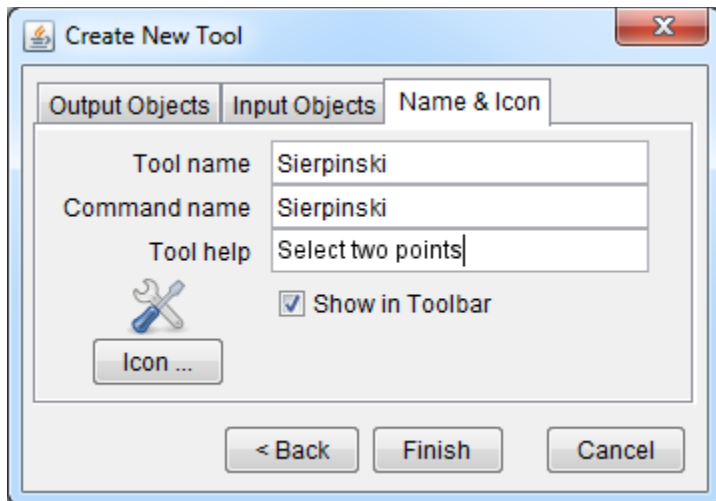
Now we have a tool to use for constructing the Sierpinski triangle. It is possible to create a better icon for the tool. Also, we can hide the points after finishing the construction for a cleaner look.
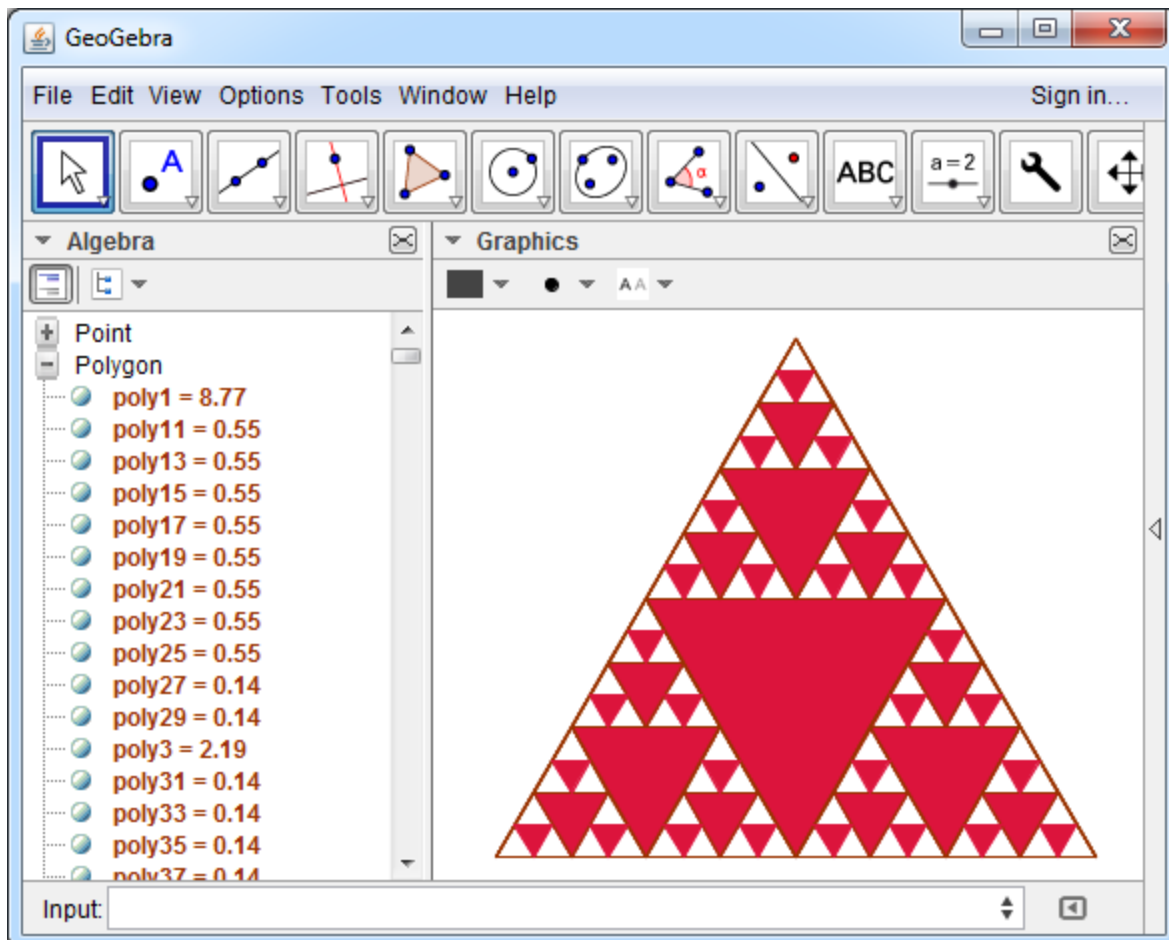
Tip 8 – Use multiple tips simultaneously

The Koch snowflake is named after the Swedish mathematician Helge von Koch (1870–1924). To construct the snowflake, start with an equilateral triangle. In the first iteration, add a triangle whose sides are one-third as long as the sides of the original triangle to the middle of each side of the original triangle. In the second iteration, we add a triangle whose sides are one-ninth as long as the sides of the original triangle to each side of the figure obtained after the first iteration. This process can be repeated indefinitely and results in the Koch snowflake. The first several iterations are shown below.
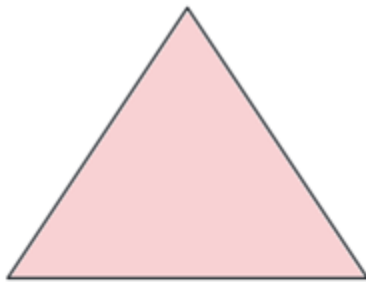
Stage 0
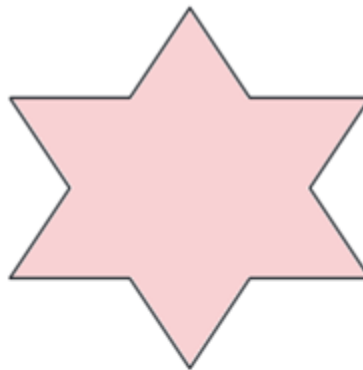
Stage 1
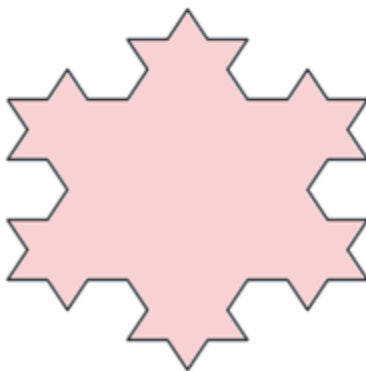
Stage 2
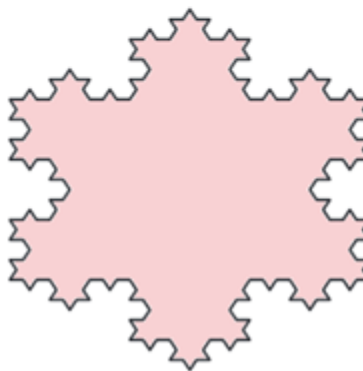
Stage 3

