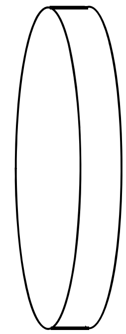
Accelerated AB Calculus

Name______________________

6.01 - A Visual Introduction to Volumes

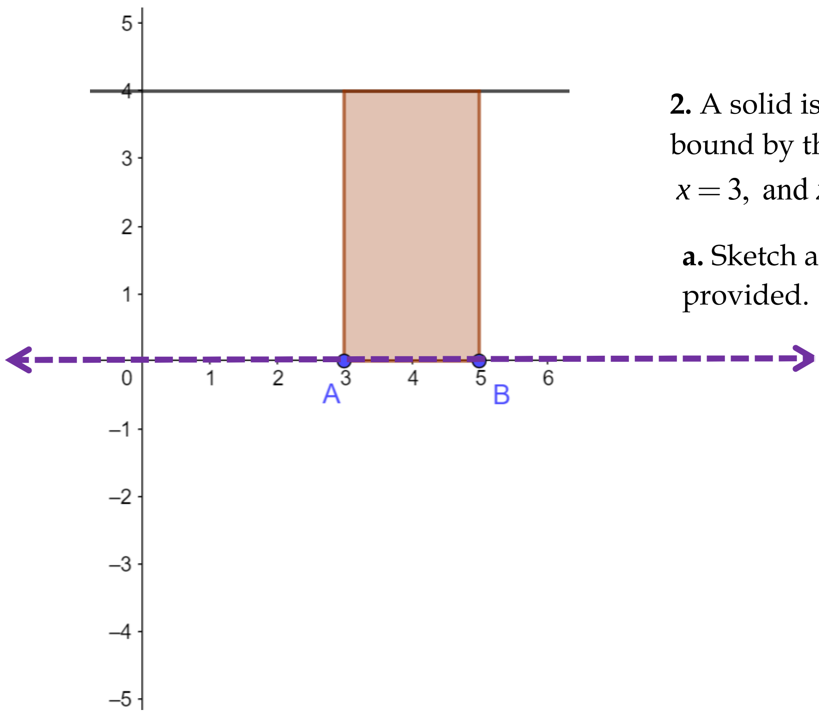
Opener/Warm-Up

1. By hand, make a sketch of a copy of the cylindrical "disk" to the right.



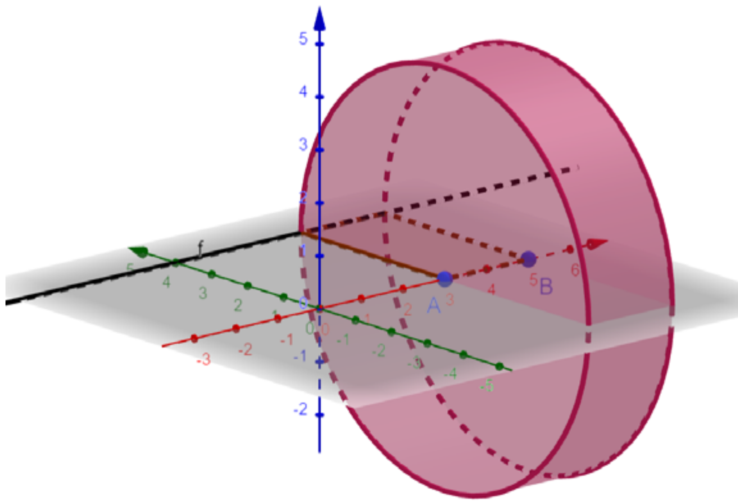
2. A solid is generated when revolving the region bound by the x-axis, the function $f(x)=4$, $x=3$, and $x=5$ around the x-axis.

a. Sketch a drawing of this solid on the grid provided.

