CHAPTER

1

Review of Real Numbers

This chapter reviews properties of real numbers and arithmetic that are necessary for success in algebra. The chapter also introduces several algebraic properties.

STUDY STRATEGY

Study Groups Throughout this book, study strategies will help you learn and be successful in this course. This chapter will focus on getting involved in a study group.

Working with a study group is an excellent way to learn mathematics, improve your confidence and level of interest, and improve your performance on quizzes and tests. When working with a group, you will be able to work through questions about the material you are studying. Also, by being able to explain how to solve a particular problem to another person in your group, you will increase your ability to retain this knowledge.

We will revisit this study strategy throughout this chapter so you can incorporate it into your study habits. See the end of Section 1.1 for tips on how to get a study group started.

1.1 Integers, Opposites, and Absolute Value

OBJECTIVES

1. Graph whole numbers on a number line.
2. Determine which is the greater of two whole numbers.
3. Graph integers on a number line.
4. Find the opposite of an integer.
5. Determine which is the greater of two integers.
6. Find the absolute value of an integer.

A **set** is a collection of objects, such as the set consisting of the numbers 1, 4, 9, and 16. This set can be written as $\{1, 4, 9, 16\}$. The braces, $\{\ \}$, are used to denote a set, and the values listed inside are said to be **elements**, or members, of the set. A set with no elements is called the **empty set** or **null set**. A **subset** of a set is a collection of some or all of the elements of the set. For example, $\{1, 9\}$ is a subset of the set $\{1, 4, 9, 16\}$. A subset also can be an empty set.

Whole Numbers

Objective 1 Graph whole numbers on a number line. For the most part, this text deals with the set of real numbers. The set of real numbers is made up of the set of rational numbers and the set of irrational numbers.

Rational Numbers

A **rational number** is a number that can be expressed as a fraction, such as $\frac{3}{4}$ and $\frac{2}{9}$. Decimal numbers that terminate, such as 2.57, and decimal numbers that repeat, such as 0.444 . . . , are also rational numbers.

Irrational Numbers

An **irrational number** is a number that cannot be expressed as a fraction, but instead is a decimal number that does not terminate or repeat. The number π is an example of an irrational number: $\pi = 3.14159\ldots$.

One subset of the set of real numbers is the set of natural numbers.

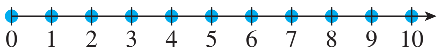
Natural Numbers

The set of **natural numbers** is the set $\{1, 2, 3, \ldots\}$.

If we include the number 0 with the set of natural numbers, we have the set of **whole numbers**.

Whole Numbers

The set of **whole numbers** is the set $\{0, 1, 2, 3, \ldots\}$. This set can be displayed on a number line as follows:

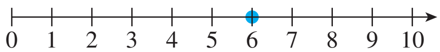


The arrow on the right-hand side of the number line indicates that the values continue to increase in this direction. There is no largest whole number, but we say that the values approach infinity (∞).

To graph any particular number on a number line, we place a point, or dot, at that location on the number line.

EXAMPLE 1 Graph the number 6 on a number line.

SOLUTION To graph any number on a number line, place a point at that number's location.



Quick Check 1

Graph the number 4 on a number line.

Inequalities

Objective 2 Determine which is the greater of two whole numbers. When comparing two whole numbers a and b, we say that a is **greater than** b, denoted $a > b$, if the number a is to the right of the number b on the number line. The number a is **less than** b, denoted $a < b$, if a is to the left of b on the number line. The statements $a > b$ and $a < b$ are called **inequalities**.

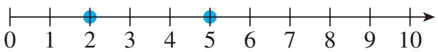
EXAMPLE 2 Write the appropriate symbol, $<$ or $>$, between the following: 6 _____ 4

SOLUTION Let's take a look at the two values graphed on a number line.

Because the number 6 is to the right of the number 4 on the number line, 6 is greater than 4. So $6 > 4$.

EXAMPLE 3 Write the appropriate symbol, $<$ or $>$, between the following: 2 _____ 5

SOLUTION Because 2 is to the left of 5 on the number line, $2 < 5$.



Quick Check 2

Write the appropriate symbol, $<$ or $>$, between the following:

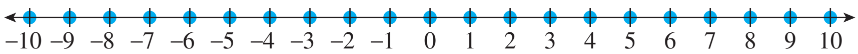
a) 8 _____ 3
b) 19 _____ 23

Integers

Objective 3 Graph integers on a number line. Another important subset of the real numbers is the set of integers.

Integers

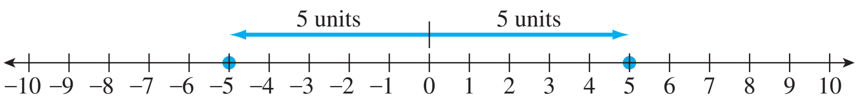
The set of **integers** is the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. We can display the set of integers on a number line as follows:

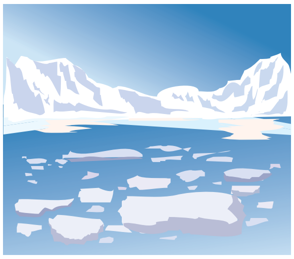


The arrow on the left side indicates that the values continue to decrease in this direction, and they are said to approach negative infinity $(-\infty)$.

Opposites

Objective 4 Find the opposite of an integer. The set of integers is the set of whole numbers together with the opposites of the natural numbers. The **opposite** of a number is a number on the other side of 0 on the number line and the same distance from 0 as that number. We denote the opposite of a real number a as $-a$. For example, -5 and 5 are opposites because both are 5 units away from 0 and one is to the left of 0 while the other is to the right of 0.

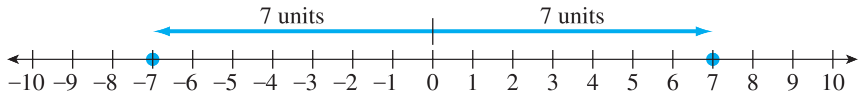




Numbers to the left of 0 on the number line are called **negative numbers**. Negative numbers represent a quantity less than 0. For example, if you have written checks that the balance in your checking account cannot cover, your balance will be a negative number. A temperature that is below 0° F, a golf score that is below par, and an elevation that is below sea level are other examples of quantities that can be represented by negative numbers.

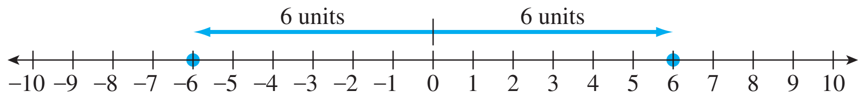
EXAMPLE 4 What is the opposite of 7?

SOLUTION The opposite of 7 is −7 because −7 also is 7 units away from 0 but is on the opposite side of 0.



EXAMPLE 5 What is the opposite of −6?

SOLUTION The opposite of −6 is 6.



The opposite of 0 is 0 itself. Zero is the only number that is its own opposite.

Quick Check 3

Find the opposite of the given integer.

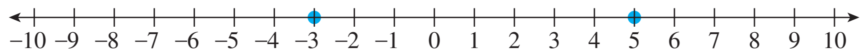
a) −13 **b)** 8

Inequalities with Integers

Objective 5 Determine which is the greater of two integers. Inequalities for integers follow the same guidelines as they do for whole numbers. If we are given two integers *a* and *b*, the number that is greater is the number that is to the right on the number line.

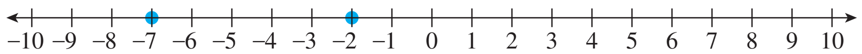
EXAMPLE 6 Write the appropriate symbol, $<$ or $>$, between the following: −3 ____ 5

SOLUTION Looking at the number line, we can see that −3 is to the left of 5; so $-3 < 5$.



EXAMPLE 7 Write the appropriate symbol, $<$ or $>$, between the following: −2 ____ −7

SOLUTION On the number line, −2 is to the right of −7; so $-2 > -7$.



Quick Check 4

Write the appropriate symbol, $<$ or $>$, between the following:

a) −14 ____ −11
b) 6 ____ −20

Absolute Values

Objective 6 Find the absolute value of an integer.

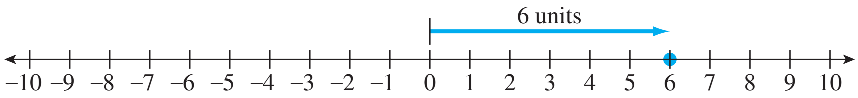
Absolute Value

The **absolute value** of a number *a*, denoted $|a|$, is the distance between *a* and 0 on the number line.

Distance cannot be negative, so the absolute value of a number *a* is always 0 or higher.

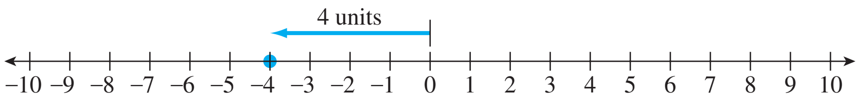
EXAMPLE 8 Find the absolute value of 6.

SOLUTION The number 6 is 6 units away from 0 on the number line, so $|6| = 6$.



EXAMPLE 9 Find the absolute value of -4.

SOLUTION The number -4 is 4 units away from 0 on the number line, so $|-4| = 4$.



Quick Check 5

Find the absolute value of -9.

BUILDING YOUR STUDY STRATEGY

Study Groups, 1 With Whom to Work? To form a study group, you must begin with this question: With whom do I want to work? Look for students who are serious about learning, who are prepared for each class, and who ask intelligent questions during class.

Look for students with whom you believe you can get along. You are about to spend a great deal of time working with this group, sometimes under stressful conditions.

If you take advantage of tutorial services provided by your college, keep an eye out for classmates who do the same. There is a strong chance that classmates who use the tutoring center are serious about learning mathematics and earning good grades.

Exercises 1.1

MyMathLab — PRACTICE · WATCH · DOWNLOAD · READ · REVIEW

Vocabulary

1. A set with no elements is called the ________.
2. A number m is ________ than another number n if it is located to the left of n on a number line.
3. The arrow on the right side of a number line indicates that the values approach ________.
4. $c > d$ if c is located to the ________ of d on a number line.

Graph the following whole numbers on a number line.

5. 7
6. 3
7. 6
8. 9
9. 2
10. 13

Write the appropriate symbol, < or >, between the following whole numbers.

11. 3 ___ 13
12. 7 ___ 9
13. 8 ___ 6
14. 12 ___ 5
15. 45 ___ 42
16. 33 ___ 37

Graph the following integers on a number line.

17. -4
18. -7
19. 5

20. −9

21. −12

22. 4

Find the opposite of the following integers.

