LESSON PLAN – How Long is a Day?

TEACHER		**DATE**		**of**	
CLASS		**YEAR**		**ROOM**	
PERIOD		**TIME**		**DURATION**	

OVERAL AIMS OF LESSON – This lesson plan was trialled with two secondary maths teachers, using GeoGebra as if they were students in the classroom setting, to gage successes, pitfalls, viability and transferability.

The overall aims for future use are for learners to understand how to use mathematical modelling via GeoGebra to solve real-world problems and appreciate the heuristics and limitations and assumptions inherent to them. Specifically: to enhance learners' visualisation and thus ability to solve A'-Level trigonometry problems.

KEY WORDS: Cosine, arccosine, tangent (function), angular speed

NEW WORDS: frame of reference, tangent (line), geodesic, great circle, meridian, equator, longitude, latitude, oblate spheroid, obliquity of the ecliptic/axial tilt, orbit, solstice, equinox (vernal/autumnal), zenith, apogee, azimuth, sidereal year, analemma.

OBJECTIVES: By the end of the lesson pupils will be able to:

All: explain the variation in day length in terms of spherical geometry (with pencil and paper) and read and interpret a time graph in GeoGebra; see the benefits and limitations of mathematical modelling;

Most: substitute algebraic expressions, solve an equation with an inverse trigonometric function and plot and fit a time graph curve to data points in GeoGebra;

Some: derive a formula for day length by visualising 2D trigonometric relations within a 3D system, using a mathematical model (on-screen and on paper).

Know, understand, have experienced key ideas. (linked to Exam syllabus)	Used, developed, gained certain skills. (linked to Exam Syllabus)	Demonstrated, developed certain values and attitudes. (linked to Exam Syllabus)

What is the key mathematical point of the lesson?

- Mathematical modelling, whilst limited in accuracy, can extract meaningful quantitative and graphical solutions to real-world problems;
- By adopting a suitable frame of reference, a complex problem can be broken down into one in which variable dependencies can be evaluated;
- 3D trigonometry can be decomposed to 2D trigonometry so that relationships purely between angles can be found by introducing lengths as "dummy variables" that ultimately cancel.

Is a numeracy element included?
Yes – BIDMAS, fractions, subtraction of complementary angles, cancelling by division.

What is the PURPOSE OF THE LESSON?
To engage students and deepen their understanding of prior knowledge by application to an innovative real-world context

What will pupil know by the end of it that they did not know at the start?

- Proportionality of angular speed with angles and arc lengths as an analogue to linear speed;
- New ways of representing 3D systems in 2D efficiently;
- How to use GeoGebra to create an interactive graph with a slider;
- Various GeoGebra user features.

Will all the pupils be expected to cover all objectives?
No – see differentiated LOs above.

POINTS FROM PREVIOUS LESSON (Refer to previous weeks evaluation)

What is the assumed learning?

- KS4-5 trigonometry – sine, cosine, tangent as functions and their inverses.
- Algebraic substitution and rearranging formulae.
- Basic GeoGebra (geometric calculator version).

Will this be tested?
Yes – via formative assessment during web book applet activities.

TEACHER ACTIVITY	**STUDENT ACTIVITY**
Starter SLIDES 1 & 2 (SNOWY TWILIGHT & STARTER QUESTION) – Think-Pair-Share and lead whole-class discussion on:	Students in groups of 2 per screen. Suggest reasons in class discussion – sketch diagrams, discuss with partner first.
• Time measures (solar or mechanical) and zones (GMT), energy saving, tilt of Earth as reason for seasons;	
• New words and links with geography, history and astrophysics;	• Suggest meanings of new words.
• Relate problem to real life – e.g. Ramadan sunset and sunrise times are published for Suhoor and Iftar, but how are they predicted and why are they not entirely adhered to by clerics in Saudi Arabia who announce the start and finish times?	
• Assumptions when modelling. E.g.: spherical Earth; circular orbit; negligible refraction; negligible solar aberration; negligible axial nutation; independence of spin from orbit (sidereal year); empirical constant accuracies.	• Suggest assumptions and why they are necessary.
WORKSHEET • Demo using the *Geoastronomy* GeoGebra book applets online on and opening the GeoGebra desktop app (if necessary installing beforehand) on screen.	
• Show how to manage animations with play button and in Algebra pane. • Pre-empt slider issues by demoing best practice – slow, methodical usage of one at a time; • **Plenty of time is needed to explore and experiment with each applet carefully – focus on slider extremes, significant values and explanations**	• Work through worksheet questions with partner – discuss with teacher. **Use PaP as much as possible to either explain or capture screen images.**
• Formatively assess work by circulating and use mini-plenaries to address what is meant by “frames	• Suggest answers during mini-plenaries. Give examples of frames of reference.