Create a custom tool

Create a custom tool which accepts two points and returns the five points necessary to add a triangle to the middle of an existing side. Enter the following commands into the input bar.

```
A = (1,1)
B = (3,1)
C = 2/3 A + 1/3 B
D = Rotate(A,2 pi/3, C)
E = 1/3 A + 2/3 B
points = {A, C, D, E, B}
```
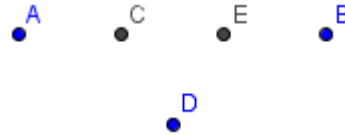
Create a custom tool by going to Tools/Create New Tool.
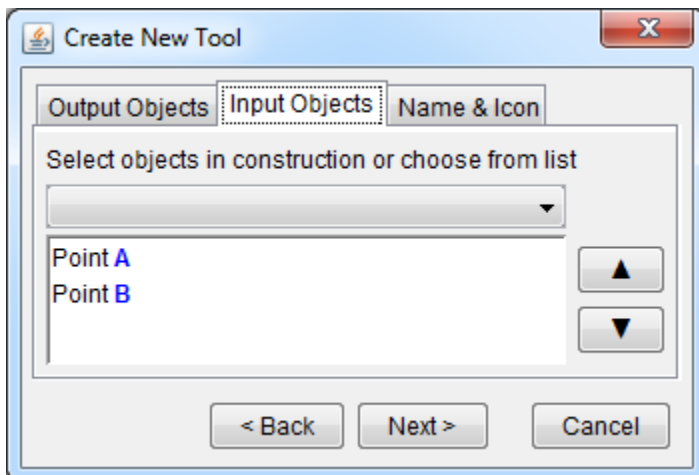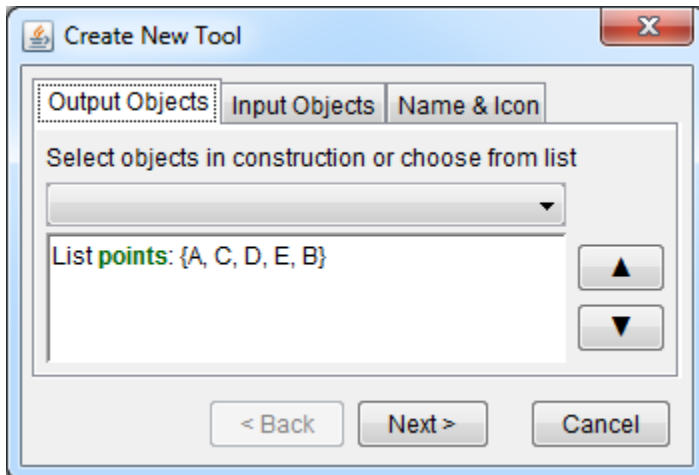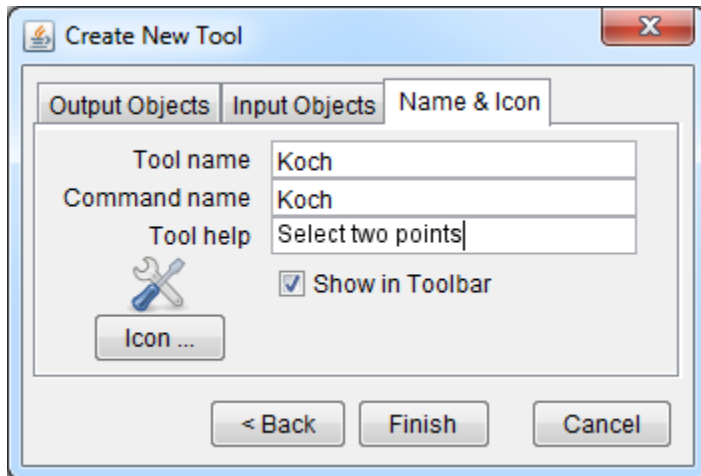
If you want, you may delete the construction used to create the tool. This will create a cleaner workspace and besides, the construction is already saved in the tool itself.

Create an equilateral triangle

Create an equilateral triangle using the regular polygon tool.
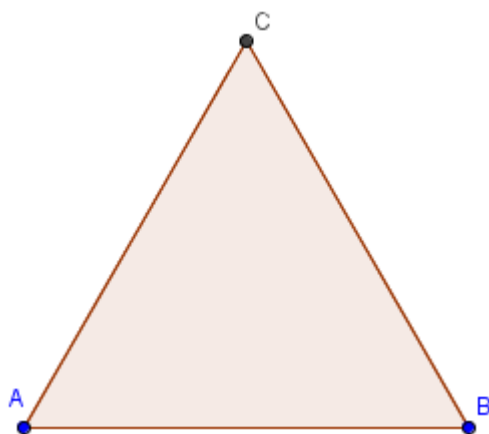
Use the sequence command in the spreadsheet

Then rather than apply the tool repeatedly by hand we use the sequence command in spreadsheet view to invoke this tool thousands of times. In column A enter the following in the first two rows and then use the fill handle to drag the formula in row 2 until you reach row 6.

	A
1	{A, B, C, A}
2	Join[Sequence[Koch[Element[A1, k], Element[A1, k + 1]], k, 1, Length[A1] - 1]]

In column B enter the following command in the first row and then drag the fill handle until you reach row 6.

	B
1	Sequence[Segment[Element[A1, k], Element[A1, k + 1]], k, 1, Length[A1] - 1]

It would be a good idea to select column A, right-click and select Show Object to hide all of the points. At this point in the construction of Koch's snowflake, all of the iterations are visible at once. To fix this, first create a slider n which takes on values in the interval [1, 6] with an increment of 1.

n = 1

Right-click on cell B1 and select Object Properties. Select the Advanced tab and in the Condition to Show Object textbox, enter the condition $n == 1$.