23. −7
24. 5
25. 22
26. −13
27. 0
28. −39

Write the appropriate symbol, < or >, between the following integers.

29. −7 ___ −9
30. −5 ___ −2
31. −13 ___ −11
32. −8 ___ −14
33. −16 ___ 0
34. 5 ___ −3
35. 9 ___ −14
36. −10 ___ 6

Find the following absolute values.

37. $|-15|$
38. $|9|$
39. $|0|$
40. $|-6|$
41. $|-7|$
42. $|-12|$
43. $-|7|$
44. $-|12|$
45. $-|-29|$
46. $-|-8|$

Write the appropriate symbol, < or >, between the following integers.

47. $|-7|$ ___ 4
48. $|-17|$ ___ $|13|$
49. −16 ___ $|-16|$
50. $|8|$ ___ $|-19|$
51. $-|-24|$ ___ $|-47|$
52. $|-8|$ ___ $-|8|$

Identify whether the given number is a member of the following sets of numbers: A. natural numbers, B. whole numbers, C. integers, D. real numbers.

53. 8
54. −6
55. 0
56. 3.14
57. −9
58. 20

Find the missing number if possible. There may be more than one number that works, so find as many as possible. There may be no number that works.

59. $|?| = 5$
60. $|?| = 18$
61. $|?| = -7$
62. $|?| = 0$
63. $|?| - 8 = 6$
64. $4 \cdot |?| + 3 = 27$

Writing in Mathematics

Answer in complete sentences.

65. A fellow student tells you that to find the absolute value of any number, make the number positive. Is this always true? Explain in your own words.

66. True or false: The opposite of the opposite of a number is the number itself.

67. If the opposite of a nonzero integer is equal to the absolute value of that integer, is the integer positive or negative? Explain your reasoning.

68. If an integer is less than its opposite, is the integer positive or negative? Explain your reasoning.

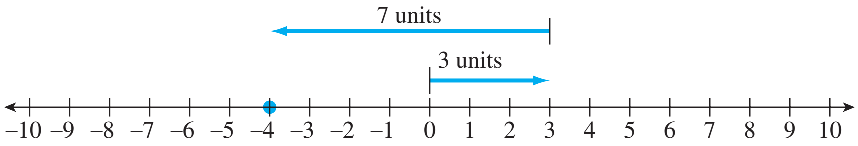
1.2 Operations with Integers

OBJECTIVES

1. Add integers.
2. Subtract integers.
3. Multiply integers.
4. Divide integers.

Addition and Subtraction of Integers

Objective 1 Add integers. Using the number line can help us learn how to add and subtract integers. Suppose we are trying to add the integers 3 and −7, which could be written as $3 + (-7)$. On a number line, we will start at 0 and move 3 units in the positive, or right, direction. Adding -7 tells us to move 7 units in the negative, or left, direction.



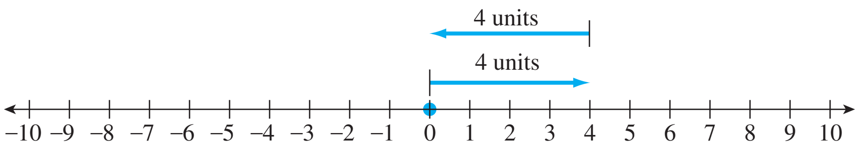
Ending up at -4 tells us that $3 + (-7) = -4$.

We can use a similar approach to verify an important property of opposites: the sum of two opposites is equal to 0.

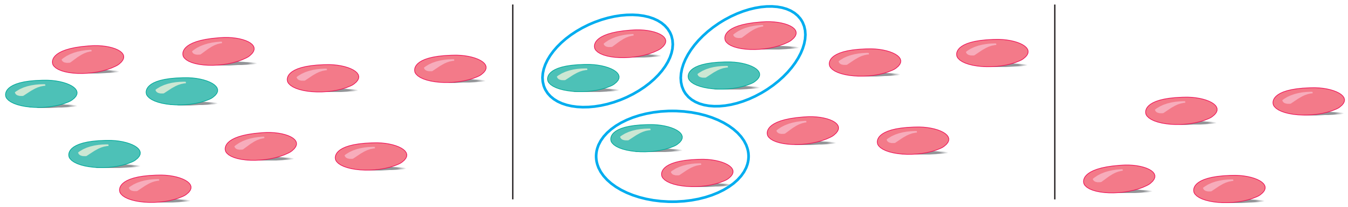
Sum of Two Opposites

For any real number a, $a + (-a) = 0$.

Suppose that we want to add the opposites 4 and -4. Using the number line, we begin at 0 and move 4 units to the right. We then move 4 units to the left, ending at 0. So $4 + (-4) = 0$.



We also can see that $3 + (-7) = -4$ through the use of manipulatives, which are hands-on tools used to demonstrate mathematical properties. Suppose we had a bag of green and red candies. Let each piece of green candy represent a positive 1 and each piece of red candy represent a negative 1. To add $3 + (-7)$, we begin by combining 3 green candies (positive 3) with 7 red candies (negative 7). Combining 1 red candy with 1 green candy has a net result of 0, as the sum of two opposites is equal to 0. So each time we make a pair of a green candy and a red candy, these two candies cancel each other's effect and can be discarded. After doing this, we are left with 4 red candies. The answer is -4.



Now we will examine another technique for finding the sum of a positive number and a negative number. In the sum $3 + (-7)$, the number 3 contributes to the sum in a positive fashion while the number -7 contributes to the sum in a negative fashion. The two numbers contribute to the sum in an opposite manner. We can think of the sum as the difference between these two contributions.

Adding a Positive Number and a Negative Number

1. **Take the absolute value of each number and find the difference between these two absolute values.** This is the difference between the two numbers' contributions to the sum.
2. **Note that the sign of the result is the same as the sign of the number that has the largest absolute value.**

For the sum $3 + (-7)$, we begin by taking the absolute value of each number: $|3| = 3$, $|-7| = 7$. The difference between the absolute values is 4. The sign of the sum is the same as the sign of the number that has the larger absolute value. In this case, –7 has the larger absolute value, so the result is negative. Therefore, $3 + (-7) = -4$.

EXAMPLE 1 Find the sum $12 + (-8)$.

SOLUTION

$\|12\| = 12;\quad \|8\| = 8$	Find the absolute value of each number.
$12 - 8 = 4$	The difference between the absolute values is 4.
$12 + (-8) = 4$	Because the number with the larger absolute value is positive, the result is positive.

Quick Check 1

Find the sum $14 + (-6)$.

Notice in the previous example that $12 + (-8)$ is equivalent to $12 - 8$, which also equals 4. What the two expressions have in common is that there is one number (12) contributes to the total in a positive fashion and a second number (8) that contributes to the total in a negative fashion.

EXAMPLE 2 Find the sum $3 + (-11)$.

SOLUTION Again, one number (3) contributes to the total in a positive way and a second number (11) contributes in a negative way. The difference between their contributions is 8 and because the number making the larger contribution is negative, the result is -8.

$$3 + (-11) = -8$$

Quick Check 2

Find the sum $4 + (-17)$.

Note that $-11 + 3$ also equals -8. The rules for adding a positive integer and a negative integer still apply when the first number is negative and the second number is positive.

Adding Two (or More) Negative Numbers

1. Total the negative contributions of each number.
2. Note that the sign of the result is negative.

EXAMPLE 3 Find the sum $-3 + (-7)$.

SOLUTION Both values contribute to the total in a negative fashion. Totaling the negative contributions of 3 and 7 results in 10, and the result is negative because both numbers are negative.

$$-3 + (-7) = -10$$

Quick Check 3

Find the sum $-2 + (-9)$.

Objective 2 Subtract integers. To subtract a negative integer from another integer, we use the following property:

Subtraction of Real Numbers

For any real numbers a and b, $a - b = a + (-b)$.

This property says that adding the opposite of b to a is the same as subtracting b from a. Suppose we are subtracting a negative integer, as in the example $-8 - (-19)$. The property for subtraction of real numbers says that subtracting -19 is the same as adding its opposite (19); so we convert this subtraction to $-8 + 19$. Remember that subtracting a negative number is equivalent to adding a positive number.

EXAMPLE 4 Subtract $6 - (-27)$.

SOLUTION

$6 - (-27) = 6 + 27$ Subtracting -27 is the same as adding 27.
$= 33$ Add.

Quick Check 4

Subtract $11 - (-7)$.

General Strategy for Adding/Subtracting Integers

- Rewrite "double signs." *Adding a negative number,* $4 + (-5)$, *can be rewritten as subtracting a positive number,* $4 - 5$. *Subtracting a negative number,* $-2 - (-7)$, *can be rewritten as adding a positive number,* $-2 + 7$.
- Look at each integer and determine whether it is contributing positively or negatively to the total.
- Add any integers contributing positively to the total, resulting in a single positive integer. In a similar fashion, add all integers that are contributing to the total negatively, resulting in a single negative integer. Finish by finding the sum of these two integers.

Rather than saying to add or subtract, the directions for a problem may state to "simplify" a numerical expression. To **simplify** an expression means to perform all arithmetic operations.

EXAMPLE 5 Simplify $17 - (-11) - 6 + (-13) - (-21) + 3$.

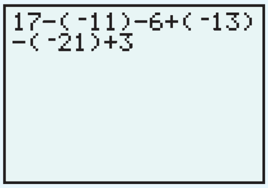
SOLUTION Begin by working on the *double signs*. This produces the following:

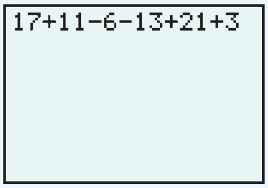
$17 - (-11) - 6 + (-13) - (-21) + 3$
$= 17 + 11 - 6 - 13 + 21 + 3$ Rewrite double signs.
$= 52 - 19$ The four integers that contribute in a positive fashion (17, 11, 21, and 3) total 52. The two integers that contribute in a negative fashion total -19.
$= 33$ Subtract.

Quick Check 5

Simplify $14 - 9 - (-22) - 6 + (-30) + 5$.

Using Your Calculator When using your calculator, you must be able to distinguish between the subtraction key and the key for a negative number. On the TI-84, the subtraction key [−] is listed above the addition key on the right side of the calculator, while the negative key [(-)] is located to the left of the [ENTER] key at the bottom of the calculator. Here are two ways to simplify the expression from the previous example using the TI-84.





In either case, pressing the [ENTER] key produces the result 33.

