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## Rational Functions Project

### Project for Rational Functions:

Goal: Analyze a Rational Functions and its graph.

Objectives:

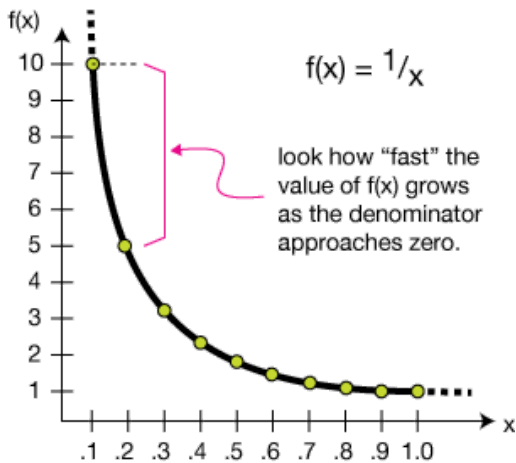
- **Recognize a rational function.**

Rational function: function that is a fraction and has the property that both its numerator and denominator are polynomials.

- **Explain why the denominator of a rational function cannot be zero thus recognizing these values as the places where vertical asymptotes occur and what they graphically look like.**

Zero is not a polynomial. By definition, Polynomial is an expression that can have constants, variables and exponents, that can be combined using addition, subtraction, multiplication and division, but:

- no division by a variable.
- a variable's exponents can only be 0,1,2,3,... etc.
- it can't have an infinite number of terms.



The figure on the left shows the upper-right quadrant of the graph of  $f(x) = 1/x$ . We know that  $x$  can never be zero, so zero doesn't appear in the domain, which is  $-\infty$  to  $\infty$ ,  $x \neq 0$ , which we also write as  $(-\infty, 0) \cup (0, \infty)$ . We say that  $x = 0$  is the location of a vertical asymptote.

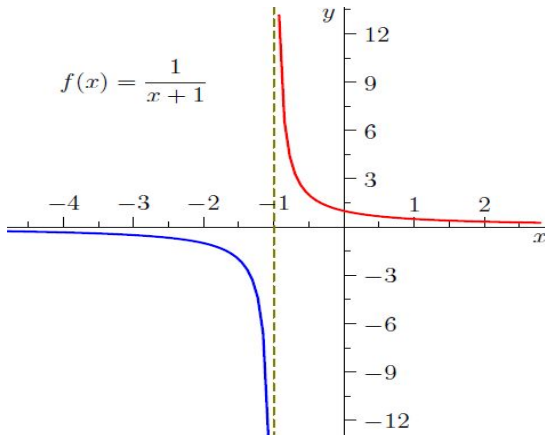
An asymptote of a curve looks like a line such that the distance between the curve and the line approaches zero as they tend to infinity.

- **Student will explain why the values where vertical asymptotes appear are excluded from domain of the function and thus the graph does not touch or cross them.**

In the case the asymptote is  $y=1/x$  the asymptote will never cross the X and Y lines since the more close the value of  $x$  is to 0 the greater will be the value of  $Y$  without touching it in any moment, since is impossible for the value of  $X$  to be 0 the value of  $X$  can be 0.0000...1 and by logic the value of  $Y$  will be 10000....000 also in the other way around the greatest the value of  $X$  will affect  $Y$  by making it very small. So by conclusion is impossible for a asymptote to touch the X and Y lines in the other cases you will need to find a imaginary X and Y lines where the asymptote will never touch, like the example given below in next question.

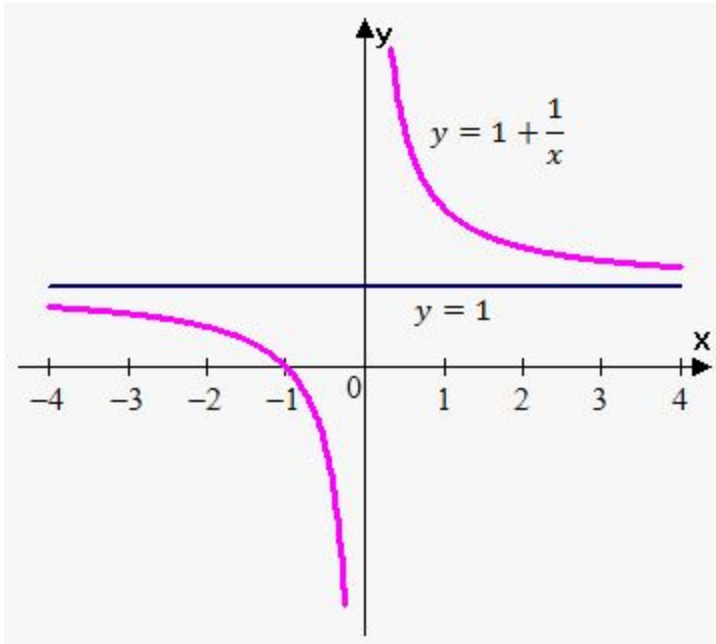
The way to find the vertical asymptotes is to take the denominator and equal it to zero. It will give me the values of  $x$  that can not be. When graphed, you can see how it avoids those values.