	B
1	{4.98, 4.98, 4.98}
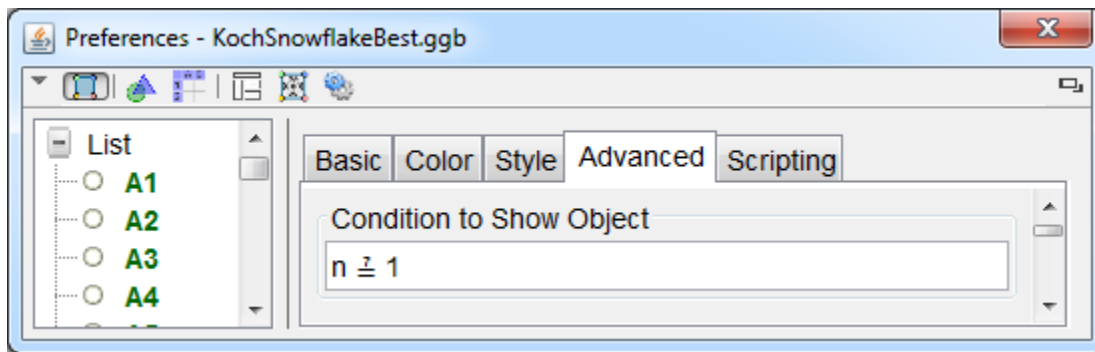
Repeat this process for cell B2 with the condition $n == 2$ and so on until your reach B6 with the condition $n == 6$. Now you can move the slider to display the desired iteration.
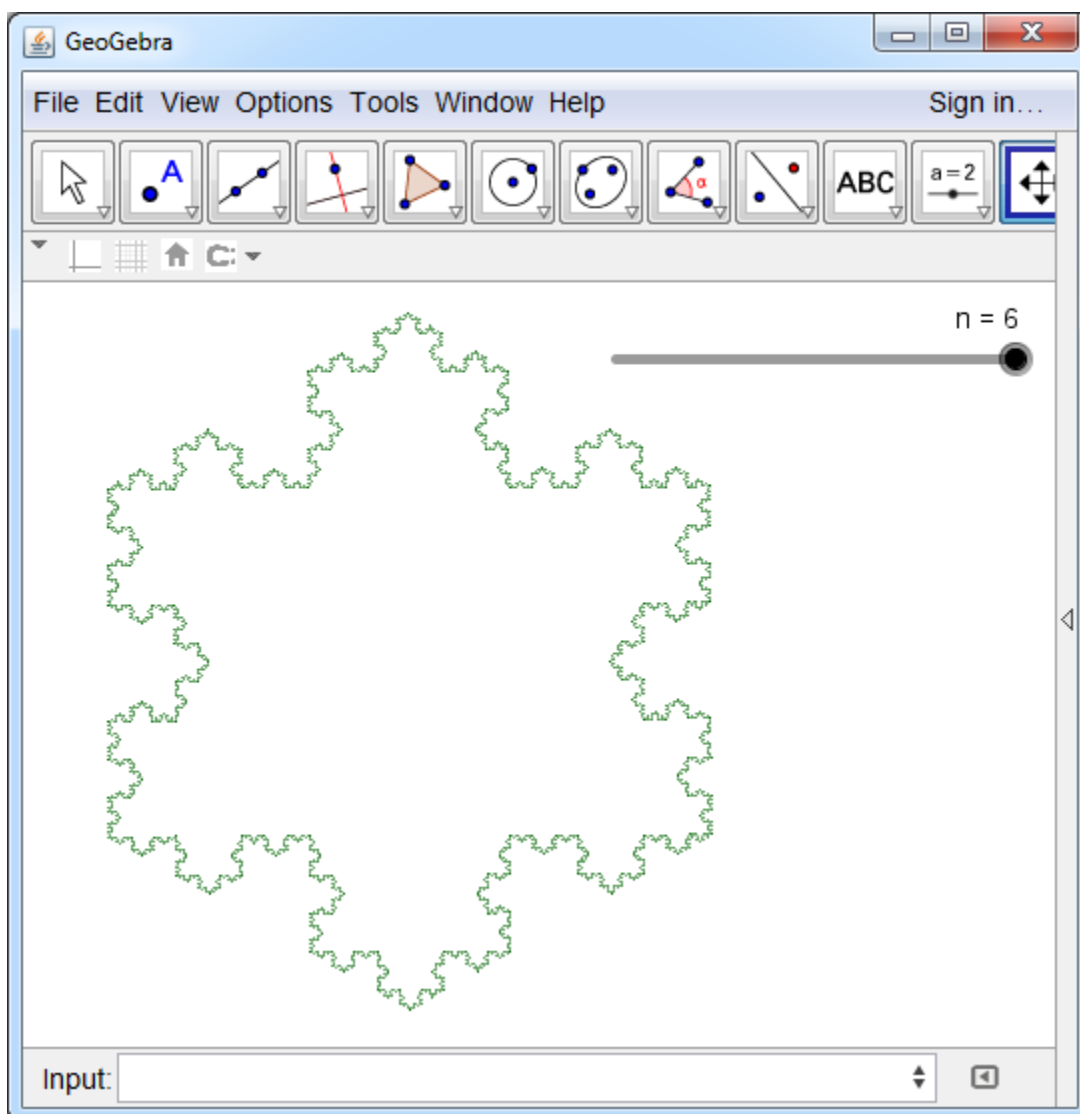
Tip 9 – Algorithmic homework