Multiplication and Division of Integers

Objective 3 Multiply integers. The result obtained when multiplying two numbers is called the **product** of the two numbers. The numbers that are multiplied are called **factors**. When we multiply two positive integers, their product also is a positive integer. For example, the product of the two positive integers 4 and 7 is the positive integer 28. This can be written as $4 \cdot 7 = 28$. The product $4 \cdot 7$ also can be written as 4(7) or (4)(7).

The product $4 \cdot 7$ is another way to represent the repeated addition of 7 four times.

$$\begin{aligned} 4 \cdot 7 &= 7 + 7 + 7 + 7 \\ &= 28 \end{aligned}$$

This concept can be used to show that the product of a positive integer and a negative integer is a negative integer. Suppose we want to multiply 4 by −7. We can rewrite this as −7 being added four times, or $(-7) + (-7) + (-7) + (-7)$. From our work earlier in this section, we know that this total is −28; so $4(-7) = -28$. Anytime we multiply a positive integer and by a negative integer, the result is negative. So $(-7)(4)$ also is equal to −28.

Products of Integers

$$(\text{Positive}) \cdot (\text{Negative}) = \text{Negative}$$
$$(\text{Negative}) \cdot (\text{Positive}) = \text{Negative}$$

EXAMPLE 6 Multiply $5(-8)$.

SOLUTION We begin by multiplying 5 and 8, which equals 40. The next step is to determine the sign of the result. Whenever we multiply a positive integer by a negative integer, the result is negative.

$$5(-8) = -40$$

Quick Check 6
Multiply $10(-6)$.

A WORD OF CAUTION Note the difference between $5 - 8$ (a subtraction) and $5(-8)$ (a multiplication). A set of parentheses *without* a sign in front of them is used to imply multiplication.

Product	Result
$(3)(-5)$	-15
$(2)(-5)$	-10
$(1)(-5)$	-5
$(0)(-5)$	0
$(-1)(-5)$	?
$(-2)(-5)$	?

The product of two negative integers is a positive integer. Let's try to understand why this is true by considering the example $(-2)(-5)$. Examine the table to the left, which shows the products of some integers and -5.

Notice the pattern in the table. Each time the integer multiplied by -5 decreases by 1, the product increases by 5. As we go from $0(-5)$ to $(-1)(-5)$, the product should increase by 5. So $(-1)(-5) = 5$ and, by the same reasoning, $(-2)(-5) = 10$.

Product of Two Negative Integers

$$(\text{Negative}) \cdot (\text{Negative}) = \text{Positive}$$

EXAMPLE 7 Multiply $(-9)(-8)$.

SOLUTION We begin by multiplying 9 and 8, which equals 72. The next step is to determine the sign of the result. Whenever we multiply a negative integer by a negative integer, the result is positive.

$$(-9)(-8) = 72$$

Quick Check 7
Multiply $(-7)(-9)$.

Products of Integers

- If a product contains an *odd number* of negative factors, the result is negative.
- If a product contains an *even number* of negative factors, the result is positive.

The main idea behind this principle is that every two negative factors multiply to be positive. If there are three negative factors, the product of the first two is a positive number. Multiplying this positive product by the third negative factor produces a negative product.

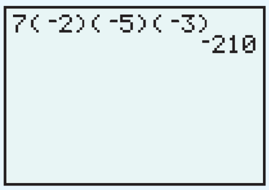
EXAMPLE 8 Multiply $7(-2)(-5)(-3)$.

SOLUTION Because there are three negative factors, the product will be negative.

$$7(-2)(-5)(-3) = -210$$

Quick Check 8
Multiply $-4(-10)(5)(-2)$.

Using Your Calculator Here is how the screen looks when you are using the TI-84 to multiply the expression in the previous example:



Notice that parentheses can be used to indicate multiplication without using the ☒ key.

Before continuing on to division, let's consider multiplication by 0. Any real number multiplied by 0 is 0; this is the multiplication property of 0.

Multiplication Property of 0

For any real number x,

$$0 \cdot x = 0$$
$$x \cdot 0 = 0.$$

Objective 4 Divide integers. When dividing one number called the **dividend** by another number called the **divisor**, the result obtained is called the **quotient** of the two numbers:

The statement "6 divided by 3 is equal to 2" is true because the product of the quotient and the divisor, $2 \cdot 3$, is equal to the dividend 6.

$$6 \div 3 = 2 \longleftrightarrow 2 \cdot 3 = 6$$

When we divide two integers that have the same sign (both positive or both negative), the quotient is positive. When we divide two integers that have different signs (one negative, one positive), the quotient is negative. Note that this is consistent with the rules for multiplication.

Quotients of Integers

(Positive) ÷ (Positive) = Positive (Positive) ÷ (Negative) = Negative
(Negative) ÷ (Negative) = Positive (Negative) ÷ (Positive) = Negative

EXAMPLE 9 Divide $(-54) \div (-6)$.

SOLUTION When we divide a negative number by another negative number, the result is positive.

$$(-54) \div (-6) = 9 \quad \text{Note that } -6 \cdot 9 = -54.$$

EXAMPLE 10 Divide $(-33) \div 11$.

SOLUTION When we divide a negative number by a positive number, the result is negative.

$$(-33) \div 11 = -3$$

Quick Check 9
Divide $72 \div (-8)$.

Whenever 0 is divided by any integer (except 0), the quotient is 0. For example, $0 \div 16 = 0$. We can check that this quotient is correct by multiplying the quotient by the divisor. Because $0 \cdot 16 = 0$, the quotient is correct.

Division by Zero

Whenever an integer is divided by 0, the quotient is said to be **undefined**.

Use the word **undefined** to state that an operation cannot be performed or is meaningless. For example, $41 \div 0$ is undefined. Suppose there was a real number a for which $41 \div 0 = a$. In that case, the product $a \cdot 0$ would be equal to 41. Because the product of 0 and any real number is equal to 0, such a number a does not exist.

BUILDING YOUR STUDY SKILLS

Study Groups, 2 When to Meet Once you have formed a study group, determine where and when to meet. It is a good idea to meet at least twice a week for at least an hour per session. Consider a location where quiet discussion is allowed, such as the library or tutorial center.

Some groups like to meet the hour before class, using the study group as a way to prepare for class. Other groups prefer to meet the hour after class, allowing them to go over material while it is fresh in their minds. Another suggestion is to meet at a time when your instructor is holding office hours.

Exercises 1.2

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Vocabulary

1. When finding the sum of a positive integer and a negative integer, the sign of the result is determined by the sign of the integer with the ________ absolute value.
2. The sum of two negative integers is a(n) ________ integer.
3. Subtracting a negative integer can be rewritten as adding a(n) ________ integer.
4. The product of a positive integer and a negative integer is a(n)________ integer.
5. The product of a negative integer and a negative integer is a(n) ________ integer.
6. If a product contains a(n) ________ number of negative integers, the product is negative.
7. In a division problem, the number you divide by is called the ________.
8. Division by 0 results in a quotient that is ________.

Add.

9. $8 + (-13)$
10. $16 + (-11)$
11. $6 + (-33)$
12. $52 + (-87)$
13. $(-4) + 5$
14. $-9 + 2$
15. $-14 + 22$
16. $(-35) + 50$
17. $-5 + (-6)$
18. $-9 + (-9)$

Subtract.

19. $8 - 6$
20. $13 - 9$
21. $5 - 11$
22. $4 - 12$
23. $(-5) - 3$
24. $(-9) - 6$
25. $-9 - 13$
26. $-47 - 16$
27. $36 - (-25)$
28. $64 - (-19)$
29. $-42 - (-33)$
30. $-27 - (-60)$

Simplify.

31. $8 - 13 - 6$
32. $-6 + 12 - 20$
33. $-9 + 7 - 4$
34. $-5 - 8 + 23$
35. $6 - (-16) + 5$
36. $18 - 21 - (-62)$
37. $4 + (-15) - 13 - (-25)$
38. $-13 + (-12) - (-1) - 29$
39. A mother with \$30 in her purse paid \$22 for her family to go to a movie. How much money did she have remaining?
40. A student had \$60 in his checking account prior to writing an \$85 check to the bookstore for books and supplies. What is his account's new balance?
41. The temperature at 6 A.M. in Fargo, North Dakota, was $-8°$ C. By 3 P.M., the temperature had risen by 12° C. What was the temperature at 3 P.M.?
42. If a golfer completes a round at 3 strokes under par, her score is denoted -3. A professional golfer had rounds of -4, -2, 3, and -6 in a recent tournament. What was her total score for this tournament?
43. Dylan drove from a town located 400 feet below sea level to another town located 1750 feet above sea level. What was the change in elevation traveling from one town to another?
44. After withdrawing \$80 from her bank using an ATM card, Alycia had \$374 remaining in her savings account.

How much money did Alycia have in her account prior to withdrawing the money?

Multiply.

45. $7(-6)$

46. $-4(9)$

47. $-8 \cdot 5$

48. $-6(-8)$

49. $-15(-12)$

50. $-11 \cdot 17$

51. $82(-1)$

52. $-1 \cdot 19$

53. $-6 \cdot 0$

54. $0(-240)$

55. $-6(-3)(5)$

56. $-2(-4)(-8)$

57. $5 \cdot 3(-2)(-6)$

58. $-7 \cdot 2(-7)(-2)$

Divide if possible.

59. $45 \div (-5)$

60. $56 \div (-7)$

61. $-36 \div 6$

62. $-91 \div 13$

63. $-32 \div (-8)$

64. $-75 \div (-5)$

65. $126 \div (-9)$

66. $-420 \div 14$

67. $0 \div (-13)$

68. $0 \div 11$

69. $29 \div 0$

70. $-15 \div 0$

Mixed Practice, 71–82

Simplify.

71. $-11(-12)$

72. $126 \div (-6)$

73. $5 - 13$

74. $5(-13)$

75. $17 - (-11) - 49$

76. $8(-7)(-6)$

77. $-432 \div 3$

78. $-5 \cdot 17$

79. $9(-24)$

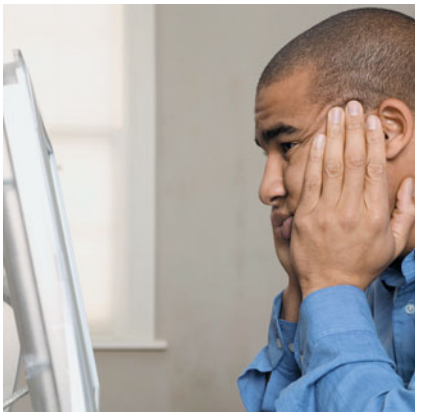
80. $9 + (-24)$

81. $5 \cdot 3(-17)(-29)(0)$

82. $-16 + (-11) - 42 - (-58)$

83. A group of 4 friends went out to dinner. If each person paid \$23, what was the total bill?

84. Three friends decided to start investing in stocks together. In the first year, they lost a total of \$13,500. How much did each person lose?



85. Tina owns 400 shares of a stock that dropped in value by \$3 per share last month. She also owns 500 shares of a stock that went up by \$2 per share last month. What is Tina's net income on these two stocks for last month?

86. Mario took over as the CEO for a company that lost \$20 million dollars in 2007. The company lost three times as much in 2008. The company went on to lose \$13 million more in 2009 than it had lost in 2008. How much money did Mario's company lose in 2009?

87. When a certain integer is added to -34, the result is -15. What is that integer?

88. Thirty-five less than a certain integer is -13. What is that integer?

89. When a certain integer is divided by -8, the result is 16. What is that integer?

90. When a certain integer is multiplied by -4 and that product is added to 22, the result is -110. What is that integer?

True or False (If false, give an example that shows why the statement is false.)

91. The sum of two integers is always an integer.

92. The difference of two integers is always an integer.

93. The sum of two whole numbers is always a whole number.

94. The difference of two whole numbers is always a whole number.

Writing in Mathematics

Explain each of the following in your own words.

95. Explain why subtracting a negative integer from another integer is the same as adding the opposite of that integer to it. Use the example $11 - (-5)$ in your explanation.

96. Explain why a positive integer times a negative integer produces a negative integer.

97. Explain why a negative integer times another negative integer produces a positive integer.

98. Explain why $7 \div 0$ is undefined.

1.3 Fractions

OBJECTIVES

1. Find the factor set of a natural number.
2. Determine whether a natural number is prime.
3. Find the prime factorization of a natural number.
4. Simplify a fraction to lowest terms.
5. Change an improper fraction to a mixed number.
6. Change a mixed number to an improper fraction.

Factors

Objective 1 Find the factor set of a natural number. To factor a natural number, express it as the product of two natural numbers. For example, one way to factor 12 is to rewrite it as $3 \cdot 4$. In this example, 3 and 4 are said to be factors of 12. The collection of all factors of a natural number is called its **factor set**. The factor set of 12 can be written as $\{1, 2, 3, 4, 6, 12\}$ because $1 \cdot 12 = 12$, $2 \cdot 6 = 12$, and $3 \cdot 4 = 12$.

EXAMPLE 1 Write the factor set for 18.

SOLUTION Because 18 can be factored as $1 \cdot 18$, $2 \cdot 9$, and $3 \cdot 6$, its factor set is $\{1, 2, 3, 6, 9, 18\}$.

Quick Check 1
Write the factor set for 36.

Prime Numbers

Objective 2 Determine whether a natural number is prime.

Prime Numbers

A natural number is **prime** if it is greater than 1 and its only two factors are 1 and itself.

For instance, the number 13 is prime because its only two factors are 1 and 13. The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. The number 8 is not prime because it has factors other than 1 and 8, namely, 2 and 4. A natural number greater than 1 that is not prime is called a **composite** number. The number 1 is considered to be neither prime nor composite.

EXAMPLE 2 Determine whether the following numbers are prime or composite:

a) 26 **b)** 37

SOLUTION

a) The factor set for 26 is $\{1, 2, 13, 26\}$. Because 26 has factors other than 1 and itself, it is a composite number.

b) Because the number 37 has no factors other than 1 and itself, it is a prime number.

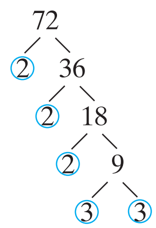
Quick Check 2

Determine whether the following numbers are prime or composite.

a) 57
b) 47
c) 48

Prime Factorization

Objective 3 Find the prime factorization of a natural number. When we rewrite a natural number as a product of prime factors, we obtain the **prime factorization** of the number. The prime factorization of 12 is $2 \cdot 2 \cdot 3$ because 2 and 3 are prime numbers and $2 \cdot 2 \cdot 3 = 12$. A **factor tree** is a useful tool for finding the prime factorization of a number. Here is an example of a factor tree for 72.

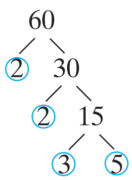


$72 = 2 \cdot 36$; 2 is prime.
$36 = 2 \cdot 18$; 2 is prime.
$18 = 2 \cdot 9$; 2 is prime.
$9 = 3 \cdot 3$; both factors are prime.

The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$. We could have begun by rewriting 72 as $8 \cdot 9$ and then factored those two numbers. The process for creating a factor tree for a natural number is not unique, although the prime factorization for the number is unique.

EXAMPLE 3 Find the prime factorization of 60.

SOLUTION



The prime factorization is $2 \cdot 2 \cdot 3 \cdot 5$.

Quick Check 3

Find the prime factorization of 63.

Fractions

Objective 4 Simplify a fraction to lowest terms. Recall from Section 1.1 that a rational number is a real number that can be written as the quotient (or ratio) of two integers, the second of which is not zero. An irrational number is a real number that cannot be written this way, such as the number π.

Rational numbers are often expressed using fraction notation such as $\frac{3}{7}$. Whole numbers such as 7 can be written as a fraction whose denominator is 1: $\frac{7}{1}$. The number on the top of the fraction is called the **numerator**, and the number on the bottom of the fraction is called the **denominator**.

$$\frac{\text{numerator}}{\text{denominator}}$$

If the numerator and denominator do not have any common factors other than 1, the fraction is said to be in **lowest terms**.

To simplify a fraction to lowest terms, begin by finding the prime factorization of both the numerator and denominator. Then divide the numerator and the denominator by their common factors.

EXAMPLE 4 Simplify $\frac{18}{30}$ to lowest terms.

SOLUTION

$$\frac{18}{30} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 5}$$ Find the prime factorization of the numerator and denominator. $18 = 2 \cdot 3 \cdot 3$, $30 = 2 \cdot 3 \cdot 5$.

$$= \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot 3}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5}$$ Divide out common factors.

$$= \frac{3}{5}$$ Simplify.

EXAMPLE 5 Simplify $\frac{4}{24}$ to lowest terms.

SOLUTION

$$\frac{4}{24} = \frac{2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 3}$$ Find the prime factorization of the numerator and denominator.

$$\frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 3}$$ Divide out common factors.

$$= \frac{1}{6}$$ Simplify.

Quick Check 4

Simplify to lowest terms:

a) $\frac{45}{210}$ **b)** $\frac{24}{384}$

A WORD OF CAUTION It is customary to leave off the denominator of a fraction if it is equal to 1. For example, rather than writing $\frac{19}{1}$, we usually write 19. However, we cannot omit a numerator that is equal to 1. In the previous example, it would have been a mistake if we had written 6 instead of $\frac{1}{6}$.

Mixed Numbers and Improper Fractions

Objective 5 Change an improper fraction to a mixed number. An **improper fraction** is a fraction whose numerator is greater than or equal to its denominator, such as $\frac{7}{4}$, $\frac{400}{150}$, $\frac{8}{8}$, and $\frac{35}{7}$. (In contrast, a proper fraction's numerator is smaller than its denominator.) An improper fraction is often converted to a **mixed number**, which is the sum of a whole number and a proper fraction. For example, the improper fraction $\frac{14}{3}$ can be represented by the mixed number $4\frac{2}{3}$, which is equivalent to $4 + \frac{2}{3}$.

To convert an improper fraction to a mixed number, begin by dividing the denominator into the numerator. The quotient is the whole number portion of the mixed number. The remainder becomes the numerator of the fractional part, while the denominator of the fractional part is the same as the denominator of the improper function.

$$\begin{array}{r} 4 \\ 3\overline{)14} \\ -12 \\ \hline 2 \end{array}$$

4 ← Whole-number portion of mixed number

Denominator of fractional portion of mixed number → 3

2 ← Numerator of fractional portion of mixed number

EXAMPLE 6 Convert the improper fraction $\frac{71}{9}$ to a mixed number.

SOLUTION Begin by dividing 9 into 71, which divides in 7 times with a remainder of 8.

$$\begin{array}{r} 7 \\ 9\overline{)71} \\ \underline{-63} \\ 8 \end{array}$$

The mixed number for $\frac{71}{9}$ is $7\frac{8}{9}$.

Quick Check 5

Convert the improper fraction $\frac{121}{13}$ to a mixed number.

Objective 6 Change a mixed number to an improper fraction. Often we have to convert a mixed number such as $2\frac{7}{15}$ into an improper fraction before proceeding with arithmetic operations.

Rewriting a Mixed Number as an Improper Fraction

- Multiply the whole number part of the mixed number by the denominator of the fractional part of the mixed number.
- Add this product to the numerator of the fractional part of the mixed number.
- The sum is the numerator of the improper fraction. The denominator stays the same.

$2\frac{7}{15}$ ← Add product to numerator; Multiply

$$2\frac{7}{15} = \frac{37}{15}$$

EXAMPLE 7 Convert the mixed number $5\frac{4}{7}$ to an improper fraction.

SOLUTION Begin by multiplying $5 \cdot 7 = 35$. Add this product to 4 to produce a numerator of 39.

$$5\frac{4}{7} = \frac{39}{7}$$

Quick Check 6

Convert the mixed number $8\frac{1}{6}$ to an improper fraction.

BUILDING YOUR STUDY STRATEGY

Study Groups, 3 Where to Meet

- Some study groups prefer to meet off campus in the evening. One good place to meet is at a coffee shop with tables large enough to accommodate everyone, provided that the surrounding noise is not too distracting.
- Some groups take advantage of study rooms at public libraries.
- Other groups like to meet at members' homes. This typically provides a comfortable, relaxing atmosphere in which to work.

Exercises 1.3

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Vocabulary

1. The collection of all factors of a natural number is called its ________.
2. A natural number greater than 1 is ________ if its only factors are 1 and itself.
3. A natural number greater than 1 that is not prime is called a(n) ________ number.
4. Define the prime factorization of a natural number.
5. The numerator of a fraction is the number written on the ________ of the fraction.
6. The denominator of a fraction is the number written on the ________ of the fraction.
7. A fraction is in lowest terms if its numerator and denominator contain no ________ other than 1.
8. A fraction whose numerator is less than its denominator is called a(n) ________ fraction.
9. A fraction whose numerator is greater than or equal to its denominator is called a(n) ________ fraction.
10. An improper fraction can be rewritten as a whole number or as a(n) ________.

11. Is 7 a factor of 247?
12. Is 13 a factor of 273?
13. Is 6 a factor of 4836?
14. Is 9 a factor of 32,057?
15. Is 15 a factor of 2835?
16. Is 103 a factor of 1754?

Write the factor set for the following numbers.

17. 48
18. 60
19. 27
20. 15
21. 20
22. 16
23. 81
24. 64
25. 31
26. 103
27. 91
28. 143

Write the prime factorization of the following numbers. (If the number is prime, state this.)

29. 18
30. 20
31. 42
32. 36
33. 39
34. 50
35. 27
36. 32
37. 125
38. 49
39. 29
40. 76
41. 99
42. 90
43. 31
44. 209
45. 120
46. 109

Simplify the following fractions to lowest terms.

47. $\frac{10}{16}$
48. $\frac{35}{42}$
49. $\frac{9}{45}$
50. $\frac{38}{2}$
51. $\frac{168}{378}$
52. $\frac{60}{84}$
53. $\frac{27}{64}$
54. $\frac{66}{154}$
55. $\frac{160}{176}$
56. $\frac{56}{45}$
57. $\frac{49}{91}$
58. $\frac{72}{140}$

Convert the following mixed numbers to improper fractions.

59. $3\frac{4}{5}$
60. $7\frac{2}{9}$
61. $2\frac{16}{17}$
62. $6\frac{5}{14}$
63. $13\frac{8}{11}$
64. $17\frac{16}{33}$

Convert the following improper fractions to whole numbers or mixed numbers.

65. $\frac{39}{5}$
66. $\frac{56}{8}$
67. $\frac{101}{7}$
68. $\frac{12}{4}$
69. $\frac{141}{19}$
70. $\frac{109}{8}$

71. List four fractions that are equivalent to $\frac{3}{4}$.

72. List four fractions that are equivalent to $1\frac{2}{3}$.

73. List four whole numbers that have at least three different prime factors.

74. List four whole numbers greater than 100 that are prime.

Writing in Mathematics

Answer in complete sentences.

75. Describe a real-world situation involving fractions. Describe a real-world situation involving mixed numbers.

76. Describe a situation in which you should convert an improper fraction to a mixed number.

1.4 Operations with Fractions

OBJECTIVES

1. **Multiply fractions and mixed numbers.**
2. **Divide fractions and mixed numbers.**
3. **Add and subtract fractions and mixed numbers with the same denominator.**
4. **Find the least common multiple (LCM) of two natural numbers.**
5. **Add and subtract fractions and mixed numbers with different denominators.**

Multiplying Fractions

Objective 1 Multiply fractions and mixed numbers. To multiply fractions, we multiply the numerators together and multiply the denominators together. When multiplying fractions, we may simplify any individual fraction, as well as divide out a common factor from a numerator and a different denominator. Dividing out a common factor in this fashion is often referred to as **cross-canceling**.

EXAMPLE 1 Multiply $\frac{4}{11} \cdot \frac{5}{6}$.

SOLUTION The first numerator (4) and the second denominator (6) have a common factor of 2 that we can eliminate through division.

$$\frac{4}{11} \cdot \frac{5}{6} = \frac{\overset{2}{\cancel{4}}}{11} \cdot \frac{5}{\underset{3}{\cancel{6}}} \quad \text{Divide out the common factor 2.}$$

$$= \frac{2}{11} \cdot \frac{5}{3} \quad \text{Simplify.}$$

$$= \frac{10}{33} \quad \text{Multiply the two numerators and the two denominators.}$$

Quick Check 1

Multiply $\frac{10}{63} \cdot \frac{9}{16}$.

EXAMPLE 2 Multiply $3\frac{1}{7} \cdot \frac{14}{55}$.

SOLUTION When multiplying a mixed number by another number, convert the mixed number to an improper fraction before proceeding.

$$3\frac{1}{7} \cdot \frac{14}{55} = \frac{22}{7} \cdot \frac{14}{55}$$ Convert $3\frac{1}{7}$ to the improper fraction $\frac{22}{7}$.

$$= \frac{\overset{2}{\cancel{22}}}{\underset{1}{\cancel{7}}} \cdot \frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{55}}}$$ Divide out the common factors 11 and 7.

$$= \frac{4}{5}$$ Multiply.

Quick Check 2

Multiply $2\frac{2}{3} \cdot 8\frac{5}{8}$.

Dividing Fractions

Objective 2 Divide fractions and mixed numbers.

Reciprocal

When we invert a fraction such as $\frac{3}{5}$ to $\frac{5}{3}$, the resulting fraction is called the **reciprocal** of the original fraction.

Consider the fraction $\frac{a}{b}$, where a and b are nonzero real numbers. The reciprocal of this fraction is $\frac{b}{a}$. Notice that if we multiply a fraction by its reciprocal, such as $\frac{a}{b} \cdot \frac{b}{a}$, the result is 1. This property will be important in Chapter 2.

Reciprocal Property

For any nonzero real numbers a and b, $\frac{a}{b} \cdot \frac{b}{a} = 1$.

To divide a number by a fraction, invert the divisor and then multiply.

EXAMPLE 3 Divide $\frac{16}{25} \div \frac{22}{15}$.

SOLUTION

$$\frac{16}{25} \div \frac{22}{15} = \frac{16}{25} \cdot \frac{15}{22}$$ Invert the divisor and multiply.

$$= \frac{\overset{8}{\cancel{16}}}{\underset{5}{\cancel{25}}} \cdot \frac{\overset{3}{\cancel{15}}}{\underset{11}{\cancel{22}}}$$ Divide out the common factors 2 and 5.

$$= \frac{24}{55}$$ Multiply.

Quick Check 3

Divide $\frac{12}{25} \div \frac{63}{10}$.

A WORD OF CAUTION When dividing a number by a fraction, we must invert the divisor (not the dividend) before dividing out a common factor from a numerator and a denominator.

When performing a division involving a mixed number, begin by rewriting the mixed number as an improper fraction.

EXAMPLE 4 Divide $2\frac{5}{8} \div 3\frac{3}{10}$.

SOLUTION Begin by rewriting each mixed number as an improper fraction.

$$2\frac{5}{8} \div 3\frac{3}{10} = \frac{21}{8} \div \frac{33}{10} \quad \text{Rewrite each mixed number as an improper fraction.}$$

$$= \frac{21}{8} \cdot \frac{10}{33} \quad \text{Invert the divisor and multiply.}$$

$$= \frac{\overset{7}{\cancel{21}}}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{11}{\cancel{33}}} \quad \text{Divide out common factors.}$$

$$= \frac{35}{44} \quad \text{Multiply.}$$

Quick Check 4

Divide $\frac{20}{21} \div 2\frac{2}{3}$.

Adding and Subtracting Fractions

Objective 3 Add and subtract fractions and mixed numbers with the same denominator. To add or subtract fractions that have the same denominator, we add or subtract the numerators, placing the result over the common denominator. Make sure you simplify the result to lowest terms.

EXAMPLE 5 Subtract $\frac{3}{8} - \frac{9}{8}$.

SOLUTION The two denominators are the same (8), so we subtract the numerators. When we subtract $3 - 9$, the result is -6. Although we may leave the negative sign in the numerator, it often appears in front of the fraction itself.

$$\frac{3}{8} - \frac{9}{8} = -\frac{6}{8} \quad \text{Subtract the numerators.}$$

$$= -\frac{3}{4} \quad \text{Simplify to lowest terms.}$$

Quick Check 5

Subtract $\frac{17}{20} - \frac{5}{20}$.

When performing an addition involving a mixed number, begin by rewriting the mixed number as an improper fraction.

EXAMPLE 6 Add $3\frac{5}{12} + 2\frac{11}{12}$.

SOLUTION Begin by rewriting each mixed number as an improper fraction.

$$3\frac{5}{12} + 2\frac{11}{12} = \frac{41}{12} + \frac{35}{12} \quad \text{Rewrite } 3\frac{5}{12} \text{ and } 2\frac{11}{12} \text{ as improper fractions.}$$

$$= \frac{76}{12} \quad \text{Add the numerators.}$$

$$= \frac{19}{3} \quad \text{Simplify to lowest terms.}$$

$$= 6\frac{1}{3} \quad \text{Rewrite as a mixed number.}$$

It is not necessary to rewrite the result as a mixed number, but this is often done when you perform arithmetic operations on mixed numbers.

Quick Check 6

Add $6\frac{1}{8} + 5\frac{5}{8}$.

Objective 4 Find the least common multiple (LCM) of two natural numbers. Two fractions are said to be **equivalent fractions** if they have the same numerical value and both can be simplified to the same fraction when simplified to lowest terms. To add or subtract two fractions with different denominators, we must first convert them to equivalent fractions with the same denominator. To do this, we find the **least common multiple (LCM)** of the two denominators. This is the smallest number that is a multiple of both denominators. For example, the LCM of 4 and 6 is 12 because 12 is the smallest multiple of both 4 and 6.

To find the LCM for two numbers, begin by factoring them into their prime factorizations.

Finding the LCM of Two or More Numbers

- Find the prime factorization of each number.
- Find the common factors of the numbers.
- Multiply the common factors by the remaining factors of the numbers.

EXAMPLE 7 Find the LCM of 24 and 30.

SOLUTION Begin with the prime factorizations of 24 and 30.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
$$30 = 2 \cdot 3 \cdot 5$$
$$24 = \textcircled{2} \cdot 2 \cdot 2 \cdot \textcircled{3}$$
$$30 = \textcircled{2} \cdot \textcircled{3} \cdot 5$$

The common factors are 2 and 3. Additional factors are a pair of 2's as well as a 5. So to find the LCM, multiply the common factors (2 and 3) by the additional factors (2, 2, and 5).

$$2 \cdot 3 \cdot 2 \cdot 2 \cdot 5 = 120$$

The least common multiple of 24 and 30 is 120.

Quick Check 7

Find the least common multiple of 18 and 42.

Another technique for finding the LCM for two numbers is to start listing the multiples of the larger number until we find a multiple that also is a multiple of the smaller number. For example, the first few multiples of 6 are

$$6: 6, 12, 18, 24, 30, \ldots$$

The first multiple listed that also is a multiple of 4 is 12, so the LCM of 4 and 6 is 12.

Objective 5 Add and subtract fractions and mixed numbers with different denominators. When adding or subtracting two fractions that do not have the same denominator, we first find a common denominator by finding the LCM of the two denominators. Then convert each fraction to an equivalent fraction whose denominator is that common denominator. Once we rewrite the two fractions so they have the same denominator, we can add (or subtract) as done previously in this section.

Adding or Subtracting Fractions with Different Denominators

- Find the LCM of the denominators.
- Rewrite each fraction as an equivalent fraction whose denominator is the LCM of the original denominators.
- Add or subtract the numerators, placing the result over the common denominator.
- Simplify to lowest terms if possible.

EXAMPLE 8 Add $\frac{5}{12} + \frac{9}{14}$.

SOLUTION The prime factorization of 12 is $2 \cdot 2 \cdot 3$, and the prime factorization of 14 is $2 \cdot 7$. The two denominators have a common factor of 2. If we multiply this common factor by the other factors of these two numbers, 2, 3, and 7, we see that the LCM of these two denominators is 84. Begin by rewriting each fraction as an equivalent fraction whose denominator is 84. Multiply the first fraction by $\frac{7}{7}$ and the second fraction by $\frac{6}{6}$. Because $\frac{7}{7}$ and $\frac{6}{6}$ are both equal to 1, we do not change the value of either fraction.

$$\frac{5}{12} + \frac{9}{14} = \frac{5}{12} \cdot \frac{7}{7} + \frac{9}{14} \cdot \frac{6}{6}$$ Multiply the first fraction's numerator and denominator by 7. Multiply the second fraction's numerator and denominator by 6.

$$= \frac{35}{84} + \frac{54}{84}$$ Multiply.

$$= \frac{89}{84}$$ Add.

This fraction is already in lowest terms.

Quick Check 8

Add $\frac{4}{9} + \frac{2}{15}$.

When performing an addition or a subtraction involving a mixed number, we can begin by rewriting the mixed number as an improper fraction.

EXAMPLE 9 Subtract $4\frac{1}{3} - \frac{3}{4}$.

SOLUTION Begin by rewriting $4\frac{1}{3}$ as an improper fraction.

$$4\frac{1}{3} - \frac{3}{4} = \frac{13}{3} - \frac{3}{4}$$ Rewrite $4\frac{1}{3}$ as an improper fraction.

$$= \frac{13}{3} \cdot \frac{4}{4} - \frac{3}{4} \cdot \frac{3}{3}$$ The LCM of the denominators is 12. Multiply the first fraction by $\frac{4}{4}$. Multiply the second fraction by $\frac{3}{3}$.

$$= \frac{52}{12} - \frac{9}{12}$$ Multiply.

$$= \frac{43}{12}$$ Subtract.

$$= 3\frac{7}{12}$$ Rewrite as a mixed number.

Quick Check 9

Subtract $5\frac{1}{5} - 3\frac{5}{6}$.

BUILDING YOUR STUDY STRATEGY

Study Groups, 4 Going Over Homework A study group can go over homework assignments together. It is important that each group member work on the assignment before arriving at the study session. If you struggled with a problem or could not do it at all, ask for help or suggestions from your group members.

If there was a problem that you seem to understand better than the members of your group do, share your knowledge; explaining how to do a certain problem increases your chances of retaining that knowledge until the exam and beyond.

At the end of each session, quickly review what the group accomplished.

Exercises 1.4

MyMathLab PRACTICE WATCH DOWNLOAD READ REVIEW

Vocabulary

1. Before multiplying by a mixed number, convert it to a(n) ________________.
2. When a fraction is inverted, the result is called its ________________.
3. Explain how to divide by a fraction.
4. When fractions are added or subtracted, they must have the same ________________.
5. The smallest number that is a multiple of two numbers is called their ________________.
6. A board is cut into two pieces that measure $4\frac{1}{6}$ feet and $3\frac{3}{8}$ feet, respectively. Which operation will give the length of the original board?

   a) $4\frac{1}{6} + 3\frac{3}{8}$ **b)** $4\frac{1}{6} - 3\frac{3}{8}$

   c) $4\frac{1}{6} \cdot 3\frac{3}{8}$ **d)** $4\frac{1}{6} \div 3\frac{3}{8}$

Multiply. Your answer should be in lowest terms.

7. $\frac{3}{8} \cdot \frac{4}{27}$ **8.** $\frac{6}{35} \cdot \frac{25}{29}$

9. $\frac{20}{21} \cdot \left(-\frac{77}{90}\right)$ **10.** $-\frac{9}{30} \cdot \frac{28}{42}$

11. $4\frac{2}{7} \cdot \frac{14}{25}$ **12.** $-3\frac{5}{9} \cdot 2\frac{1}{6}$

13. $5 \cdot 6\frac{3}{10}$ **14.** $8 \cdot \frac{7}{12}$

15. $\frac{2}{3} \cdot \frac{8}{9}$ **16.** $-\frac{12}{35}\left(-\frac{14}{99}\right)$

Divide. Your answer should be in lowest terms.

17. $\frac{6}{25} \div \frac{8}{45}$ **18.** $\frac{15}{32} \div \frac{9}{20}$

19. $-\frac{22}{56} \div \frac{33}{147}$ **20.** $-\frac{24}{91} \div \left(-\frac{9}{39}\right)$

21. $\frac{17}{40} \div \frac{1}{2}$ **22.** $\frac{7}{30} \div \left(-\frac{1}{5}\right)$

23. $\frac{4}{11} \div 3\frac{1}{5}$ **24.** $3\frac{3}{8} \div 6$

25. $-2\frac{4}{5} \div 6\frac{2}{3}$ **26.** $7\frac{1}{9} \div 13\frac{3}{8}$

Add or subtract.

27. $\frac{7}{15} + \frac{4}{15}$ **28.** $\frac{1}{8} + \frac{5}{8}$

29. $\frac{5}{9} + \frac{4}{9}$ **30.** $\frac{3}{10} + \frac{9}{10}$

31. $\frac{2}{5} - \frac{4}{5}$ **32.** $\frac{17}{18} - \frac{5}{18}$

33. $\frac{9}{16} - \frac{3}{16}$ **34.** $\frac{13}{42} - \frac{29}{42}$

Find the LCM of the given numbers.

35. 10, 15 **36.** 8, 12

37. 12, 42 **38.** 9, 30

39. 16, 80 **40.** 16, 27

41. 8, 10, 14 **42.** 20, 35, 50

Simplify. Your answer should be in lowest terms.

43. $\frac{4}{5} + \frac{3}{4}$ **44.** $\frac{4}{7} + \frac{1}{4}$

45. $\frac{7}{10} + \frac{5}{8}$ **46.** $\frac{3}{4} + \frac{5}{6}$

47. $6\frac{1}{5} + 5$ **48.** $3 + 8\frac{3}{7}$

49. $6\frac{2}{3} + 5\frac{1}{6}$ **50.** $11\frac{4}{9} + 5\frac{1}{3}$

51. $\frac{2}{3} - \frac{7}{15}$ **52.** $\frac{1}{2} - \frac{7}{9}$

53. $\frac{3}{4} - \frac{2}{7}$ **54.** $\frac{5}{8} - \frac{5}{6}$

55. $7\frac{1}{2} - 3\frac{1}{4}$ **56.** $12\frac{2}{3} - 6\frac{2}{5}$

57. $12\frac{3}{10} - 9$ **58.** $6 - 4\frac{3}{4}$

59. $-\frac{5}{9} - \frac{7}{12}$ **60.** $-\frac{9}{10} - \frac{11}{14}$

61. $-\frac{9}{16} + \frac{5}{24}$ **62.** $-\frac{3}{8} + \frac{13}{24}$

63. $\frac{6}{7} - \left(-\frac{8}{15}\right)$ **64.** $\frac{1}{12} - \left(-\frac{19}{30}\right)$

65. $-\frac{4}{15} + \left(-\frac{13}{18}\right)$ **66.** $-\frac{17}{24} + \left(-\frac{25}{42}\right)$

67. $\frac{3}{16} + \frac{9}{20} - \frac{11}{12}$

68. $\frac{10}{21} - \frac{13}{18} + \frac{8}{15}$

Mixed Practice, 69–88

Simplify.

69. $\frac{8}{9} \cdot \frac{3}{5}$

70. $\frac{3}{4} + \frac{7}{10}$

71. $\frac{7}{30} \div \frac{35}{48}$

72. $\frac{12}{35} \cdot \frac{14}{27}$

73. $\frac{1}{6} - \frac{7}{8}$

74. $\frac{7}{24} - \frac{29}{40}$

75. $\frac{19}{30} + \frac{11}{18}$

76. $3\frac{1}{5} \cdot 4\frac{3}{8}$

77. $13 \div \frac{1}{8}$

78. $\frac{3}{5} - \frac{2}{3} - \frac{7}{10}$

79. $3\frac{4}{7} + 6\frac{3}{5} - 8$

80. $12\frac{1}{3} + 7\frac{1}{6} - 5\frac{1}{2}$

81. $\frac{15}{56} + \left(-\frac{16}{21}\right)$

82. $\frac{7}{12} - \left(-\frac{23}{30}\right)$

83. $-\frac{9}{13} + \frac{19}{36}$

84. $-\frac{3}{8} - \frac{81}{100}$

Find the missing number.

85. $\frac{11}{24} + \frac{5}{?} = \frac{13}{12}$

86. $\frac{?}{10} - \frac{1}{3} = \frac{1}{6}$

87. $\frac{10}{21} \cdot \frac{?}{75} = \frac{4}{45}$

88. $\frac{11}{40} + \frac{?}{40} = \frac{9}{10}$

89. Bruce is fixing a special dinner for his girlfriend. The three recipes he is preparing call for $\frac{1}{2}$ cup, $\frac{3}{4}$ cup, and $\frac{1}{3}$ cup of flour, respectively. In total, how much flour does Bruce need to make these three recipes?

90. Sue gave birth to twins. One of the babies weighed $4\frac{7}{8}$ pounds at birth, and the other baby weighed $5\frac{1}{4}$ pounds. Find the total weight of the twins at birth.

91. A chemist has $\frac{23}{40}$ fluid ounce of a solution. If she needs $\frac{1}{8}$ fluid ounce of the solution for an experiment, how much of the solution will remain?

92. A popular weed spray concentrate recommends using $1\frac{1}{4}$ tablespoons of concentrate for each quart of water. How much concentrate needs to be mixed with 6 quarts of water?

93. A board that is $4\frac{1}{5}$ feet long needs to be cut into 6 pieces of equal length. How long will each piece be?

94. Ross makes a batch of hot sauce that will be poured into bottles that hold $5\frac{3}{4}$ fluid ounces. If Ross has 115 fluid ounces of hot sauce, how many bottles can he fill?