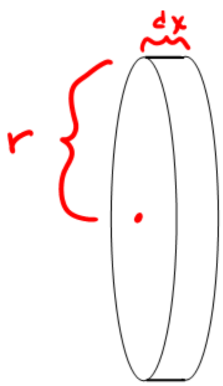


Axis of Revolution = x-axis

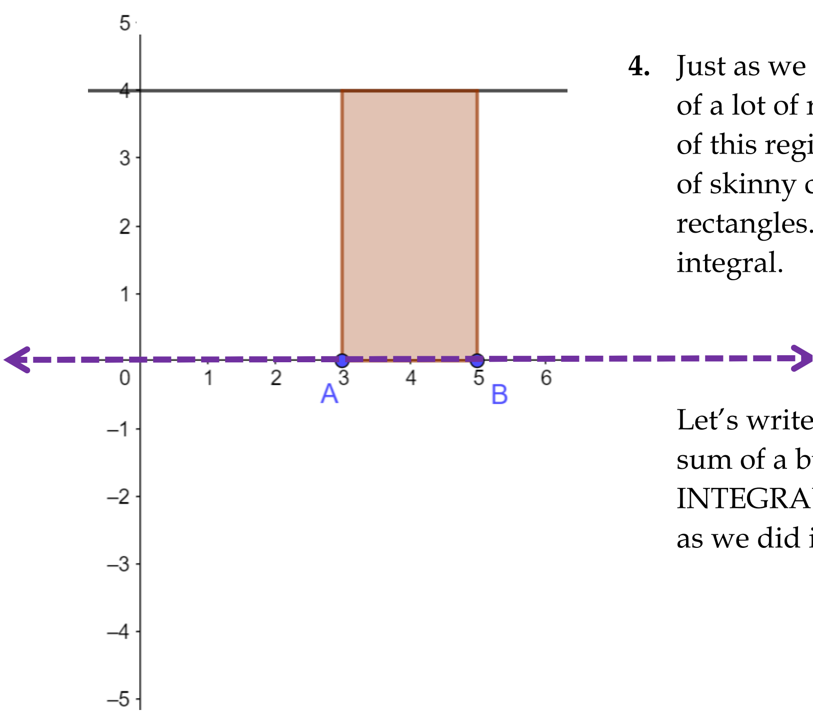
b. Find the volume of this solid.



Here is a computer drawn version of that "disk" from a different angle (the z-axis point upward).



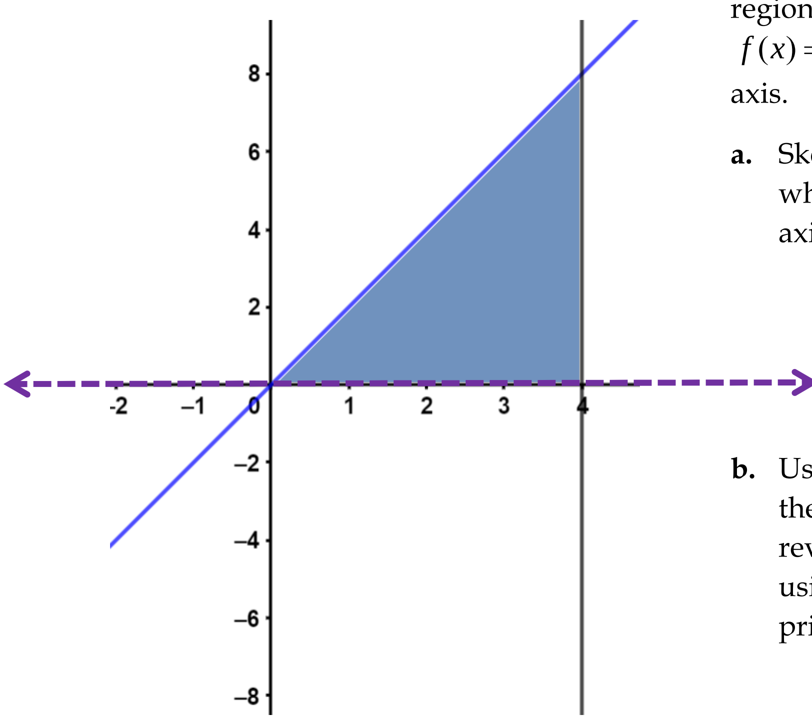
3. Write an expression for the volume of the disk on the left.



Axis of Revolution = x-axis

4. Just as we thought of the area of the region as a sum of a lot of really, really skinny rectangles, the volume of this region can also be thought of as the sum of a lot of skinny cylinders based upon revolving those same rectangles.   Hence… it can be thought of as an integral.

Let's write the volume of that solid in question 2 as a sum of a bunch of skinny cylinders (i.e. an INTEGRAL!!!) and show that we get the same results as we did in part b.