In this tip, we will learn how to create a GeoGebra applet that asks students to determine the slope of a line from its graph. Students will have the opportunity to enter their answers and will be given feedback on their performance.
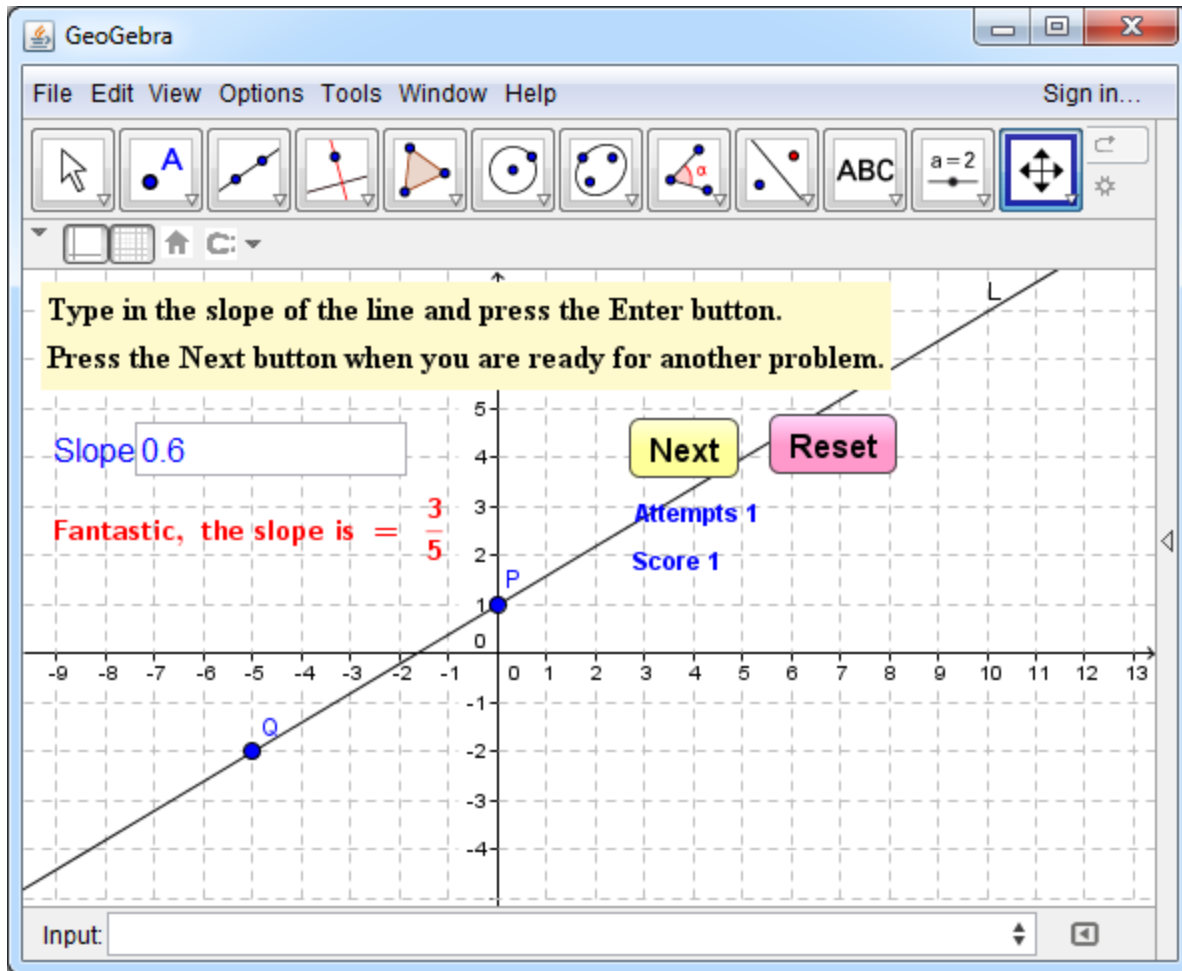

Enter the following commands in the input bar to create two randomly generated points with a line passing through them. Note that the *x*-coordinates are handled differently that the *y*-coordinates since we do not want the slope to be undefined.

```
xList = Shuffle[Sequence[k, k, -5, 5]]
P = (Element[xList, 1], RandomBetween[-5, 5])
Q = (Element[xList, 2], RandomBetween[-5, 5])
L = Line[P, Q]
m = ?
```

Use the Input Box tool to create the input box.

Enter a caption and link the input box to the slope *m*.
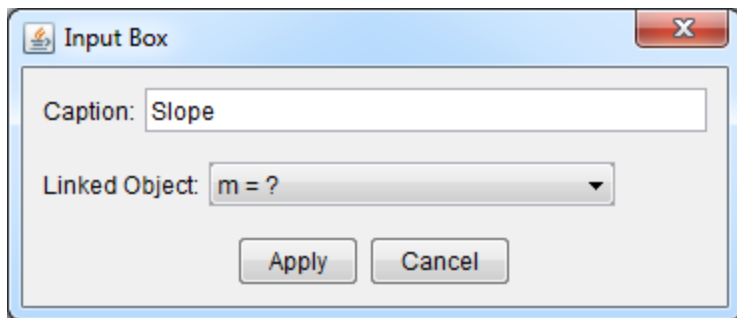
Check to see if the student's response is correct using the Boolean variable correct.

```
correct = (m == Slope[L])
```

Use the Text tool to create two textboxes, one for when the student provides an incorrect value for the slope and another for when the student provides the correct value.
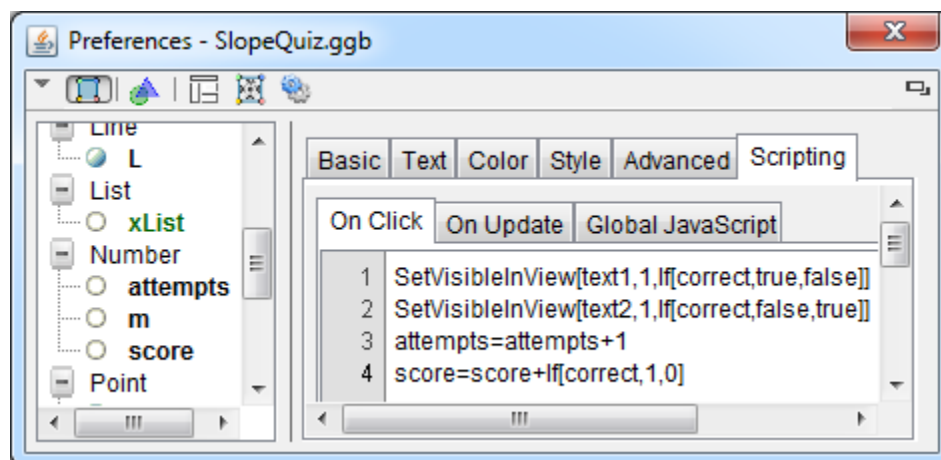
Text ☒

Edit

\text{Sorry, your answer is incorrect. Try again.}

☑ LaTeX formula ▾ | Symbols ▾ | Objects ▾

| π | | | | | | | | | |

Preview

Sorry, your answer is incorrect. Try again.

