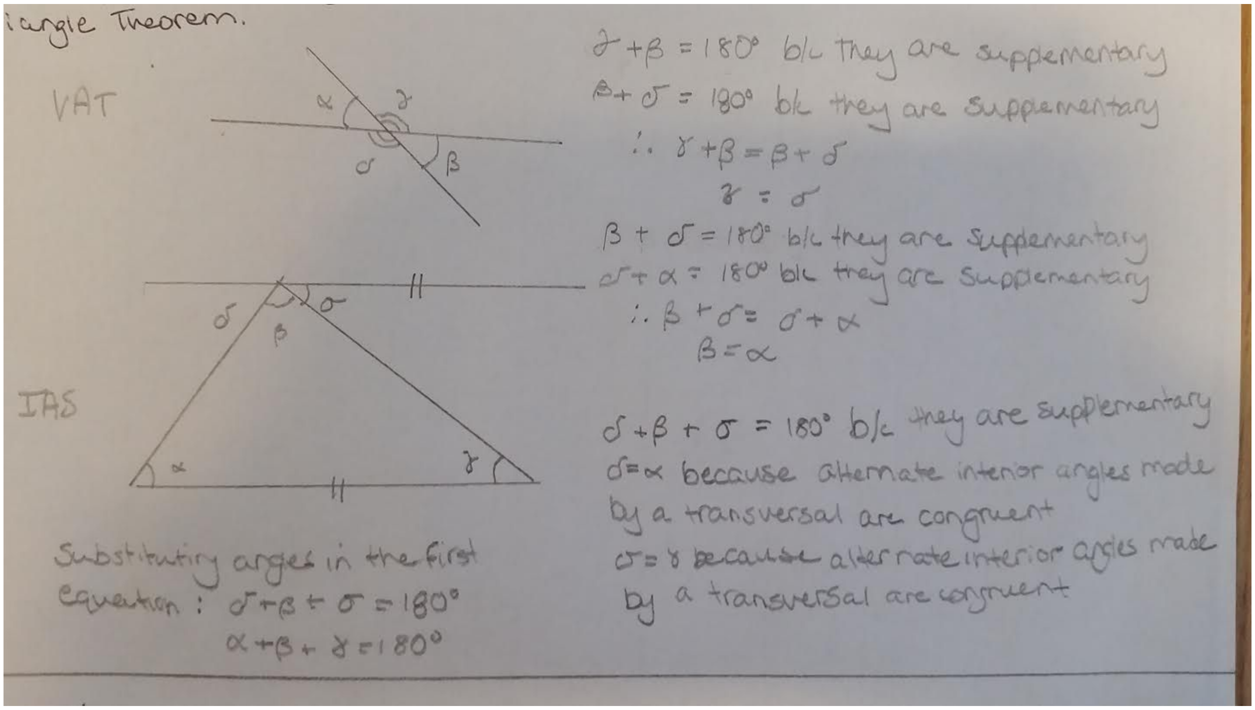
MTH 362 Study Guide Exam 1

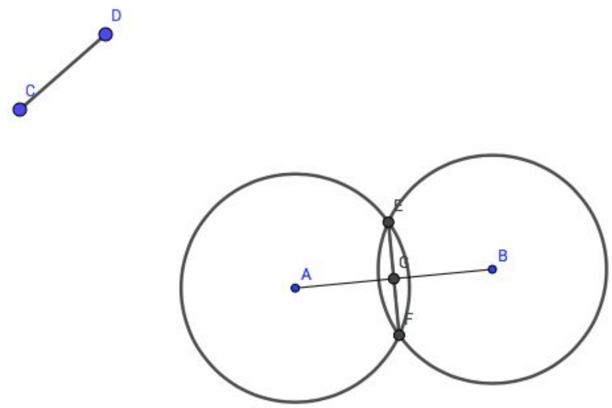
System of Euclidean Geometry

1. Describe the building blocks of Euclidean geometry.
    a. Point, line, and plane - undefined terms used to create definitions. Definitions are used to create postulates. Postulates are used to create theorems. Theorems are conclusions.
2. Who was the first to propose geometry as a system and when was it?
    a. Euclid 300 BC
3. Look at the HS CCSS and identify theorems, the proof of which should be discussed in high schools.
    a. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
    b. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
    c. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
4. Prove the Vertical Angles Theorem. Prove the Interior Angle Sum in a Triangle Theorem

