

$$\textcircled{39} \quad V = \iint_R (3-x) \, dA$$

$$V = \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (3-x) \, dy \, dx \Rightarrow 3y - xy \Big|_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}$$

$$3(\sqrt{9-x^2}) - x(\sqrt{9-x^2}) + 3\sqrt{9-x^2} - x\sqrt{9-x^2}$$

$$= 6\sqrt{9-x^2} - 2x\sqrt{9-x^2}$$

$$= \int_{-3}^3 (6\sqrt{9-x^2} - 2x\sqrt{9-x^2}) \, dx$$

$$= 6 \int \sqrt{9-x^2} \, dx \quad \leadsto \text{usando } x = 3 \operatorname{seno}(t)$$

$$= 6 \int \sqrt{9 - (3 \operatorname{seno}(t))^2} \cdot 3 \cos(t) \, dt$$

$$= 6 \int 3 \sqrt{9 - 9 \operatorname{seno}(t)^2} \cos(t) \, dt$$

$$= 6 \int 3 \sqrt{9 - 9 \operatorname{seno}(t)^2} \cos(t) \, dt$$

$$6 \int 3 \sqrt{9(1 - \operatorname{seno}(t)^2)} \cos(t) \, dt$$

$$\text{Usando } 1 - \operatorname{sen}(t)^2 = \cos(t)^2$$

$$6 \int 3 \sqrt{9 \cos(t)^2} \cos(t) dt$$

$$6 \int 3 \cdot 3 \cos(t) \cos(t) dt$$

$$6 \int 9 \cos(t)^2 dt \Rightarrow 6 \cdot 9 \int \cos(t)^2 dt$$

$$= 54 \int \frac{1 + \cos(2t)}{2} dt \Rightarrow 54 \cdot \frac{1}{2} \int 1 + \cos(2t) dt$$

$$27 \int 1 + \cos(2t) dt \Rightarrow 27 \left(t + \frac{\operatorname{sen}(2t)}{2} \right)$$

$$t = \operatorname{arc} \operatorname{sen} \left(\frac{x}{3} \right)$$

$$= 27 \left(\operatorname{arc} \operatorname{sen} \left(\frac{x}{3} \right) + \frac{\operatorname{sen} \left(2 \operatorname{arc} \operatorname{sen} \left(\frac{x}{3} \right) \right)}{2} \right)$$

$$= 27 \operatorname{arc} \operatorname{sen} \left(\frac{x}{3} \right) + 3x \sqrt{9 - x^2} + \frac{2(9 - x^2) \sqrt{9 - x^2}}{3}$$

$$= \left(27 \operatorname{arc} \operatorname{sen} \left(\frac{x}{3} \right) + 3x \sqrt{9 - x^2} + \frac{2(9 - x^2) \sqrt{9 - x^2}}{3} \right) \Big|_{-3}^3$$

$$= 27 \arcsin\left(\frac{3}{3}\right) + 3 \cdot 3\sqrt{9-3^2} + \frac{2(9-3^2)\sqrt{9-3^2}}{3} -$$

$$\left(27 \arcsin\left(-\frac{3}{3}\right) + 3(-3)\sqrt{9-(-3)^2} + \frac{2(9+3^2)\sqrt{9+3^2}}{3} \right)$$

$$= 27 \arcsin(1) + 9\sqrt{9-3^2} + \frac{2(9-9)\sqrt{9-3^2}}{3} -$$

$$\left(27 \arcsin(-1) - 9\sqrt{9+3^2} + \frac{2(9-9)\sqrt{9+3^2}}{3} \right)$$

$$27 \cdot \frac{\pi}{2} + 9\sqrt{9-9} + \frac{2 \cdot 0 \sqrt{9-9}}{3} - \left(27 \cdot \left(-\frac{\pi}{2}\right) - 9\sqrt{9-9} \right) +$$

$$\frac{2 \cdot 0 \sqrt{9+3^2}}{3}$$

$$= \frac{27\pi}{2} + 9\sqrt{0} + \frac{0}{3} - \left(-\frac{27\pi}{2} - 9\sqrt{0} + \frac{0}{3} \right)$$

$$= \frac{27\pi}{2} + \frac{27\pi}{2} = \frac{54\pi}{2} = 27\pi //$$