? Help | OK | Cancel
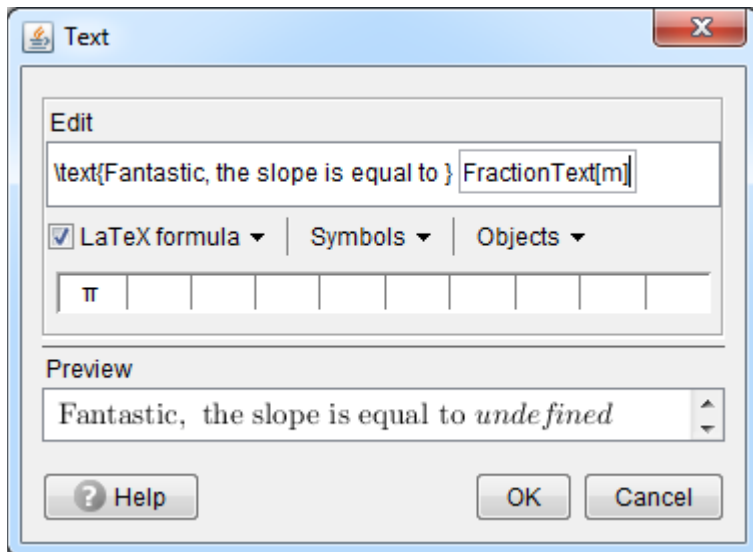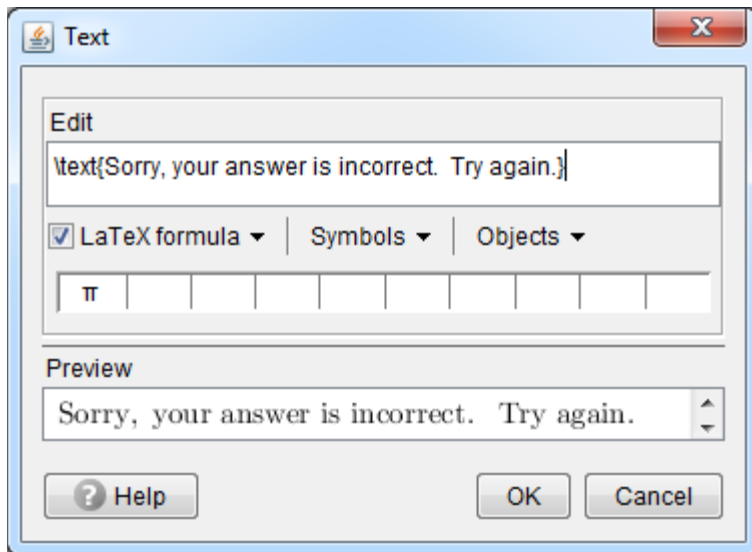
Fantastic, the slope is equal to *unde fined*

Sorry, your answer is incorrect. Try again.

Enter the following commands in the input bar to initialize two variables, attempts and score to track student performance.

```
attempts = 0
score = 0
```

At this point, both messages display all of the time. To fix this, right-click on the input box, select Object Properties, click on the On Click tab and add the code shown below to selectively display the textboxes and update the attempts and the score. Note that the attempts variable is updated after each use of the input box and the score variable is updated only after successful attempts.

Slope ?

Preferences - SlopeQuiz.ggb ☒

Line
  ● L
List
  ○ xList
Number
  ○ attempts
  ○ m
  ○ score
Point

Basic | Text | Color | Style | Advanced | Scripting

On Click | On Update | Global JavaScript

```
1  SetVisibleInView[text1,1,If[correct,true,false]]
2  SetVisibleInView[text2,1,If[correct,false,true]]
3  attempts=attempts+1
4  score=score+If[correct,1,0]
```
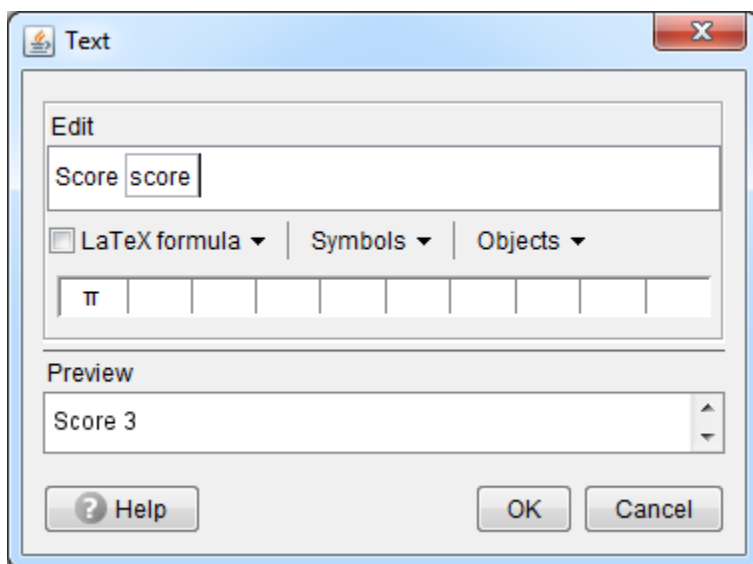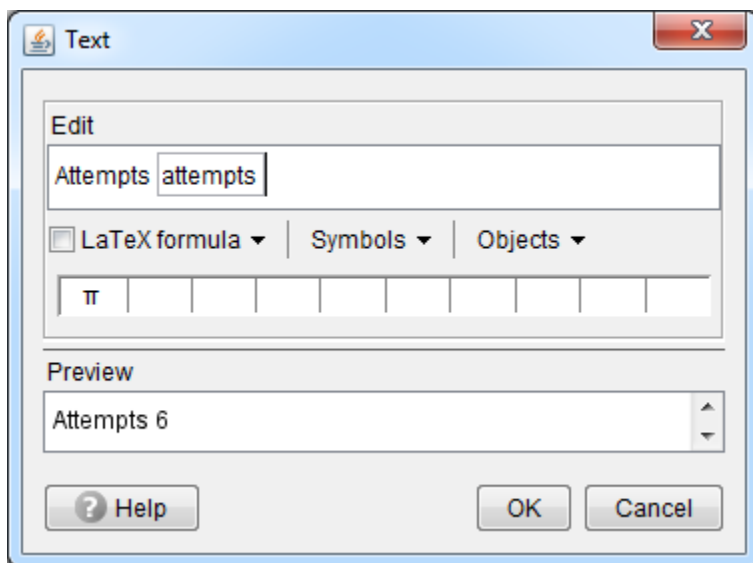
We are now ready to add a next button so that students can attempt to determine the slopes of as many lines as they wish. Use the button tool to add a button. Add a caption and then add the code shown below to update the construction, initialize the slope *m* to undefined and hide both of the text boxes.