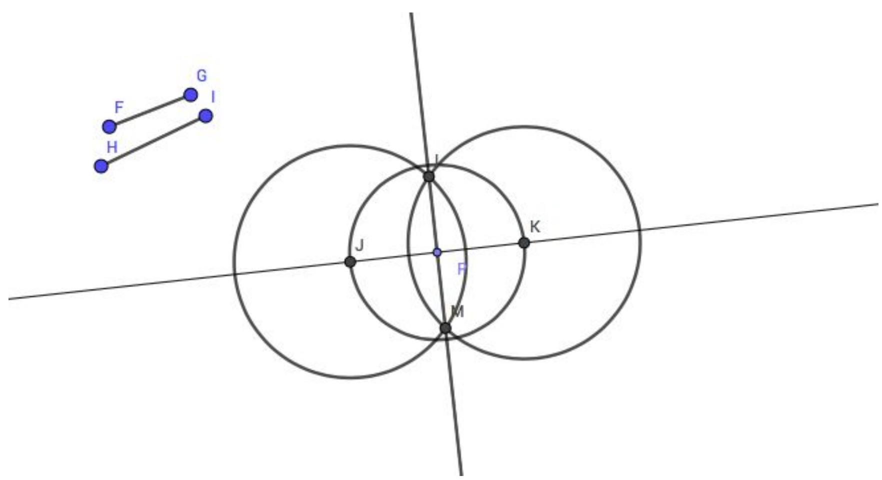


Constructions

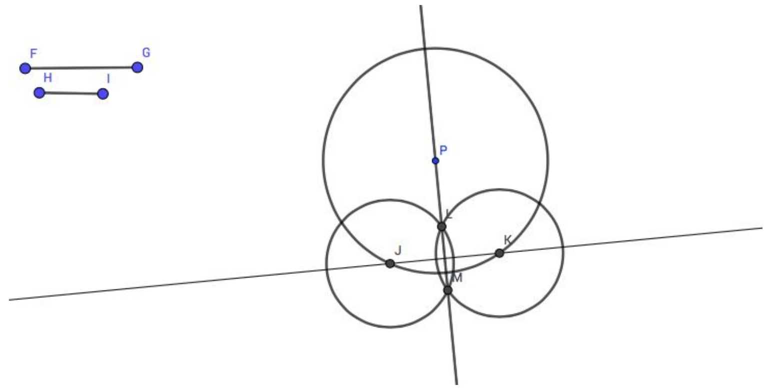
1. Discuss what we mean by elementary Euclidean constructions and how they are different from constructions that are possible by using technology
    a. These are constructions only done using a compass and a straight edge. These constructions are not dynamic like TI-Nspire and Geogebra.
2. Name two tools of elementary Euclidean constructions. Which Euclid's postulates are referring these tools?
    a. Compass & Straight edge. Draw a straight line from any point to any point, produce a finite straight line continuously in a straight line, to describe a circle with any center and radius, #5
3. Elementary constructions
    a. Midpoint



