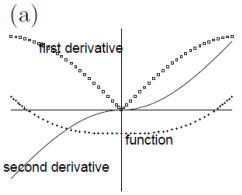
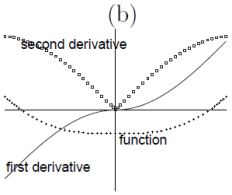
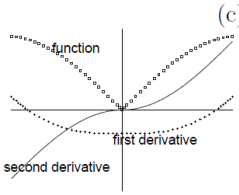
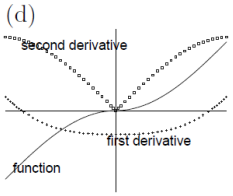
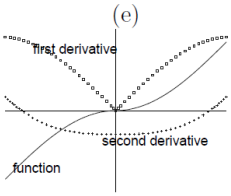
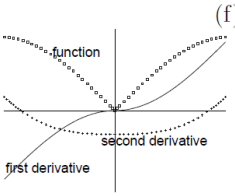
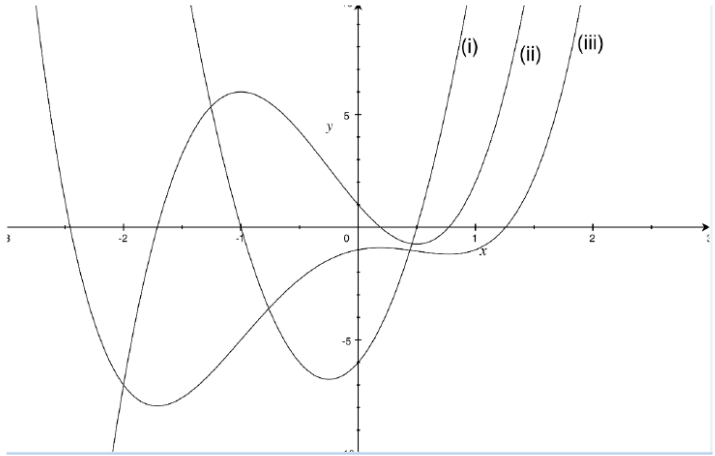
Afgeleide betekenis en toepassingen

Virga Jessecollege **Toets WISKUNDE**
Hasselt **leerkracht: Karel Appeltans**
Datum: **schooljaar……**
Klas: V **Afdeling:**
Naam:..... **Aantal uren wiskunde: 6**

1	3. Let $f(x)=\begin{cases} ax^3-6x, & \text{if } x\le 1 \\ bx^2+4, & \text{if } x>1 \end{cases}$ If the $f(x)$ is differentiable, then $a=$ (a) 0 (b) 1 (c) -14 (d) -24 (e) 26
2	The following shows the graph of a function $f(x)$, together with the graph of its first derivative $f'(x)$ and its second derivative $f''(x)$. Each answer shows the same three graphs, but only one is labelled correctly. Which one? (a)  (b)  (c)  (d)  (e)  (f) 
3	10. If the function $f(x)=\begin{cases} 3ax^2+2bx+1, & x\le 1 \\ ax^4-4bx^2-3x, & x>1 \end{cases}$ is differentiable for all real values of x, then $b=$ A) $\frac{-11}{4}$ B) $\frac{1}{4}$ C) $\frac{-7}{16}$ D) 0 E) $\frac{-1}{4}$ F) $\frac{11}{4}$
4	**3.** [points] For each of the problems below, circle all of the correct answers. If none of the answer choices provided are correct, circle NONE OF THESE. **a.** [points] Let $s(t)=\begin{cases} t^3+8t^2+6t & \text{if } t\le c \\ 4t^2+2t & \text{if } t>c \end{cases}$ For which of the following values of c is $s(t)$ differentiable on $(-\infty,\infty)$?

5

6. The curves (i), (ii), and (iii) in the graph below are the graphs of a function f and its first and second derivatives. Which curve is f, which is f', and which is f''? Explain.