95. A craftsperson is making a rectangular picture frame. Each of two sides will be $\frac{5}{6}$ of a foot long, while each of the other two sides will be $\frac{2}{3}$ of a foot long. If the craftsperson has one board that is $2\frac{3}{4}$ feet long, is this enough to make the picture frame? Explain.

96. A pancake recipe calls for $1\frac{1}{3}$ cups of whole wheat flour to make 12 pancakes. How much flour is needed to make 48 pancakes?

97. A pancake recipe calls for $1\frac{1}{3}$ cups of whole wheat flour to make 12 pancakes. How much flour is needed to make 6 pancakes?

Writing in Mathematics

Answer in complete sentences.

98. Explain, using your own words, the difference between dividing a number in half and dividing a number by one-half.

99. Explain why you cannot divide a common factor of 2 from the numbers 4 and 6 in the expression $\frac{4}{7} \cdot \frac{6}{11}$.

100. Explain why it would be a bad idea to rewrite fractions with a common denominator before multiplying them.

1.5 Decimals and Percents

OBJECTIVES

1. **Perform arithmetic operations with decimals.**
2. **Rewrite a fraction as a decimal number.**
3. **Rewrite a decimal number as a fraction.**
4. **Rewrite a fraction as a percent.**
5. **Rewrite a decimal as a percent.**
6. **Rewrite a percent as a fraction.**
7. **Rewrite a percent as a decimal.**

Decimals

Rational numbers also can be represented using **decimal notation**. The decimal 0.23 is equivalent to the fraction $\frac{23}{100}$, or twenty-three hundredths. The digit 2 is in the tenths place, and the digit 3 is in the hundredths place. The following chart shows several place values for decimals:

0.123456 . . .

- 1: Tenths
- 2: Hundredths
- 3: Thousandths
- 4: Ten Thousandths
- 5: Hundred Thousandths
- 6: Millionths

Objective 1 Perform arithmetic operations with decimals. Here is a brief summary of arithmetic operations using decimals.

- To add or subtract two decimal numbers, align the decimal points and add or subtract as you would with integers.

$$3.96 + 12.072$$

$$\begin{array}{r} 3.96 \\ +\ 12.072 \\ \hline 16.032 \end{array}$$

- To multiply two decimal numbers, multiply them as you would integers. The total number of decimal places in the two factors shows how many decimal places are in the product.

$$-2.09 \cdot 3.1$$

In this example, the two factors have a total of three decimal places, so the product must have three decimal places. Multiply these two numbers as if they were 209 and 31 and then insert the decimal point in the appropriate place, leaving three digits to the right of the decimal point.

$$\begin{array}{r} -209 \\ \times\ 31 \\ \hline -6479 \end{array} \qquad \begin{array}{r} -2.09 \\ \times\ 3.1 \\ \hline -6.479 \end{array}$$

3 places

- To divide two decimal numbers, move the decimal point in the divisor to the right so that it becomes an integer. Then move the decimal point in the other number (dividend) to the right by the same number of spaces. The decimal point in the answer will be aligned with this new location of the decimal point in the dividend.

$8.24 \div 0.4$

$0.4\overline{)8.24}$ Begin by moving *each* decimal point one place to the right.

$0.4\overline{)8.24}$

$$\begin{array}{r} 20.6 \\ 4\overline{)82.4} \end{array}$$ Then perform the division.

Rewriting Fractions as Decimals and Decimals as Fractions

Objective 2 Rewrite a fraction as a decimal number. To rewrite any fraction as a decimal, we divide its numerator by its denominator. The fraction line is simply another way to write "÷".

EXAMPLE 1 Rewrite the fraction $\frac{5}{8}$ as a decimal.

SOLUTION To rewrite this fraction as a decimal, divide 5 by 8.

Now begin the division, adding a decimal point after the 5 and 0's to the end of the dividend until there is no remainder.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -4.8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$$\frac{5}{8} = 0.625$$

Quick Check 1

Rewrite the fraction $\frac{3}{4}$ as a decimal.

EXAMPLE 2 Rewrite the fraction $\frac{23}{30}$ as a decimal.

SOLUTION When we divide 23 by 30, the result is a decimal that does not terminate (0.76666 . . .). The pattern continues repeating the digit 6 forever. This is an example of a **repeating decimal**. We may place a bar over the repeating digit(s) to denote a repeating decimal.

$$\frac{23}{30} = 0.7\overline{6}$$

Quick Check 2

Rewrite the fraction $\frac{5}{18}$ as a decimal.

Objective 3 Rewrite a decimal number as a fraction. Suppose we want to rewrite a decimal number such as 0.48 as a fraction. This decimal is read as "forty-eight hundredths" and is equivalent to the fraction $\frac{48}{100}$. Simplifying this fraction shows that $0.48 = \frac{12}{25}$.

EXAMPLE 3 Rewrite the decimal 0.164 as a fraction in lowest terms.

SOLUTION This decimal ends in the thousandths place, so start with a fraction of $\frac{164}{1000}$.

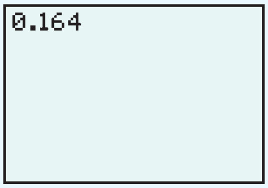
$$0.164 = \frac{164}{1000} \quad \text{Rewrite as a fraction whose denominator is 1000.}$$
$$= \frac{41}{250} \quad \text{Simplify to lowest terms.}$$

Quick Check 3

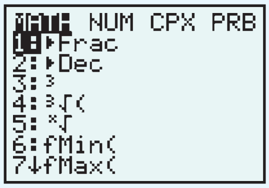
Rewrite the decimal 0.425 as a fraction in lowest terms.

Using Your Calculator To rewrite a decimal as a fraction using the TI-84, key the decimal, press the MATH key, and select option 1. The following screens show how to rewrite the decimal 0.164 as a fraction using the TI-84, as in Example 3.

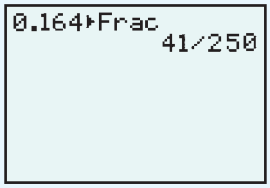
Key the decimal.



Press the MATH key.



Result



Percents

Objective 4 Rewrite a fraction as a percent. **Percents** (%) are used to represent numbers as parts of 100. One percent, which can be written as 1%, is equivalent to 1 part of 100, or $\frac{1}{100}$, or 0.01. The fraction $\frac{27}{100}$ is equivalent to 27%. Percents, decimals, and fractions are all ways to write a rational number. We will learn to convert back and forth between percents and fractions as well as between percents and decimals.

Rewriting Fractions and Decimals as Percents

To rewrite a fraction or a decimal as a percent, we multiply it by 100%. Because 100% is equal to 1 this will not change the value of the fraction or decimal number.

EXAMPLE 4 Rewrite as a percent: a) $\frac{2}{5}$, b) $\frac{3}{8}$.

SOLUTION

a) Begin by multiplying by 100% and simplifying.

$$\frac{2}{\underset{1}{\cancel{5}}} \cdot \overset{20}{\cancel{100}}\% = 40\%$$

b) Again, multiply by 100%. Occasionally, as in this example, we will end up with an improper fraction, which can be changed to a mixed number.

$$\frac{3}{\underset{2}{\cancel{8}}} \cdot \overset{25}{\cancel{100}}\% = \frac{75}{2}\%, \text{ which can be rewritten as } 37\frac{1}{2}\%.$$

Quick Check 4

a) Rewrite $\frac{7}{10}$ as a percent.
b) Rewrite $\frac{21}{40}$ as a percent.

Objective 5 Rewrite a decimal as a percent.

EXAMPLE 5 Rewrite 0.3 as a percent.

SOLUTION When we multiply a decimal by 100, the result is the same as moving the decimal point two places to the right.

$$0.30.$$

$$0.3 \cdot 100\% = 30\%$$

Quick Check 5

Rewrite 0.42 as a percent.

Rewriting Percents as Fractions and Decimals

Objective 6 Rewrite a percent as a fraction. To rewrite a percent as a fraction or a decimal, we can divide it by 100 and omit the percent sign. When rewriting a percent as a fraction, we may choose to multiply by $\frac{1}{100}$ rather than dividing by 100.

EXAMPLE 6 Rewrite as a fraction: a) 44%, b) $16\frac{2}{3}\%$.

SOLUTION

a) Begin by multiplying by $\frac{1}{100}$ and omitting the percent sign.

$$\overset{11}{\cancel{44}} \cdot \frac{1}{\underset{25}{\cancel{100}}} = \frac{11}{25}$$

b) Rewrite the mixed number $16\frac{2}{3}$ as an improper fraction $\left(\frac{50}{3}\right)$, multiply by $\frac{1}{100}$, and simplify.

$$\frac{\overset{1}{\cancel{50}}}{3} \cdot \frac{1}{\underset{2}{\cancel{100}}} = \frac{1}{6}$$

Quick Check 6

a) Rewrite 35% as a fraction.
b) Rewrite $11\frac{2}{3}\%$ as a fraction.

Objective 7 Rewrite a percent as a decimal. In the next example, we will rewrite percents as decimals rather than as fractions.

EXAMPLE 7 Rewrite as a decimal: a) 56%, b) 143%.

SOLUTION

a) Begin by dropping the percent sign and dividing by 100. Keep in mind that dividing a decimal number by 100 is the same as moving the decimal point two places to the left.

$$.56.$$

$$56 \div 100 = 0.56$$

b) When a percent is greater than 100%, its equivalent decimal must be greater than 1.

$$143 \div 100 = 1.43$$

Quick Check 7

a) Rewrite 8% as a decimal. **b)** Rewrite 240% as a decimal.

BUILDING YOUR STUDY STRATEGY

Study Groups, 5 Three Questions Another way to structure a group study session is to have each member bring a list of three questions to the meeting. The questions can be about specific homework problems or about topics or procedures that have been covered in class. Once the members have asked their questions, the group should attempt to come up with answers that each member understands. If the group cannot answer a question, see your instructor at the beginning of the next class or during office hours, asking him or her for an explanation.

Exercises 1.5

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Vocabulary

1. The first place to the right of a decimal point is the ____________ place.
2. The third place to the right of a decimal point is the ____________ place.
3. To rewrite a fraction as a decimal divide the ____________ by the ____________.
4. To rewrite a fraction as a percent ____________ it by 100%.
5. To rewrite a percent as a fraction ____________ it by 100 and omit the percent sign.
6. Percents are used to represent numbers as parts of ____________.

Simplify the following decimal expressions.