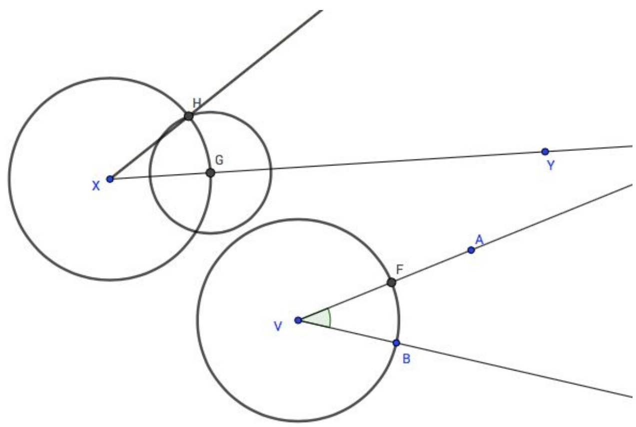
Perpendicular Line 1



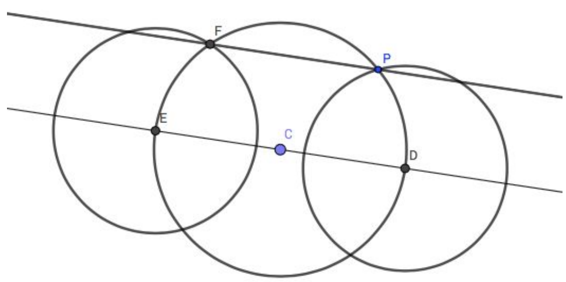
Perpendicular Line 2



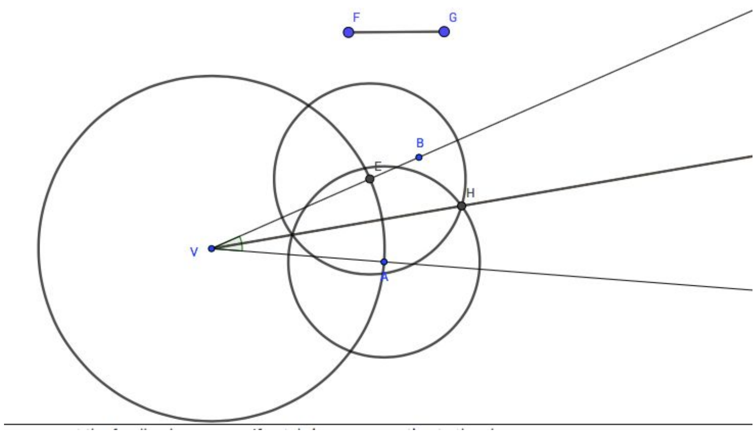
Transferring an Angle



Parallel Line



Angle Bisector



4. List several different tools and methods to make formal geometric constructions (Compass & ruler, paper-folding, technology). Pick a construction and demonstrate it using 3 different methods. Briefly discuss pros/cons, similarities/differences of these methods.

a. Pros of Technology: Dynamic
b. Con of Technology: You have to teach students how to use the software, Wi-Fi does not work on your calculator so you keep losing your files!!
c. Pro of Paper Folding: Good for hands-on learners, tangible
d. Pro of Compass: Good for hands-on learners, tangible
e. Cons of paper folding and compass: not Dynamic

5. Can Elementary constructions mentioned above be done by paper folding? Are there some that cannot be done by paper folding? Be specific and explain your reasoning.

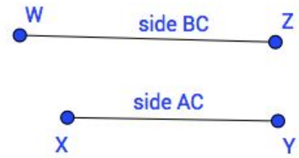
a. All of them can be done with paper folding except angle transfer. Angles can't be replicated by folding and therefore can't be replicated.

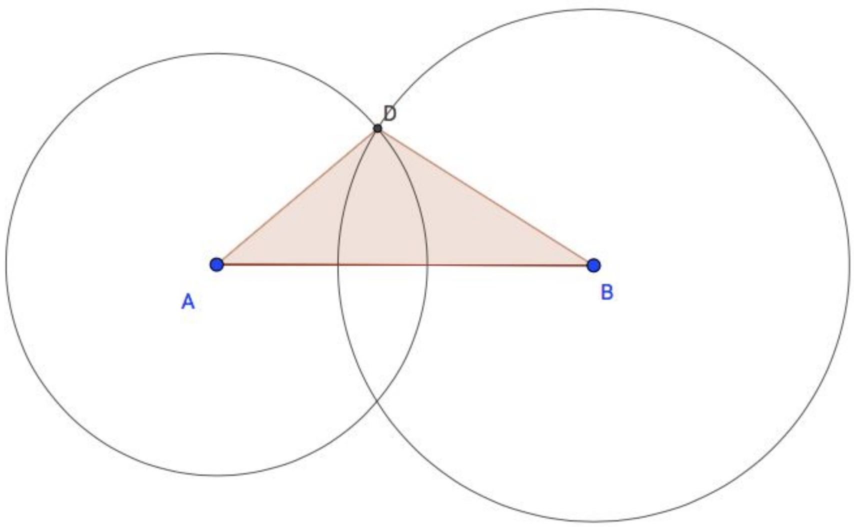
6. Construct the following:

A,b,c

Construct ΔABC such that AC ≅ XY and BC ≅ WZ.

This is a self-check applet that provides instant feedback. Make sure you construct the object exactly as it appears here (i.e. use polygon tool to draw the final triangle, don't just connect points with line segments).

