Sección 3,1

(27)

$y_p \rightarrow$ Solución particular de $\underbrace{y'' + py' + qy = f(x)}_{\text{No homogénea}}$

$y_c \rightarrow$ solución de ecuación homogénea asociado

Mostrar que:

$y = y_c + y_p$ es solución de

Entonces

$$y_p'' + py_p' + qy_p = f(x)$$

$$y_c'' + py_c' + qy_c = 0$$

$$(y_p + y_c)'' + p(y_p + y_c)' + Q(y_p + y_c) =$$

$$y_p'' + y_c'' + py_p' + py_c' + Qy_p + Qy_c =$$

$$\underbrace{y_p'' + py_p' + Qy_p}_{f(x)} + \underbrace{y_c'' + py_c' + Qy_c}_{0} = f(x)$$

$\boxed{y_p + y_c \text{ es solución}}$

(32) $A(x)y'' + B(x)y' + C(x)y = 0 \qquad A(x) \neq 0$

a) $W(y_1, y_2) = W$ Demuestre que

$$A(x)\frac{dW}{dx} = (y_1)Ay_2'' - (y_2)Ay_1''$$

$W(y_1, y_2) \neq 0 \rightarrow$ porque y_1 y y_2 son soluciones de la ecuación diferencial homogénea asociada

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \quad y_1 y_2' - y_1' y_2 = W$$

$$\frac{dW}{dx} = \frac{d}{dx}(y_1 y_2' - y_1' y_2)$$

$$= (\cancel{y_1' y_2'} + y_1 + y_2'') - (y_1'' y_2 + \cancel{y_1' y_2'})$$

$$A(x)\frac{dW}{dx} = (y_1 y_2'' - y_1'' y_2)A(x)$$

$$A(x)\frac{dW}{dx} = A(x)y_1 y_2'' - A(x)y_1'' y_2$$

$$Ay_1'' + By_1' + Cy_1 = 0 \rightarrow Ay_1'' = -By_1' - Cy_1$$

$$Ay_2'' + By_2' + Cy_2 = 0 \rightarrow Ay_2'' = -By_2' - Cy_2$$

$$A(x)\frac{dw}{dx} = (y_1)(-By_2' - Cy_2) - (y_2)(-By_1' - Cy_1)$$

$$A(x)\frac{dw}{dx} = -By_1y_2' - \cancel{Cy_1y_2} + By_1'y_2 + \cancel{Cy_1y_2}$$

$$A(x)\frac{dw}{dx} = -B(y_1y_2' - y_1'y_2)$$

$$A(x)\frac{dw}{dx} = -B(x)\,w(x)$$

b)

$$w(x) = Ke^{\left(-\int \frac{B(x)}{A(x)}dx\right)}$$

$$\underbrace{\frac{1}{\cancel{A(x)}}\cdot\left(\cancel{A(x)}\frac{dw}{dx} + B(x)\,w\right)}_{\text{Ec. Lineal primer orden}} = 0$$

$$\boxed{\begin{array}{l} p(x) = \frac{B(x)}{A(x)} \\ Q(x) = 0 \\ \rho(x) = e^{\int p(x)dx} \end{array}}$$

$$e^{\int \frac{B(x)}{A(x)}dx}\frac{dw}{dx} + e^{\int \frac{B(x)}{A(x)}dx}\frac{B(x)}{A(x)}w = 0$$

$$\int \cancel{dx}\frac{d}{\cancel{dx}}\left(we^{\int \frac{B}{A}dx}\right) = \int 0\,dx$$

$$we^{\int \frac{B}{A}dx} + C_1 = C_2$$

$$we^{\int \frac{B}{A}dx} = K$$

$$\omega(x) = Ke^{-\int \frac{B}{A} dx}$$

• Para que $\omega = 0$ entonces $K = 0$

• $e^{-\int \frac{B}{A} dx} \neq 0$ siempre

(51)

$$ax^2 y'' + bxy' + cy = 0 \qquad v = \ln x$$

a)

$$\frac{dv}{dx} = \frac{1}{x}$$

$$y' = \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx} = \frac{dy}{dv} \cdot \frac{1}{x}$$

$$y'' = \frac{d^2y}{dx} = \frac{d^2y}{dv^2} \cdot \frac{dv}{dx} \frac{1}{x} - \frac{dy}{dv} \cdot \frac{1}{x^2}$$

$$= \frac{d^2y}{dv^2} \cdot \frac{1}{x^2} - \frac{dy}{dv} \cdot \frac{1}{x^2}$$

$$a\cancel{x^2}\left(\frac{d^2y}{dv^2} \cdot \frac{1}{\cancel{x^2}} - \frac{dy}{dv} \cdot \frac{1}{\cancel{x^2}}\right) + b\cancel{x}\left(\frac{dy}{dv} \cdot \frac{1}{\cancel{x}}\right) + cy = 0$$

$$a\frac{d^2y}{dv^2} - a\frac{dy}{dv} + b\frac{dy}{dv} + cy = 0$$

$$a\frac{d^2y}{dv^2} + (b-a)\frac{dy}{dv} + cy = 0$$

b)

$$ar^2 + (b-a)r + c = 0 \rightarrow r_1 \text{ y } r_2 \text{ como reales y } \neq$$

$$y(v) = C_1 e^{r_1 v} + C_2 e^{r_2 v}$$

$$y(x) = C_1 e^{r_1 \ln x} + C_1 e^{r_2 \ln x} = C_1 x^{r_1} + C_2 x^{r_2}$$

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$$x^2y'' + 2xy' - 12y = 0$$

$$ax^2y'' + bxy' + cy = 0$$

$a = 1 \quad b = 2 \quad c = -12$

$v = \ln x$

$$a\frac{d^2y}{dv} + (b-a)\frac{dy}{dv} + cy = 0$$

$$(1)\frac{d^2y}{dv} + (2-1)\frac{dy}{dv} - 12y = 0$$

$$r^2 + r - 12 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r_1 = 3 \quad r_2 = -4$$

$$y(v) = C_1 e^{3v} + C_2 e^{-4v}$$

$$y(x) = C_1 e^{3\ln x} + C_2 e^{-4\ln x}$$

$$y(x) = C_1 x^3 + C x^{-4}$$

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$$x^2y'' + xy' = 0$$

$$ax^2y'' + bxy' + cy = 0$$

$a = 1 \quad b = 1 \quad c = 0$

$v = \ln x$

$$\frac{a\,d^2y}{dv} + (b-a)\frac{dy}{dv} + cy = 0$$

$$(1)\frac{d^2y}{dv} + \cancel{(1-1)\frac{dy}{dv}} + \cancel{(0)y} = 0$$

$$r^2 = 0 \quad r_1 = 0 \quad r_2 = 0$$

$$y(v) = C_1 e^{rv} + C_2 e^{rv} v$$

$$y(v) = (C_1 + C_2 v) e^{rv}$$

$$y(x) = C_1 + C_2 \ln x$$