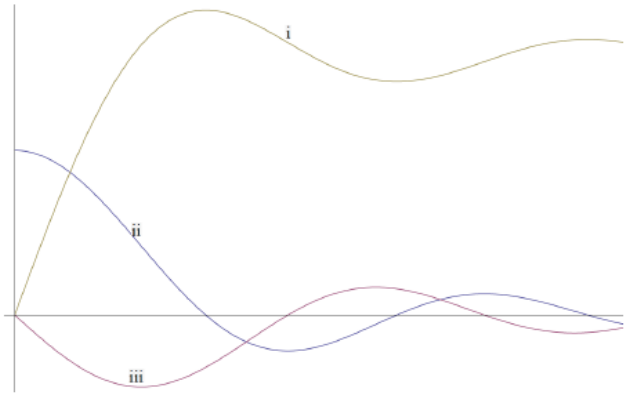
A) $(i)\ f \quad (ii)\ f' \quad (iii)\ f''$
B) $(i)\ f \quad (ii)\ f'' \quad (iii)\ f'$
C) $(i)\ f' \quad (ii)\ f \quad (iii)\ f''$
D) $(i)\ f' \quad (ii)\ f'' \quad (iii)\ f$
E) $(i)\ f'' \quad (ii)\ f \quad (iii)\ f'$
F) $(i)\ f'' \quad (ii)\ f' \quad (iii)\ f$

6

Example 2.47. Find the linearization of $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$.

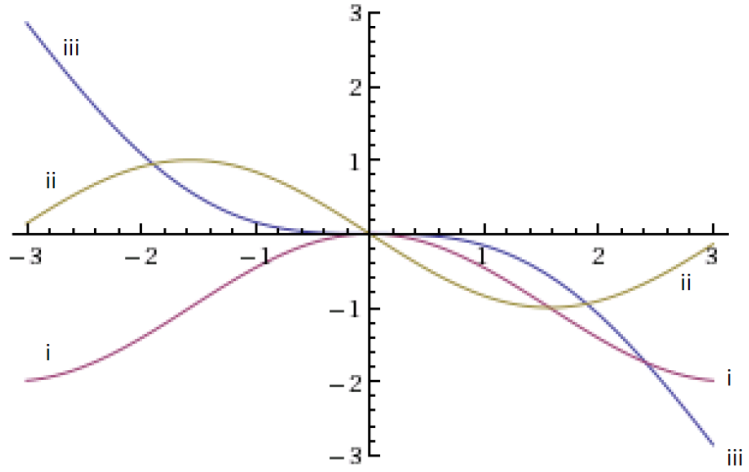
7

8. The curves (i), (ii), and (iii) in the graph below are the graphs of a function f and its first and second derivatives. Which curve is f, which is f', and which is f''? Explain.



A) (i) f (ii) f' (iii) f''
B) (i) f (ii) f'' (iii) f'
C) (i) f' (ii) f (iii) f''
D) (i) f' (ii) f'' (iii) f
E) (i) f'' (ii) f (iii) f'
F) (i) f'' (ii) f' (iii) f

8



A) (i) f (ii) f' (iii) f''
B) (i) f (ii) f'' (iii) f'
C) (i) f' (ii) f'' (iii) f
D) (i) f' (ii) f (iii) f''
E) (i) f'' (ii) f' (iii) f
F) (i) f'' (ii) f (iii) f'

9

16. The function f is defined on all the reals such that $f(x)=\begin{cases} x^2+kx-3 & for\ x\le 1 \\ 3x+b & for\ x>1 \end{cases}$

For which of the following values of k and b will the function f be both continuous and differentiable on its entire domain?

(A) k= -1, b= -3

(B) k= 1, b= 3

(C) k=1, b=4

(D) k=1, b= -4

(E) k= -1, b=6

10

Example: a) Find the linear approximation of $f(x)=\sqrt{x}$ at $x=16$.
b) Use it to approximate $\sqrt{15.9}$.

11

Find an approximation to $\sqrt[3]{8.01}$.

12

(points) Find the linear approximation to $f(x)=-3x^2+5x-8$ at $x=2$ and use it to approximate the value of $f(x)$ at $x=2.1$. [illegible] estimation or under estimation? [illegible]

13

. [· points] To aid in Elphaba's escape, Walt has concocted a supplement that will make her stronger and more agile. The concentration of the supplement in Elphaba's system, in mg/ml, t minutes after it is administered is given by the following formula:

$$T(t) = \begin{cases} at^3 & 0 \le t \le 5 \\ b(t-6)^2 + 10 & 5 < t \le 7 \end{cases}$$

where a and b are constants.

a. [points] Given that $T(t)$ is differentiable, find a and b. Give your answers in exact form.

14

2. Let a be a real number, and $f(x) = \begin{cases} 2x - 4a^2 & \text{if } x \le 2 \\ a(x^2 - 6x + 8) & \text{if } x > 2 \end{cases}$.

(a) [5 MARKS] Determine the value(s) of a — if any — for which f is continuous.

(b) [5 MARKS] Explain why f is not differentiable at $x = 2$ if $a = -2$.