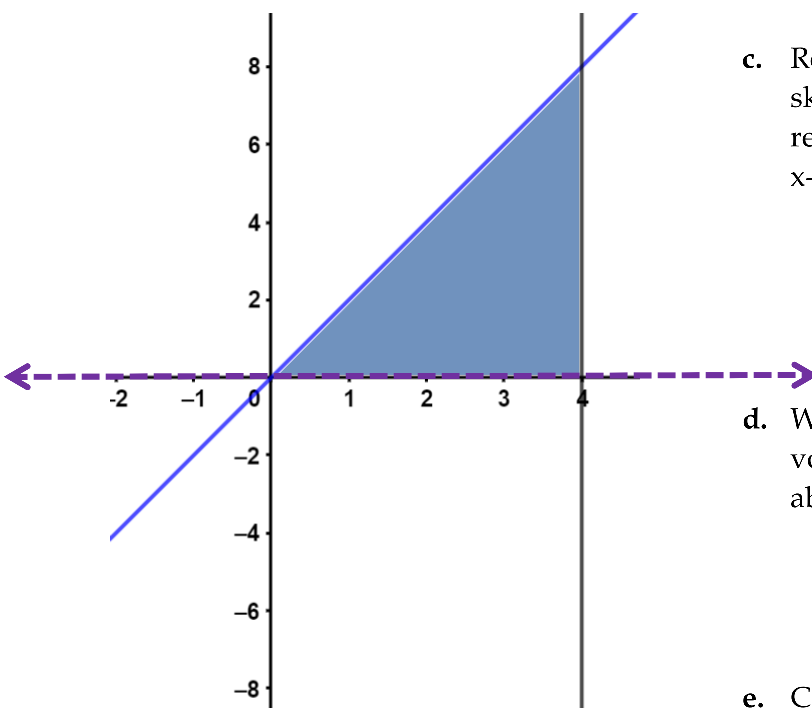
5. A solid is generated when revolving the region bound by the x-axis, the function $f(x) = 2x$, $y-axis$, and $x = 4$ around the x-axis.

a. Sketch a picture of the solid that is generated when this region is revolved around the x-axis.

Axis of Revolution = x-axis

b. Using your knowledge of geometry, what is the shape of the solid that is generated from revolving this region? Calculate its volume using no calculus, but only geometry principles.

Same problem…. Now a Calculus Based Solution.



c. Reflect the region over the x-axis and then sketch an example of a single arbitrary rectangle that has been revolved about the x-axis to form one of our "disks."

Axis of Revolution = x-axis

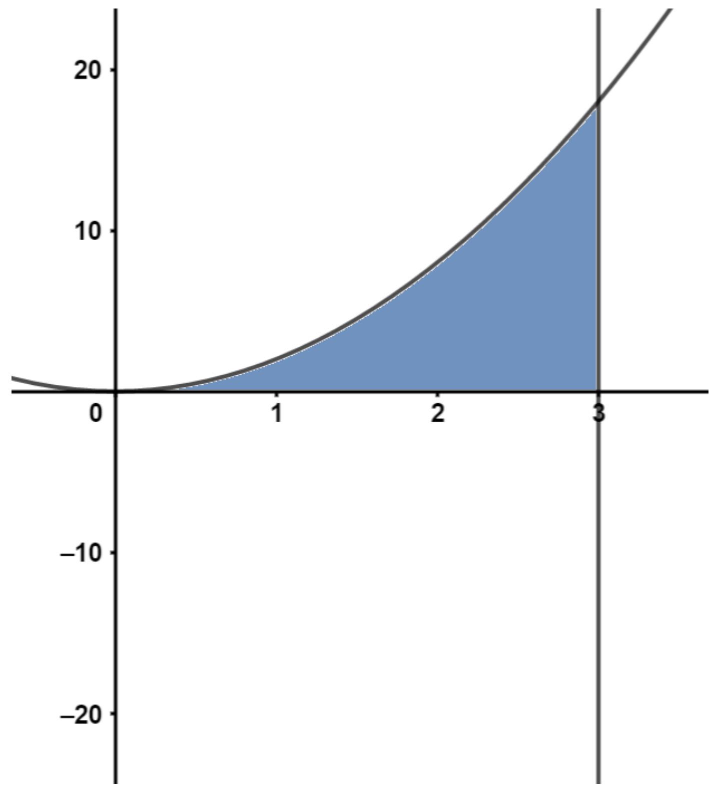
d. Write the integral that represent the volume of the solid formed by rotating R about the x-axis and then calculate it.

e. Compare you results to the geometric solution.

Suppose the region doesn't produce a familiar shape like a cylinder or a cone or a sphere?

We can use this technique of slicing it up into little cylinders that we can calculate!

6. Let R be the region bounded by the x-axis, y-axis, the line $x = 3$, and $y = 2x^2$. A solid is formed by rotating R about the x-axis.



a. Reflect the region over the x-axis and then sketch an example of a single arbitrary rectangle that has been revolved about the x-axis to form one of our "disks."

b. Write the integral that represents the volume of the solid formed by rotating R about the x-axis and then calculate it.