- Student will demonstrate what the graph of the function does as it approaches the vertical asymptote from the left and right.



As it is shown in the graph, as it reaches the vertical asymptote from left or right, it completely avoids crossing it. It goes up and down respectively. In the graph, it avoids the  $x = -1$ , and doesn't cross it. When it gets close, it goes up, or down, without touching it and increasing the value of  $Y$  infinitely.

- Student will be able to graphically recognize what a horizontal asymptote is. (is the x point that a parabola would never touch on a graph)



As it is shown in the graph, the horizontal asymptote is related by the function where  $f(x) = 1 + \frac{1}{x}$  is affected by adding 1, or any other number. For make it easier let's value that as a  $b$ . If the function is  $f(x) = b + \frac{1}{x}$ , the value that is in the variable  $b$  will be the horizontal asymptote. In case the value of  $b$  was a 5 the asymptote will never touch the imaginary line of  $y = 5$ .

**Learning Objective:** You will explain why the denominator of a rational function cannot be zero thus recognizing these values as the places where vertical asymptotes occur and graphically what vertical asymptotes look like and mean.

**Learning Activity:** You will use a word problem showing a real world application of rational functions (given), with a grading rubric to explain the possible effects of dividing by zero. The following is an actual mathematical model used for Cost-Benefit analysis. The model is a rational function. Read the situation and analyze what the solution should be using the algebraic techniques we have studied.

# Rational Functions Project

## Project for Rational Functions:

**Application of Rational Functions:** If you want to know why it is important to understand Rational Functions, consider the following.

This application is a Cost-Benefit Model. A utility company burns coal to generate electricity. The cost  $C$  (in dollars) of removing  $p$  amount (percent) of the smokestack pollutants is given by:



$$C(p) = \frac{80,000p}{(100-p)}$$

**Is it possible for the company to remove 100 percent of the pollutants? Discuss why or why not, and support your response by using algebraic analysis on the given model. Remember to write in complete sentences.**

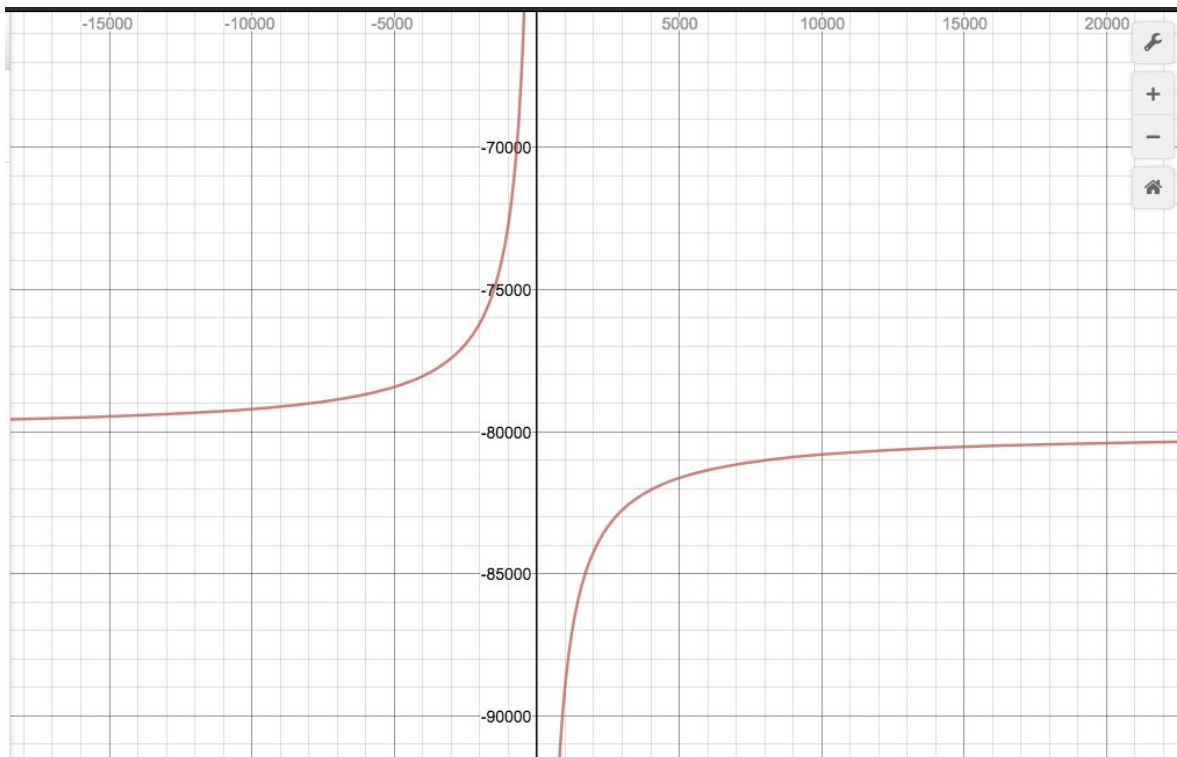
Is impossible since as we have seen before the value of the denominator of a rational graph can't be 0, you may remove 99.99999...999 percent but never the 100% since that will make the function as  $C(p) = 80,000(100)/0$ , and this is something that is called as an invalid operation.