OK

Button

Caption: Next

GeoGebra Script:

```
1  UpdateConstruction[]
2  m = ?
3  SetVisibleInView[text1,1,false]
4  SetVisibleInView[text2,1,false]
```

Apply   Cancel
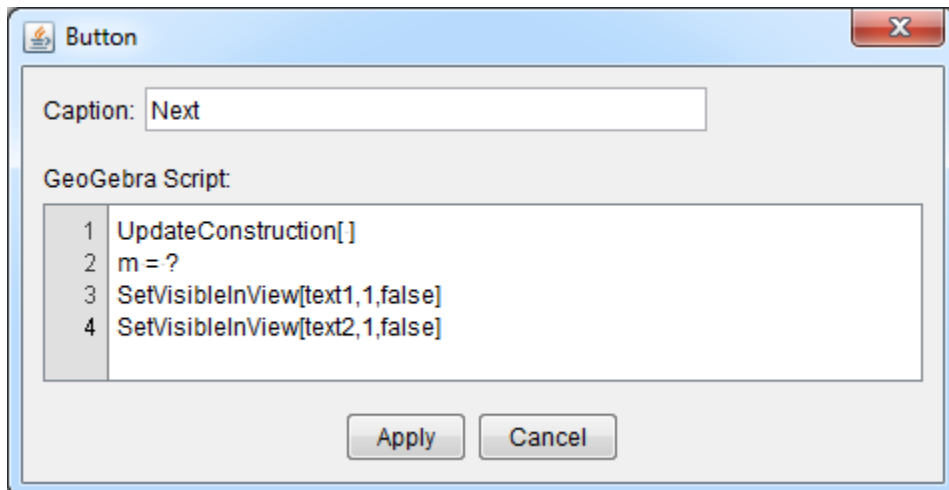
Use the button tool to add a reset button, so students can reset the applet. Add a caption and then add the code shown below to reset the attempts and score variables, reset the slope *m* to undefined and hide the text boxes.

Button

Caption: Reset

GeoGebra Script:

```
1  attempts = 0
2  score = 0
3  m = ?
4  SetVisibleInView[text1,1,false]
5  SetVisibleInView[text2,1,false]
```

Apply   Cancel
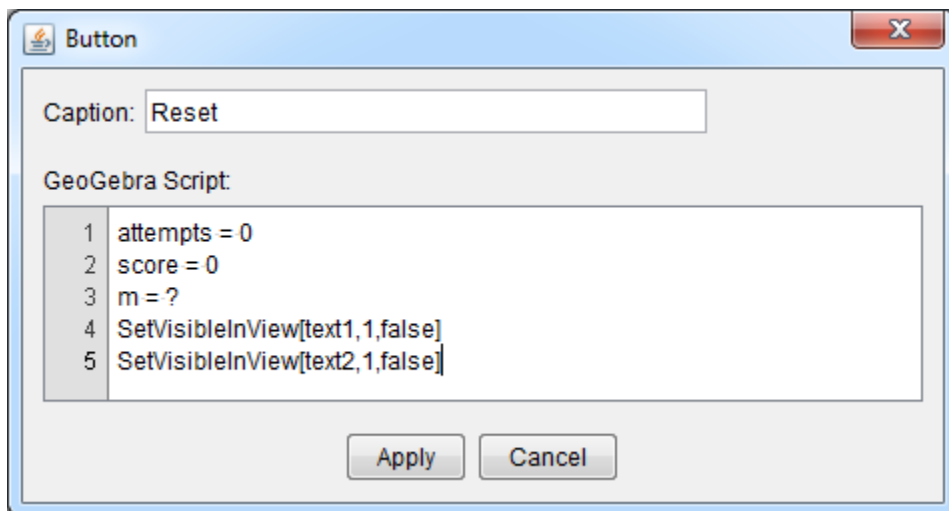
Use the Text tool to create two additional textboxes to display the number of attempts and the score.

ABC

Text

Edit

Attempts attempts

☐ LaTeX formula ▾ | Symbols ▾ | Objects ▾

π

Preview

Attempts 6

Help | OK | Cancel

Text

Edit

Score score

☐ LaTeX formula ▾ | Symbols ▾ | Objects ▾

π

Preview

Score 3

Help | OK | Cancel

The slope quizzing applet is complete.

Tip 10 – Locus tool

The locus is a set of points whose location satisfies a set of conditions. For example, a circle can be defined as the locus of all points equidistant from a given point.

Example: The parabola can be defined geometrically as the locus of all points equidistant from a fixed line called the directrix and a fixed point called the focus. To construct the parabola using GeoGebra, first create a line (a) and a point F.
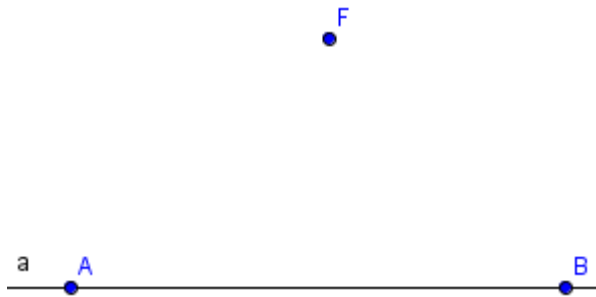
Then construct a point P on the parabola as follows.

1. Construct a point C on the line.
2. Construct a line (b) perpendicular to line (a) through point C.
3. Construct line segment FC.
4. Construct a perpendicular bisector (d) of segment FC.
5. Find the intersection point P of lines (b) and (d).
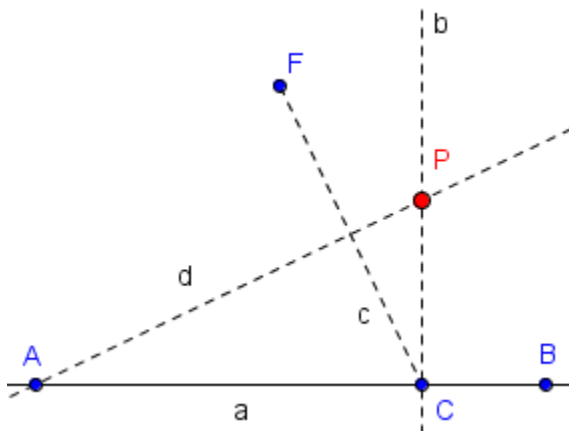
To clean up the construction do the following. This reveals the isosceles triangle

1. Hide lines (b) and (d).
2. Add line segments FP and PC.
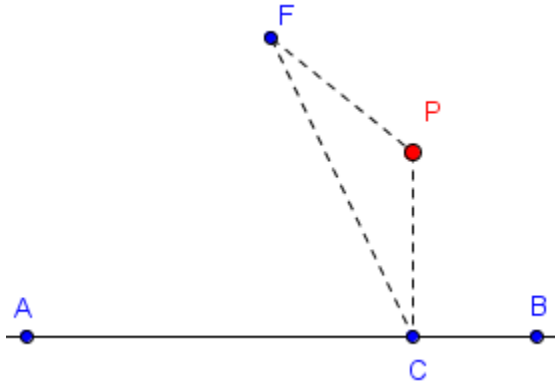
Use the locus tool to construct the parabola. This tool is under the Perpendicular Line tools. To use the locus tool, select point P (locus) and then point C (movable point).
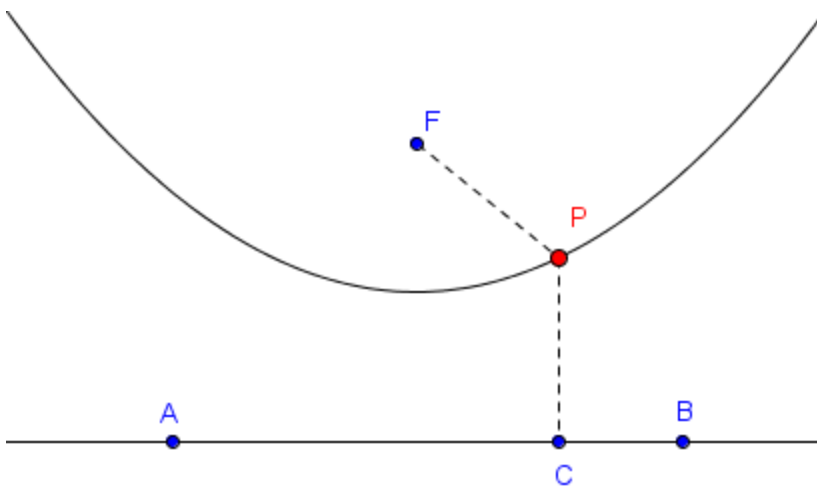
Example: Function and its derivative.
Enter a function into the input bar.

```
f(x) = x^3 - x
```

Place a point A on the *x*-axis and create a point B on the *x*-axis with the same *x*-coordinate as A and whose *y*-coordinate is the derivative of the function $f(x)$.

```
B = (x(A), f'(x(A)))
```
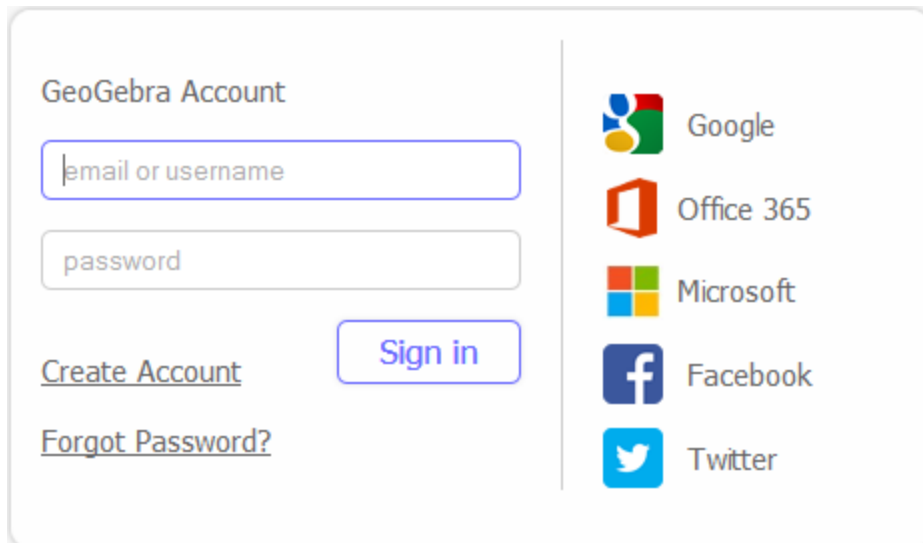
Use the locus tool to construct the graph of the derivative. When using the tool, first click on the locus point B and then on the movable point A.
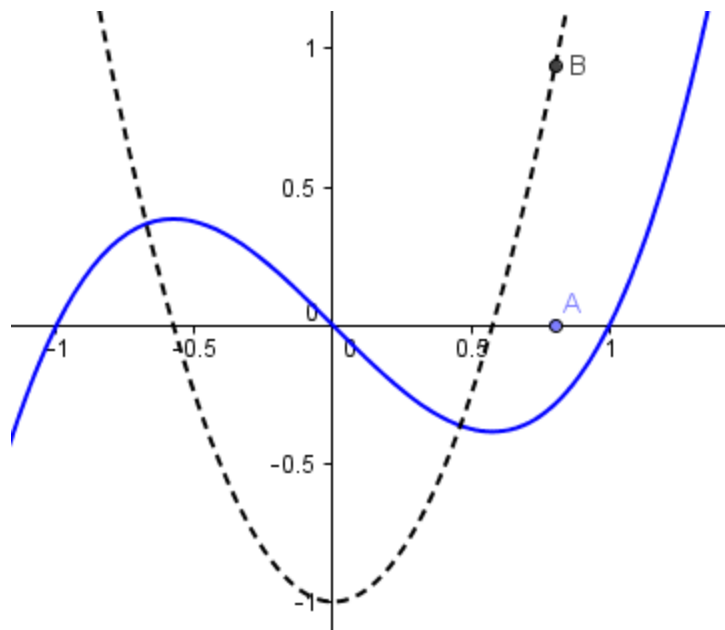
If you right-click on point A and select Animation On, you will see point B trace out the derivative.
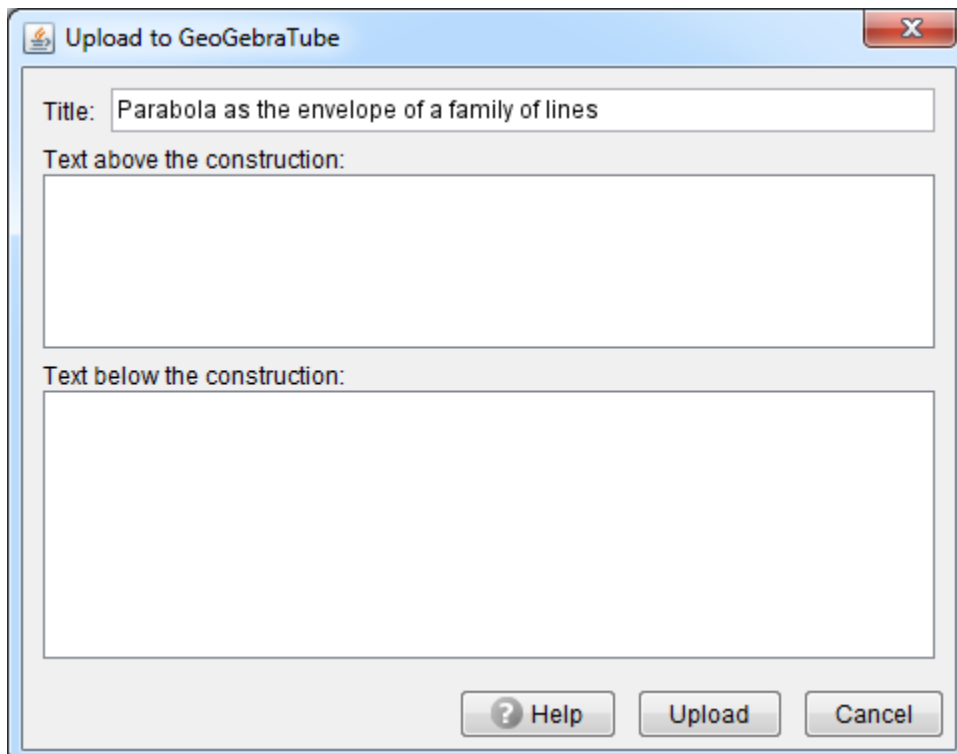
Tip 11 – GeoGebraTube

GeoGebraTube has thousands of GeoGebra applets available for people to download and use. In addition, GeoGebraTube is a convenient way to share GeoGebra applets with students and use them for in class demonstrations. To upload a GeoGebra applet, you first create an account.

Then use the File/Export/Dynamic Worksheet as Webpage.

Enter the Tile and any text you want to appear above or below the construction and then press Upload. Complete the section on Information for Students and Information for Teachers and then press Save.

Information for Students

This information will be shown on the student worksheet. Please provide a short explanation for students above the applet, and specific tasks below the applet.

Description or explanation for students appearing above the applet (optional)

Please provide some short description or explanation for your students that should be shown above the applet.

| **B** | *I* | *f(x) =* | www | ⫶☰ | ☰ | ⫶☰ | ▮ |

Advanced Settings...

Questions or tasks for students appearing below the applet (optional)

Please provide some specific questions or tasks for your students that should be shown below the applet.

| **B** | *I* | *f(x) =* | www | ⫶☰ | ☰ | ⫶☰ | ▮ |

Use the sliders to adjust the focal point of the parabola (a) and the number of lines drawn (2n+1).

Continue

Information for other Teachers

This information will be shown on the material page. It is intended for other teachers who will be able to use and comment on your material.

Title

The title of your material is shown on the material page and on the student worksheet.

Parabola as the envelope of a family of lines

Description

This description will be visible on the material page, but it will not be shown on the student worksheet.

B *I*   *f(x) =*   www   ☷   ☷   ☷   ▮

A family of straight lines can be used to give the appearance of a parabola.

Language

Please choose the main language of your material.

English ▾

Target Group (Age)

The age of the students this material is intended for.

Age: 12   —   18

Tags

Tags are key words to describe your material and help with searching. You and other users can add new tags later.

envelope parabola

Visibility

Please choose if you want to share this material with others or keep it private

◉ **Public** - Everyone can find and view this material. Thank you for sharing it with the community!

◯ **Shared with Link** - Only users who have the link can view this material.

◯ **Private** - Only you can find and view this material.

Back to Student Information   Save

The GeoGebra applet is now available to the world and can be used in a browser and downloaded.

Parabola as the envelope of a family of lines
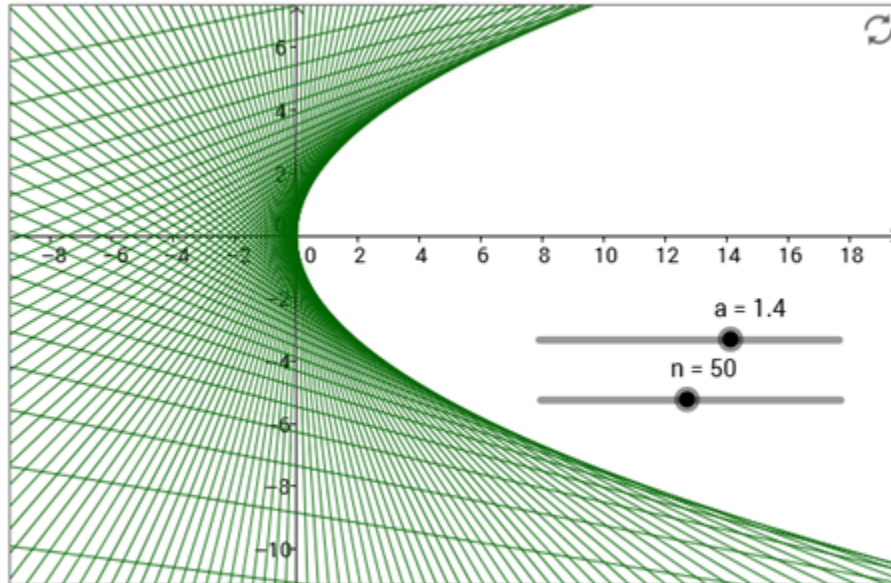
a = 1.4

n = 50

Use the sliders to adjust the focal point of the parabola (a) and the number of lines drawn (2n+1).

Created with GeoGebra – Shared by Joseph Manthey – Share or copy