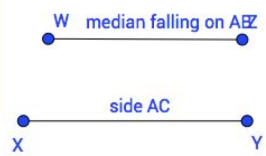


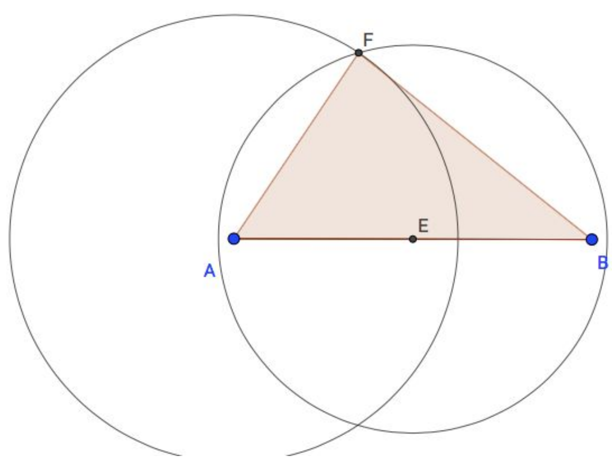


A,b,mb

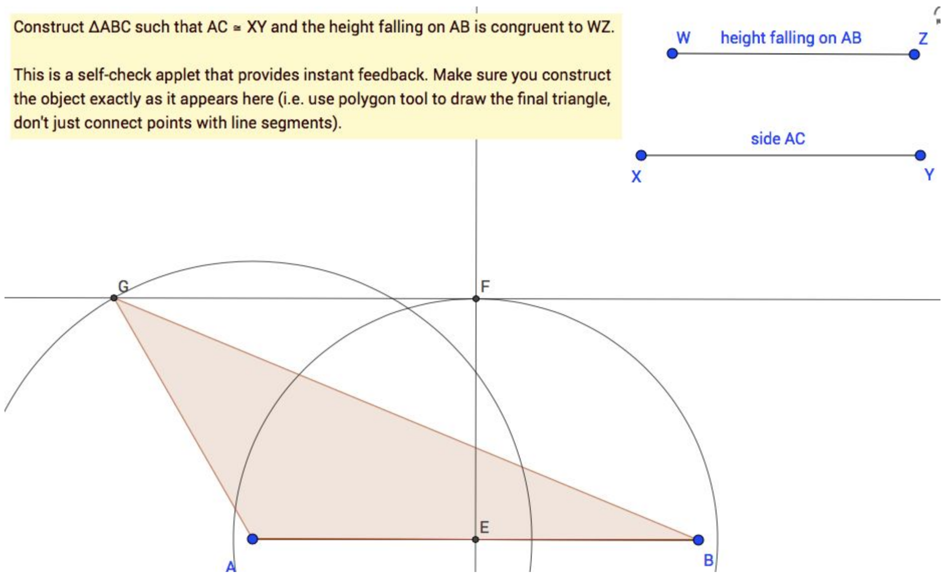
Construct ΔABC such that AC ≅ XY and the median falling on AB is congruent to WZ.

This is a self-check applet that provides instant feedback. Make sure you construct the object exactly as it appears here (i.e. use polygon tool to draw the final triangle, don't just connect points with line segments).