of reference", common errors and best practice hacks. • When the majority of students are on question 6, ask them to Think-Pair-Share and feed back on answer 6.f.	• Consider question 6.f. in pairs and share findings in class discussion.
SLIDE 3 (TWITTER POST OF SUNSETS) • Discuss the photos in terms of the compass on the applet. Ask: are they taken at the same time every day (as was suggested online)? and explore reasoning.	
Ensure that **some** are working on questions 8.d. and 9.e., provide guidance and divide into workstreams if necessary.	**Some** will be working on 8.d. and 9.e. to derive the two key formulae. If stuck, discuss with partner to be able to accurately describe the key issue preventing completion of the task.
SLIDE 4 (PHOTO OF STONEHENGE SOLSTICE) • Discuss ancient astronomy, druids and their technology (trilithon mystery). • For question 11 - ask students to describe summer solstice in terms of their applet sliders. • Ask students to demo other key annual events and describe the relevance of the yellow arrow/band.	• Demo solstice on screen. • Demo annual events and note and explain slider values. • Analyse yellow arrow/band and verbalise its significance.
SLIDE 5 (SPIN & ORBIT VARIABLES) – Lead whole-class discussion to: • Discuss ranges of values certain variable have (e.g. T) and why. • Discuss the analogue of angular speed in terms of prior learning and methods for memorising its computation (e.g. S-D-T triangle). • Ask those who were stuck on 8.d. and 9.e. how they would use this information to complete these questions.	• Suggest aide memoire for angular speed computations.
SLIDE 6 (IMPORTANT EQUATIONS) • Clarify all variables on screen and how they are calculated. • Give the whole class the opportunity to work out the final formula for Δt. • Discuss the strategies required (algebraic substitution followed by making Δt the subject). • Discuss the meaning of arccos and what type of function it is (inverse; non-injective). What other values can it have and why (what is the meaning of $360^{\circ}n$+answer in terms of days?).	• Suggest meanings and values for variables in question. • Work out the substitution and rearrangement.
SLIDE 7 (WHICH CAPITAL CITY?) • Explain that students are going to work out which capital city the data they have could be for. • First ask students to try question 12-19 in their pairs and to find and import the spreadsheet data into GeoGebra. • Highlight the automated syntax correction of "arccos" to "ACOS" by GeoGebra. • Discuss the importance of BIDMAS and keeping track of open and closed brackets.	• Attempt worksheet questions 12-19 in pairs.

• Demo answers on screen with discussion on question 13 over what should be y and x in the function plot. • Discuss best strategy for question 19 using their slider. • Note that the answer to question 19 (London or Nursultan) is another example of more than one solution (as with non-injective functions)	• Use best-fitting GPS latitude in Google Maps or Google which capital city is at this latitude.
Plenary SLIDE 8 (WHAT DID YOU LEARN?) – students to feed back learning outcomes (e.g. plenary pyramid, three stars and a wish). Ask students to recall the formulae derived and explain the meaning and ranges of variables.	• Provide feedback and answer questions when prompted.
Homework SLIDE 9 (2 QUESTIONS) – Urge students to look over the relevant applets (ask which these might be) and **draw/sketch diagrams to understand problems**. SLIDE 10 (UNIVERSE PIC AND QUOTE) – leave up for students to ruminate on as they pack up.	• Complete homework prior to commencement of next worksheet.

DIFFERENTIATION: (Support materials) (Extension materials)

Attach worksheets.

What are your exemplars/worked examples?
Provided by applets – diagrams, interactive frames of reference

How will your teaching promote progression in understanding of the topic/concept?
Real-world context and re-exposure to prior learning new varieties of trigonometric problem (in line with Bruner's Spiral Curriculum)

RESOURCES

What equipment will you be using?

Laptop (Apple MacBook Air) connected to internet with large LG TV screen; PowerPoint slides; GeoGebra web book applet and desktop apps; scientific calculators; PaP.

Recording equipment: Nikon D3500 DSLR; Android phone video app

What links will be made with the next lesson?
Same theme will be revisited in the context of making a sundial – alternative solutions to the day length formula will be found using different mathematical models and these will be critiqued in terms of relative strengths and weaknesses.