Pæne stykkevise tredjegradspolynomiumsfunktioner med heltallige og rationelle ekstrema.

Lad $f(x) = \begin{cases} f_1(x), x \leq x_0 \\ f_2(x), x > x_0 \end{cases}$

Forskrifterne for f_1 *og* f_2 *står nedenfor.*

Lad $x_0, l_1\, l_2, r_1, r_2, m$ *og* $n \in \mathbb{Z}, \frac{l_1}{l_2} \neq x_0, \frac{r_1}{r_2} \neq x_0, \frac{r_1}{r_2} < x_0, \frac{l_1}{l_2} < x_0, s \in \mathbb{N}, x_0 < 3mns, x_0 < 0.$

m *og* n *er ikke begge nul,* $l_2 \neq 0, r_2 \neq 0.$

Da er $f(x)$ *differentiabel i* x_0, $f_2(x)$ *har tre rødder, to ekstremumspunkter og et vendepunkt, som alle er heltallige.* ($f_2(x)$ *er et pænt tredjegradspolynomium*)

$f_1(x)$ *har to ekstremumspunkter i hhv.* $x = \frac{r_1}{r_2}$ *og* $x = \frac{l_1}{l_2}$.

Det følger at $f(x)$ *har to heltallige ekstremumspunkter, to rationelle ekstremumspunkter to rationelle vendepunkter og mindst tre heltallige rødder.*

$$f(x) = -216 l_1 l_2^2 m^3 n r_1 r_2^2 s^2 x + 216 l_1 l_2^2 m^3 n r_2^3 s^2 x_0 x - 540 l_1 l_2^2 m^2 n^2 r_1 r_2^2 s^2 x + 540 l_1 l_2^2 m^2 n^2 r_2^3 s^2 x_0 x + 36 l_1 l_2^2 m^2 r_1 r_2^2 s x^2 - 36 l_1 l_2^2 m^2 r_2^3 s x_0 x^2 - 216 l_1 l_2^2 m n^3 r_1 r_2^2 s^2 x + 216 l_1 l_2^2 m n^3 r_2^3 s^2 x_0 x + 144 l_1 l_2^2 m n r_1 r_2^2 s x^2 - 144 l_1 l_2^2 m n r_2^3 s x_0 x^2 + 36 l_1 l_2^2 n^2 r_1 r_2^2 s x^2 - 36 l_1 l_2^2 n^2 r_2^3 s x_0 x^2 - 12 l_1 l_2^2 r_1 r_2^2 x^3 + 12 l_1 l_2^2 r_2^3 x_0 x^3 + 216 l_2^3 m^3 n r_1 r_2^2 s^2 x_0 x - 216 l_2^3 m^3 n r_2^3 s^2 x_0^2 x + 540 l_2^3 m^2 n^2 r_1 r_2^2 s^2 x_0 x - 540 l_2^3 m^2 n^2 r_2^3 s^2 x_0^2 x - 36 l_2^3 m^2 r_1 r_2^2 s x_0 x^2 + 36 l_2^3 m^2 r_2^3 s x_0^2 x^2 + 216 l_2^3 m n^3 r_1 r_2^2 s^2 x_0 x - 216 l_2^3 m n^3 r_2^3 s^2 x_0^2 x - 144 l_2^3 m n r_1 r_2^2 s x_0 x^2 + 144 l_2^3 m n r_2^3 s x_0^2 x^2 - 36 l_2^3 n^2 r_1 r_2^2 s x_0 x^2 + 36 l_2^3 n^2 r_2^3 s x_0^2 x^2 + 12 l_2^3 r_1 r_2^2 x_0 x^3 - 12 l_2^3 r_2^3 x_0^2 x^3$$

$$g(x) = -216 l_1 l_2^2 m^3 n r_1 r_2^2 s^2 x + 108 l_1 l_2^2 m^3 n r_2^3 s^2 x_0^2 + 108 l_1 l_2^2 m^3 n r_2^3 s^2 x^2 - 540 l_1 l_2^2 m^2 n^2 r_1 r_2^2 s^2 x + 270 l_1 l_2^2 m^2 n^2 r_2^3 s^2 x_0^2 + 270 l_1 l_2^2 m^2 n^2 r_2^3 s^2 x^2 - 36 l_1 l_2^2 m^2 r_1 r_2^2 s x_0^2 + 72 l_1 l_2^2 m^2 r_1 r_2^2 s x_0 x - 36 l_1 l_2^2 m^2 r_2^3 s x_0 x^2 - 216 l_1 l_2^2 m n^3 r_1 r_2^2 s^2 x + 108 l_1 l_2^2 m n^3 r_2^3 s^2 x_0^2 + 108 l_1 l_2^2 m n^3 r_2^3 s^2 x^2 - 144 l_1 l_2^2 m n r_1 r_2^2 s x_0^2 + 288 l_1 l_2^2 m n r_1 r_2^2 s x_0 x - 144 l_1 l_2^2 m n r_2^3 s x_0 x^2 - 36 l_1 l_2^2 n^2 r_1 r_2^2 s x_0^2 + 72 l_1 l_2^2 n^2 r_1 r_2^2 s x_0 x - 36 l_1 l_2^2 n^2 r_2^3 s x_0 x^2 + 24 l_1 l_2^2 r_1 r_2^2 x_0^3 - 36 l_1 l_2^2 r_1 r_2^2 x_0^2 x - 6 l_1 l_2^2 r_2^3 x_0^4 + 18 l_1 l_2^2 r_2^3 x_0^2 x^2 + 108 l_2^3 m^3 n r_1 r_2^2 s^2 x_0^2 + 108 l_2^3 m^3 n r_1 r_2^2 s^2 x^2 - 144 l_2^3 m^3 n r_2^3 s^2 x_0^3 - 72 l_2^3 m^3 n r_2^3 s^2 x^3 + 270 l_2^3 m^2 n^2 r_1 r_2^2 s^2 x_0^2 + 270 l_2^3 m^2 n^2 r_1 r_2^2 s^2 x^2 - 360 l_2^3 m^2 n^2 r_2^3 s^2 x_0^3 - 180 l_2^3 m^2 n^2 r_2^3 s^2 x^3 - 36 l_2^3 m^2 r_1 r_2^2 s x_0 x^2 + 12 l_2^3 m^2 r_2^3 s x_0^4 + 24 l_2^3 m^2 r_2^3 s x_0 x^3 + 108 l_2^3 m n^3 r_1 r_2^2 s^2 x_0^2 + 108 l_2^3 m n^3 r_1 r_2^2 s^2 x^2 - 144 l_2^3 m n^3 r_2^3 s^2 x_0^3 - 72 l_2^3 m n^3 r_2^3 s^2 x^3 - 144 l_2^3 m n r_1 r_2^2 s x_0 x^2 + 48 l_2^3 m n r_2^3 s x_0^4 + 96 l_2^3 m n r_2^3 s x_0 x^3 - 36 l_2^3 n^2 r_1 r_2^2 s x_0 x^2 + 12 l_2^3 n^2 r_2^3 s x_0^4 + 24 l_2^3 n^2 r_2^3 s x_0 x^3 - 6 l_2^3 r_1 r_2^2 x_0^4 + 18 l_2^3 r_1 r_2^2 x_0^2 x^2 - 12 l_2^3 r_2^3 x_0^2 x^3$$

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