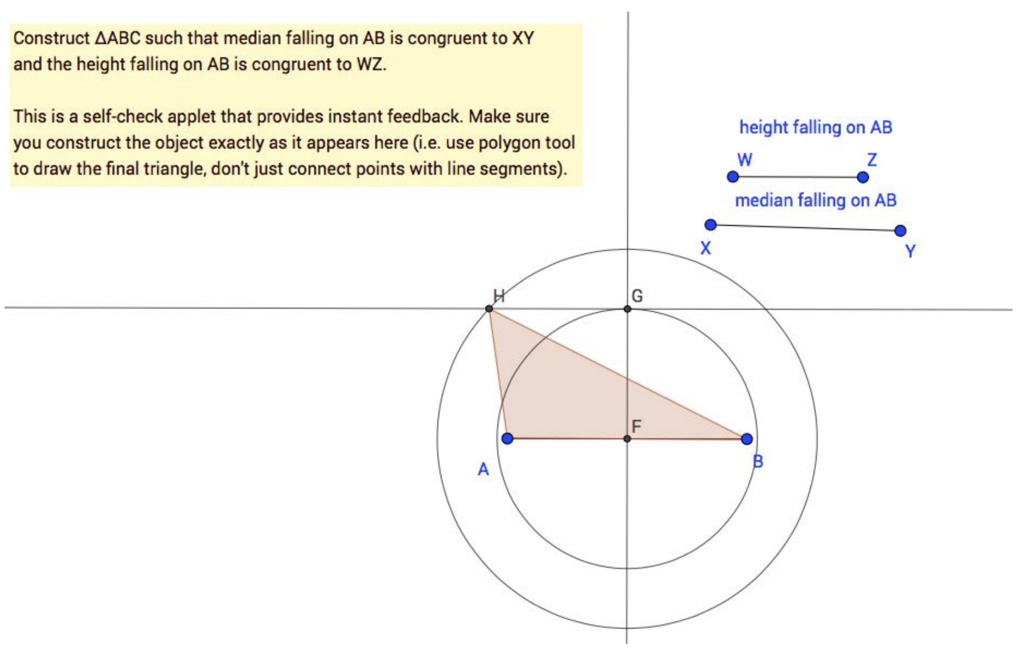




A,b,ha



A,ma,ha



Comment on the student's proof: The student did not find the height. He found the median.

There is no such triangle

Other Triangle Facts and Theorems

1. A student says “any median of a triangle is always longer than the height falling on the same side”. Comment on the student’s claim.
    a. It is correctamundo
2. Comment on a student’s claim: An acute triangle is a triangle with an acute angle.
    a. This statement is not quite correct. An acute triangle must have three acute angles not just one.

Triangle Inequality

1. Explain the difference between triangle inequality theorem and criterion
    a. Theorem: any two sides added together must be bigger than the 3rd side
       |AB|<|AC|+|BC| |BC|<|AB|+|AC| |AC|<|AB|+|BC|
    b. Criterion: don’t check all 3 conditions. Pick two shortest sides. If product of two shortest sides is longer than the longest side, a triangle is possible
2. Suggest two activities (one with manipulatives, one with technology) that will help students discover the theorem or criterion and explain why it works
    a. The colored sticks we used in class, give students different lengths and see who can make a triangle or not
    b. The applet on geogebra, have students play with it to see when they can construct a triangle at what conditions and when they cannot construct a triangle

Centers and Circles in a Triangle

1. You should be able to identify or construct the following objects in a triangle
    a. Altitudes, heights, medians
    b. Orthocenter, Circumcenter, Incenter, Centroid
    c. Circumcenter, incircle

Incenter- intersection of angle bisectors

Circumcenter- intersection of perpendicular bisectors (can be outside the triangle)

Median- line segment from midpoint to opposite vertex

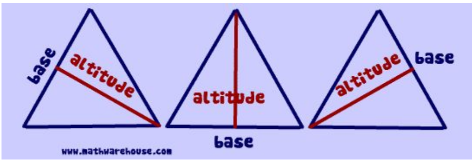
Centroid- where all medians in a triangle intersect (must be inside triangle)

Orthocenter- where the altitudes cross (can be inside or outside)

Height- perpendicular to the base

Euler’s line- the line all these points lie on https://www.geogebra.org/m/EdMBfMyW

Altitude- perpendicular from vertex to side opposite from vertex



Cool applet:

https://www.mathsisfun.com/definitions/altitude-geometry-.html

Pythagorean theorem

1. Formulate the theorem and prove it in at least 3 different ways
    a. Paper folding, cut out triangles two different ways, add up areas
2. Discuss two ways to generalize the pythagorean theorem
    a. Add up areas outside of the sides/similar shapes
    b. Formulate it for obtuse, right, and acute triangles

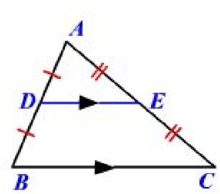
       Obtuse: a^2+b^2<c^2

       Right: a^2+b^2=c^2

       Acute: a^2+b^2>c^2

Midsegment theorem

1. Formulate the theorem and prove it
    a. In a triangle, the segment joining the midpoints of any two sides will be parallel to the third side and half its length



    b.

Midsegment quadrilateral

1. Formulate observations related to the following properties of midsegment quadrilateral and justify your observation
    a. Midsegment quadrilateral is always a parallelogram

b. Midsegment quadrilateral will be a rectangle if the original quadrilateral diagonals are perpendicular
c. Midsegment quadrilateral will be a square if the original quadrilateral is a square
d. How does the area of a midsegment quadrilateral relate to the area of the original quadrilateral? It is half the area of the original.