**What happens if the company does try to remove 100 percent of the pollutants? Will the company be successful at doing so, or will the attempt end in failure, that is, will it be too much expense for the company? Explain your thoughts and remember to write in complete sentences.**

These will end as a failure since the more pollutants they remove, the more they expend, and by getting closer to the 100% the more cost will be, if they remove by an example a 99.999 percent, this will make the cost a  $80,000(99.999)/0.001$  making the amount of expense huge, so for the company is impossible to eliminate by 100% the pollutants.

Make a graph to show what the consequences of the last question would be. Pick your scale carefully so that all the information you want to discuss is visible on the graph. Remember to label the axes and show units and tick marks. Show the vertical asymptotes as dashed lines and label them. Then discuss their impact on the company's expense (Explain). There are numerous interactive graphing resources on the Internet that can be used. Google it!

Normal Graph  $(80000p)/(100-p)$ :



If they achieve to eliminate the 100% of the pollutants, the cost will be immense. It would end in a total failure and the ruin for the company. Both lines would meet, and no curve would exist. There would be no asymptote in the graph.

*This project is slightly adapted from one written by a Professor Rust. I do not know who he or she is so I cannot give more complete credit than that.*

SOURCE: [http://graybeard.wikispaces.com/file/view/Rational Functions Project.pdf](http://graybeard.wikispaces.com/file/view/Rational+Functions+Project.pdf)

REFERENCES:

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	Criteria				Points
	4	3	2	1	
<b>Explanation</b>	A complete response with a detailed explanation showing individual insight.	Response is a clear explanation, but no personal in depth details.	Explanation is unclear.	Misses key points.	—
<b>Use Of Visuals</b>	Clear diagram or sketch with details and labeling.	Diagram or sketch with no details or labeling.	Inappropriate or unclear diagram.	No diagram or sketch.	—
<b>Mechanics</b>	No math errors. Complete sentences and properly constructed paragraphs	No <b>major</b> math errors, serious flaws in reasoning, or major grammar and sentence structure problems	May be <b>some</b> serious math errors, flaws in reasoning, or grammar and sentence structure mistakes	Major math errors, serious flaws in reasoning, major grammar and sentence structure mistakes	—
<b>Demonstrated Knowledge</b>	Shows complete understanding of the questions, mathematical ideas, and processes, gives individual insight to problem.	Shows understanding of the problem, ideas, and processes, but no individual insight added only definitions given.	Response shows some understanding of the problem.	Response shows a complete lack of understanding for the problem.	—
<b>Requirements</b>	Goes beyond the requirements of the problem, explains concepts in detail enhancing answers with own insights and reasoning.	Meets the requirements of the problem, may explain concepts by stating definitions, instead of contributing own insights.	Hardly meets the requirements of the problem.	Does not meet the requirements of the problem.	—
				<b>Total----&gt;</b>	—

Teacher Comments: