$Nice\ pair-wise\ quadratic\ polynomials.$

$Two\ quadratic\ real\ polynomials\ f(x)\ and\ g(x)\ is\ called\ a\ nice\ pair\ of\ quadratic\ polynomials\ if\ they\ fulfill\ the\ conditions\ below.$

$A: h(x) = f(x) - g(x)\ has\ two\ roots\ \rightarrow\ f(x)\ and\ g(x)\ has\ two\ intersection\ points.$

$B: f(x)\ and\ g(x)\ both\ have\ two\ roots\ and\ all\ six\ roots\ of\ f(x), g(x)\ and\ h(x)\ are\ distinct.$

$C: All\ the\ values\ associated\ with\ the\ points\ below\ are\ integers:$

$Coefficients, constants, roots\ and\ extremum\ points\ of\ f(x)\ and\ g(x). Roots\ of\ h(x).$

$A\ nice\ pair\ of\ quadratic\ polynomials\ is\ called\ a\ NPW\ pair.$

$Theorem.$

$Assume\ that\ (q, r, s, v) \in \mathbb{Z}^4\ and\ q \neq 0, r \neq 0, s \neq 0, v \neq 0$

$The\ parametrizations\ 1-5\ below\ of\ polynomials\ f(x)\ and\ g(x), which\ are\ subject\ to\ the\ attached\ conditions\ (CO), are\ subsets\ of\ the\ set\ of\ NPW\ pairs\ and\ the\ extremum\ of\ h(x)\ is\ an\ integer.$ [i]

$1: g(x) = (2q(r + s) + x)(2v(r - s) + x), f(x) = 2x\,(2\,(q\,s + r\,v) + x)$ [ii]

$CO: r \neq \pm s, v \neq \pm q, qs + r\,v \neq 0, s(q + v) + r(q - v) \neq 0$ [iii]

$2: g(x) = (2\,q(s - r) + x)(2v(r + s) + x), f(x) = 2x\,(2s\,v - 2qr + x)$ [iv]

$CO: s \neq \pm r, q \neq s, q \neq -v, sv - qr \neq 0\,, q(s - r) - v(r + s) \neq 0$ [v]

$3: g(x) = (2s(-q - v) + x)(2r(q - v) + x), f(x) = 2x\,(2(-\,q\,s\, -\, r\,v) + x)$

$CO: q \neq \pm v, r \neq \pm s, s(q + v) + r(q - v) \neq 0, qs + rv \neq 0$

$4: g(x) = (2s(v - q) + x)(2r(-q - v) + x), f(x) = 2x\,(2(vs - qr) + x)$

$CO: v \neq \pm q, vs - qr \neq 0, s(v - q) + r(q + v) \neq 0$

$5: g(x) = (2r(-q + v) + x)(2s(q + v) + x), f(x) = 2x\,(2(s\,v - qr) + x)$ [vi]

$CO: Exercise.$

$Heine\ Strømdahl, 12-06-2026.$ [vii]