15

3. (3 points) Let $f(x) = \begin{cases} x^2 + 6x + 12 & \text{if } x < -2, \\ ax + b & \text{if } x \ge -2. \end{cases}$
Find all pairs of values a and b so that f is differentiable everywhere.

16

(12 points) Suppose the function g is defined by

$$g(x) = \begin{cases} kx + (1 - 2k) & \text{if } x < 2 \\ 1 & \text{if } x = 2 \\ \sqrt{x - 1} & \text{if } x > 2, \end{cases}$$

where k is a constant.

Find k so that $g(x)$ is differentiable at $x = 2$.

17

4. A function $K(x)$ is defined as follows, where α and β are constants to be evaluated:

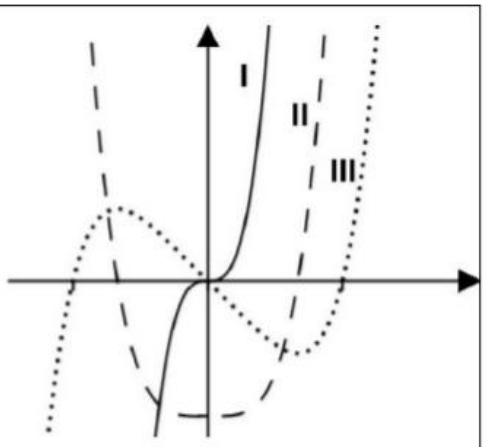
$$K(x) = \begin{cases} \alpha + x - x^2 & \text{if } x < 2 \\ x^2 - \beta(x - 2) - 4 & \text{if } x \ge 2 \end{cases}$$

(a) [8 MARKS] Showing all your work, determine all values of α and β — if any — that will make K continuous at $x = 2$.

(b) [7 MARKS] Showing all your work, determine all values of α and β — if any — that will make K differentiable at $x = 2$.

18

10. Nella figura a lato, denotati con I, II e III, sono disegnati tre grafici. Uno di essi è il grafico di una funzione f, un altro lo è della funzione derivata f' e l'altro ancora di f''.
Quale delle seguenti alternative identifica correttamente ciascuno dei tre grafici?



	f	f'	f''
A)	I	II	III
B)	I	III	II
C)	II	III	I
D)	III	II	I
E)	III	I	II

Si motivi la risposta.

19

5. Use differentials or a linear approximation to estimate $\sqrt[3]{999.4}$.

20

6. [10 MARKS] Use a "linear" or "tangent-line" approximation at $x = 4$ to compute an approximate value for $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ at $x = 3.97$.

21

(b) [8 MARKS] A particle is moving on the x axis so that its position at time t is given by

$$x(t) = 2t^3 - 7t^2 + 4t + 1\,.$$

Find

i. The times when the velocity is 0.

ii. The acceleration at each time when the velocity is 0.

22

1. Determinare i valori di a e b in modo che la funzione $g: \mathbb{R} - \{3\} \to \mathbb{R}$

$$g(x) = \begin{cases} 3 - a\,x^2 & \text{per } x \le 1 \\ \dfrac{b}{x-3} & \text{per } x > 1 \end{cases}$$

sia derivabile in tutto il suo dominio. Tracciare i grafici delle funzioni g e g'.

23	***Example 1*** Suppose that the amount of water in a holding tank at t minutes is given by $V(t) = 2t^2 - 16t + 35$. Determine each of the following. **(a)** Is the volume of water in the tank increasing or decreasing at $t = 1$ minute? [Solution] **(b)** Is the volume of water in the tank increasing or decreasing at $t = 5$ minutes? [Solution] **(c)** Is the volume of water in the tank changing faster at $t = 1$ or $t = 5$ minutes? [Solution] **(d)** Is the volume of water in the tank ever not changing? If so, when? [Solution]
24	Find the values of a and b that make the function f(x) differentiable. $f(x) = \begin{cases} \dfrac{a}{x} & x \geq 1 \\ 12 - bx^2 & x < 1 \end{cases}$
25	Let $f(x) = \sqrt{16 + x}$. First, find the linearization to $f(x)$ at $x = 0$, then use the linearization to estimate $\sqrt{15.75}$. Present your solution as a rational number.
26	Suppose we want to paint a sphere of radius 200 cm with a coat of paint .2 cm thick. Use a linear approximation to approximate the amount of paint we need to do the job.
27	EXAMPLE: A disk of radius $r = 5$ cm is to be cut from a metal sheet weighing 3 g/cm^2. If the radius is measured to within an error of $\Delta r = \pm 0.2$ cm, what is the approximate range of error in the weight? This is the kind of error-control problem from our limit analyses in