Polynomial Overview

Polynomial Operations and Factoring

(Including the miscellaneous facts not usually presented.)

While much of Page 1 is not on test, it helps understanding.

When doing algebra problems, one must work with many forms of polynomials—algebra problems are built from them. A monomial, the simplest form of a polynomial, consists of a coefficient (the numeric value) or the product of a coefficient multiplied by one or more non-constant variable(s). Here, we will discuss the different polynomial formats starting with the **monomial**, the building block for all polynomial (algebraic) forms.

$x^0 = 1, if\ x \neq 0;\ 0^0\ is\ undefined!$

1. Monomials (These are building blocks of all polynomials.)

Definitions: (a) A monomial (term), it can also be called a power product, is an algebraic expression consisting more or less of a coefficients (unique numeric values), variables, and exponents (exponents **must be** whole numbers*, not variables); (b) a polynomial which has only one term. (c) A monomial is a product of coefficient and/or various variables: i.e.,

51, 5n, 2x, 158, π, 3xy, $14x^3y$, -43, 0, xy^3z, x^0y^2z, |-7|, …

Monomials can only factor into their relatively prime or prime parts:

$14x^3y = 2 \cdot 7 \cdot x \cdot x \cdot x \cdot y$, 5•n, -1•43, 1•π, or x•y•y•y•z, …

There are some things you ***cannot*** have within a monomial: you can't add or subtract any parts; divide by a non-constant factor (variable); raise a non-constant factor to a negative or variable power; and you cannot take absolute values of non-constant factors. Examples of non-monomials:

$x + 7$, x - 3, $\frac{1}{x}$, y^{-2} or $\frac{1}{y^2}$, -15 x^{-2} y, 25 $\frac{xy}{z}$, $\frac{x^3}{x-3}$, |x|, x^y

Like Monomials have the SAME variables and exponents of those variables. The coefficients on the terms can be different.

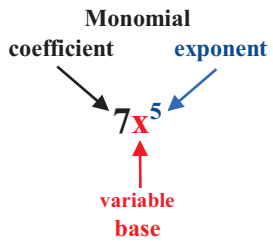
$$3z^5yx^2 + 8yx^2z^5 = (3+8)x^2yz^5 = 11x^2yz^5$$

Like monomials will simplify to single monomial term.

The Standard Form for a monomial is when there is a single numeric value, and all variables with exponents are written in alphabetical order.

The Degree of a Monomial is the total value of the exponents of all its variable elements. The degree of the monomial $3xy^5b^3$ is 9 (= 0 + 1 + 5 + 3). Or count the variables in factored form 3•x•y•y•y•y•y•b•b•b.

(Note: $3b^3xy^5$ is standard form.)



Monomials can be a single number, a single variable, a number followed by 1 or more variables:

- **The coefficient is the number multiplied by 1 or more variable(s) and/or its exponent. The coefficients of like monomials follow the rules of arithmetic.**
- **An exponent is the number of times the base is repeatedly multiplied.**

All single variables have a coefficient of 1 and an exponent of 1 which are not normally displayed.

$x = 1 \cdot x^1$

Unwritten coefficients or exponents have a value of **one**, **1**.

All monomials are **terms**, the building blocks of **expressions**.

https://www.geogebra.org/m/vmc

Definition:

$$|b| = \begin{cases} b,\ b \geq 0 & non-negative \\ -b,\ b < 0 & negative \end{cases}$$

Absolute Value is a **distance**, if the problem mentions distance specifically or by inference a distance. Basically, it removes the condition modifier signs from all numbers. The values are neither positive nor negative, they are the distance between values.

$$|^{+}7| = 7 \textbf{ and } |^{-}7| = 7$$

If a number has a sign (+ or –), it is a directed number. Directed numbers are a distance from zero on the number line. **Absolute values have no direction**.

Polynomials are formed when adding and/or subtracting monomial terms in the simplified form.

https://examples.yourdictionary.com/examples-of-monomials.html

http://content.nroc.org/DevelopmentalMath/COURSE_TEXT2_RESOURCE/U11_L2_T1_text_final.html

Natural (Counting) numbers: N ={1, 2, 3, …}

*__Whole numbers__: W = {0, 1, 2, 3, …}

Integers: Z = {… , ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, …}

∅ denotes the empty set, the set with no members

Q denotes the set of rational numbers

R denotes the set of real numbers

Rational numbers are all possible fractions including integers, $4 = \frac{4}{1}$, including terminating and repeating decimals.

Decimal values of Irrational numbers do not repeat or terminate, π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{13}$, 2.121122111222333….

Real numbers include all Rational and all Irrational numbers.

There is an additional set of numbers called the Imaginary Numbers (**not HSE tested**); $i = \sqrt{-1}$ is a factor of all.

The Real Numbers plus the Imaginary numbers are called Complex Numbers.

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Circle the valid monomials.

$9s^4t^2u^2$ $2gh + g$ $-x^5y^2z^2$ $-x^5y^{-2}z^2$ $-4x^4 - x^2$ $|-5|x^5y^2z^2$ $-5x^3y|z|$

Write the degree of each monomial.

1) $x^5y^2z^2$ ____ 2) $-9p^2q^2$ ____ 3) $2g^2h^3$ ____

4) $-6c^3$ ____ 5) $-5st^3u^3$ ____ 6) $8m^5n^0$ ____

Monomials: [1, 3, 6]
1) 9
2) 4
3) 5
4) 3
5) 7
6) 5

Not all sums are monomials, state this when found. Add the following monomials. If there is a "," replace with a "+".

1) $-y^4 + 4y^5$ 2) $11n + 9n$ 3) $8w^5, 8w^5$

4) $2s, -9s$ 5) $-12pq^3r^2, 12pq^3r^2$ 6) $5ab^2, 4a^2b$

7) $-13pqr, \frac{6}{7}pqr$ 8) $24g^3h^3, -4g^3h^3$ 9) $3ab^2, -5b^2a$

Addition
1) $4y^5 - y^4$
2) 20n
3) $16w^5$
4) -7s
5) 0
6) $4a^2b + 5ab^2$
7) $-12\frac{1}{7}pqr$
8) $20g^3h^3$
9) $-2ab^2$

Subtraction
1) $ab^5c - 7b$
2) $4v^3w^2$
3) $-33q^4r^3$
4) $-13b^2c^4d$
5) $-24pq^3r^2$
6) $-4a^2b + 5ab^2$
7) $-13\frac{6}{7}pqr$
8) $28g^3h^3$
9) $8ab^2$

Not all differences are monomials, state this when found. Subtract the following monomials. If there is a "," replace with a "–".

1) $(-7b) - (-ab^5c)$ 2) $15v^3w^2 - 11v^3w^2$ 3) $q^4r^3 - 34q^4r^3$

4) $(-16b^2c^4d) - (-3b^2c^4d)$ 5) $-12pq^3r^2, 12pq^3r^2$ 6) $5ab^2, 4a^2b$

7) $-13pqr, \frac{6}{7}pqr$ 8) $24g^3h^3, -4g^3h^3$ 9) $3ab^2, -5b^2a$

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Multiply the following. When multiplying like variable, add exponents ($d^3 \bullet d^2 = d^5$).

1) $\frac{1}{6}q^6rs \times 72q^5$

2) $-7y^2z^3 \times 7y^3z^2$

3) $\frac{5}{8}rs^3t^2 \times 64r^3st^3$

4) $-\frac{1}{2}uv^4 \times \frac{3}{4}u \times -\frac{8}{3}v^3$

5) $\frac{2}{7}c^5d \times \frac{7}{4}cd^6$

6) $-45r^3s \times \frac{7}{9}s$

7) $5m^2n^3 \times \frac{3}{-4}mn^7$

8) $-4c^3 \times \frac{2}{3}cf^5$

9) $12b^4c^3 \div 3b^4c^3$

Find the missing monomials.

1) _____ × $-8x^4y$ = $-96x^7y^2$

2) $-4v^7$ × _____ = $68v^{12}$

3) _____ × $5m^3$ = $-35m^7n$

4) $8p^6q^5$ × _____ = $-64p^{12}q^{12}$

5) $4m^3n$ ___ × ___ mn^5p = $-32m^4n^6p^4$

Multiplication
1) $12q^{11}rs$
2) $-49y^5z^5$
3) $40r^4s^4t^5$
4) u^2v^7
5) $\frac{1}{2}c^6d^7$
6) $-5r^3s^2$
7) $-\frac{15}{4}m^3n^{10}$
8) $-\frac{8}{3}c^4f^5$
9) 4

Missing Monomials
1) $12x^3y$
2) $-17v^5$
3) $-7m^4n$
4) $-8p^6q^7$
5) p^3 and -8

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