7. $4.23 + 3.62$
8. $13.89 - 2.54$
9. $-7(5.2)$
10. $69.54 \div 6$
11. $8.4 - 3.7$
12. $-7.9 + (-4.5)$
13. $13.568 \div 0.4$
14. $3.6(4.7)$
15. $-2.2 \cdot 3.65$
16. $6.2 - 15.9$
17. $13.47 - (-21.562)$
18. $5.283 \div 0.25$
19. $-6.3(3.9)(-2.25)$
20. $-4.84 \div (-0.016)$
21. $37.278 + 56.722$
22. $109.309 - 27.46 - 52.3716$

Rewrite the following fractions as decimal numbers.

23. $\frac{9}{10}$
24. $\frac{2}{5}$
25. $-\frac{23}{8}$
26. $\frac{59}{4}$
27. $\frac{13}{25}$
28. $-\frac{11}{16}$
29. $24\frac{29}{50}$
30. $7\frac{3}{20}$

Rewrite the following decimal numbers as fractions in lowest terms.

31. 0.2
32. 0.5
33. 0.85
34. 0.36
35. −0.74
36. −0.56
37. 0.375
38. 0.204
39. The normal body temperature for humans is 98.6° F. If Melody has a temperature that is 2.8° F above normal, what is her temperature?
40. Paul spent the following amounts on gifts for his wife's birthday: \$32.95, \$16.99, \$47.50, \$12.37, and \$285. How much did Paul spend on gifts for his wife?

41. The balance of Carie's checking account is \$427.36. If she writes checks for \$19.95, \$34.40, and \$148.68, what will her new balance be?

42. At the close of the stock market on Tuesday, the price for one share of Google was \$426.17. Over the next three days, the stock went down by \$9.63, up by \$14.08, and down by \$7.84. What was the price of the stock at the end of Friday's session?

43. An office manager bought 12 cases of paper. If each case cost \$21.47, what was the total cost for the 12 cases?

44. Jean gives Chris a \$20 bill and tells him to go to the grocery store and buy as many hot dogs as he can. If each package of hot dogs costs \$2.65, how many packages can Chris buy? How much change will Chris have?

Rewrite as percents.

45. $\frac{3}{4}$ **46.** $\frac{3}{5}$

47. $\frac{4}{5}$ **48.** $\frac{5}{8}$

49. $\frac{7}{8}$ **50.** $\frac{11}{12}$

51. $\frac{27}{4}$ **52.** $\frac{12}{5}$

53. 0.4 **54.** 0.6

55. 0.15 **56.** 0.87

57. 0.09 **58.** 0.03

59. 3.2 **60.** 2.75

Rewrite as fractions.

61. 84% **62.** 80%

63. 7% **64.** 2%

65. $11\frac{1}{9}\%$ **66.** $18\frac{2}{11}\%$

67. 520% **68.** 275%

Rewrite as decimals.

69. 54% **70.** 71%

71. 16% **72.** 29%

73. 7% **74.** 9%

75. 0.3% **76.** 61.3%

77. 400% **78.** 320%

79. Find three fractions that are equivalent to 0.375.

80. Find three fractions that are equivalent to 0.4.

Complete the following table.

	Fraction	Decimal	Percent
81.		0.2	
82.	$\frac{7}{40}$		
83.			32%
84.			45%
85.	$\frac{13}{8}$		
86.		0.64	

Writing in Mathematics

Answer in complete sentences.

87. Stock prices at the New York Stock Exchange used to be reported as fractions. Now prices are reported as decimals. Do you think this was a good idea? Explain.

88. Describe a real-world application involving decimals.

1.6 Basic Statistics

OBJECTIVES

1. Calculate basic statistics for a set of data.
2. Construct a histogram for a set of data.

In today's data-driven society, we often see graphs presenting data or information on television news as well as in newspapers, magazines, and online. This section focuses on the calculation of statistics used to describe a set of data as well as the creation of a histogram to represent a set of data.

Objective 1 Calculate basic statistics for a set of data. There are two basic types of statistics—those that describe the typical value for a set of data and those that describe how varied the values are. Statistics that describe the typical value for a set of data are often called **measures of center**, while statistics that describe how varied a set of data is are often called **measures of spread**.

Mean

The **mean** of a set of data is one measure of center for a set of data. To calculate the mean for a set of data, we simply add all of the values and divide by how many values there are. The mean is the arithmetic average of the set of data.

$$\text{Mean} = \frac{\text{Sum of All Values}}{\text{Number of Values}}$$

EXAMPLE 1 Eight students were asked how far they drive to school each day. Here are the results, in miles: 17, 8, 30, 1, 2, 5, 15, 10. Calculate the mean for this set of data.

SOLUTION The total of these eight values is 88.

$$\text{Mean} = \frac{88}{8} = 11$$

The mean mileage for these students is 11 miles.

Quick Check 1

Five families were asked how much they spend on groceries each month. Here are the results: \$850, \$1020, \$970, \$635, \$795. Calculate the mean for this set of data.

Median

The **median** of a set of data is another measure of center for a set of data, often used for types of data that have unusually high or low values, such as home prices and family income. To find the median for a set of data, we begin by writing the values in ascending order. If a set of data has an odd number of values, the single value in the middle of the set of data is the median. If a set of data has an even number of values, the median is the average of the two center values.

EXAMPLE 2 Find the median of the given set of values.

a) 37, 16, 59, 18, 30, 4, 75, 46, 62

b) 65, 72, 74, 81, 71, 83, 89, 82, 53, 48, 77, 65

SOLUTION

a) Begin by rewriting the values in ascending order: 4, 16, 18, 30, 37, 46, 59, 62, 75. Because there is an odd number of values (9), the median is the single value in the center of the list.

4 16 18 30 37 46 59 62 75

↑ Median (37)

The median is 37.

b) Here are the values in ascending order: 48, 53, 65, 65, 71, 72, 74, 77, 81, 82, 83, 89. Because there is an even number of values (12), the median is the average of the two values in the center of the list.

48 53 65 65 71 (72 74) 77 81 82 83 89

$$\text{Median} = \frac{72 + 74}{2} = \frac{146}{2} = 73$$

The median is 73.

Quick Check 2

Find the median of the given set of values.

a) 54, 21, 39, 16, 7, 75
b) 51, 3, 29, 60, 62, 25, 43, 102, 14

Mode and Midrange

The **mode** and **midrange** are two other measures of center for a set of data. The mode is the value that is repeated most often.

- If there are no repeated values, the set of data has no mode. The set 1, 2, 3, 4, 5 has no mode because no value is repeated.
- A set of data can have more than one mode if two or more values are repeated the same number of times. The set 5, 5, 7, 8, 8 has two modes—5 and 8.

The midrange is the average of the set's minimum value and maximum value.

$$\text{Midrange} = \frac{\text{Minimum Value} + \text{Maximum Value}}{2}$$

EXAMPLE 3 During a medical trial, the LDL cholesterol levels of 16 adult males were measured. Here are the results.

104 122 115 90 116 88 167 105 154 129 81 157 143 122 106 87

Find the mode and the midrange for this data.

SOLUTION Only one value, 122, has been repeated; so the mode is 122. To find the midrange, identify the maximum and minimum values for this set of data. The smallest value in this set is 81, and the largest value is 167.

$$\text{Midrange} = \frac{81 + 167}{2} = \frac{248}{2} = 124$$

The midrange is 124.

Quick Check 3

Here are the heights, in inches, of 10 adult females.

56	65	66	66	65	66	65	60	61	65

Find the mode and the midrange for this data.

Range

The **range** is a measure of spread for a set of data, showing how varied the values are. To find the range of a set of values, we subtract the minimum value in the set from the maximum value in the set.

$$\text{Range} = \text{Maximum Value} - \text{Minimum Value}$$

EXAMPLE 4 Here are the test scores of 9 math students.

65	75	96	91	78	81	73	92	61

Find the range of these test scores.

SOLUTION To find the range, identify the maximum and minimum values for this set of data. The maximum value in this set is 96, and the minimum value is 61.

$$\text{Range} = 96 - 61 = 35$$

The range is 35 points.

Quick Check 4

Here are the heights, in centimeters, of 8 adult males.

165	168	174	179	182	159	171	180

Find the range for this data.

Histogram

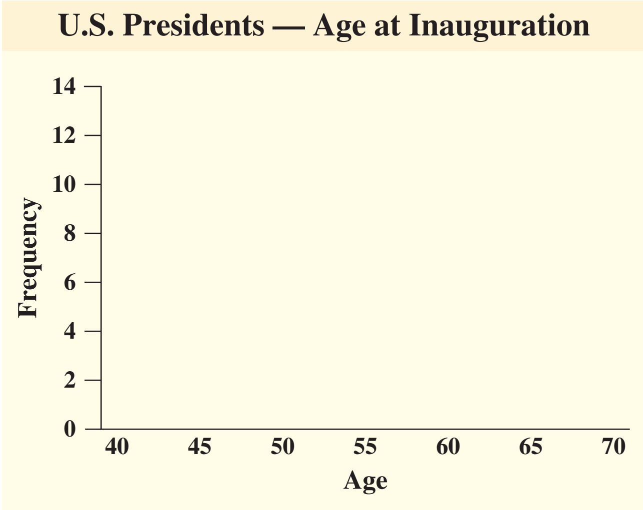
Objective 2 Construct a histogram for a set of data.

A **histogram** is a graph that can be used to show how a set of data is distributed, giving an idea of where the data values are centered as well as how they are dispersed. To construct a histogram, we begin with a **frequency distribution**, which divides the data into groups, called **classes**, and lists how many times each class is represented in the set of data. Here is a frequency distribution showing the ages of U.S. Presidents at inauguration.

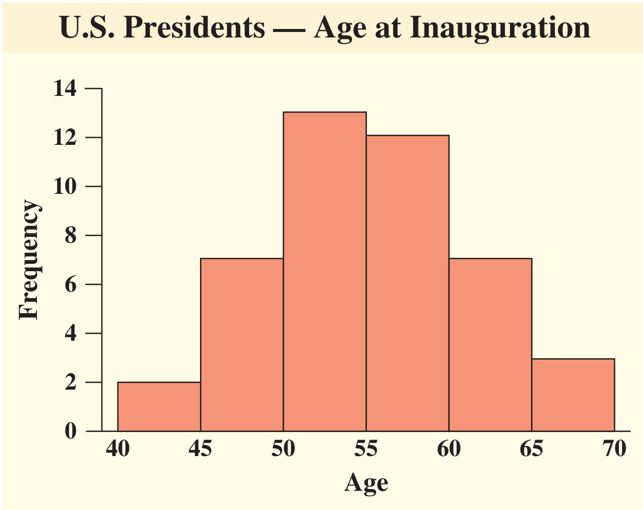
Age	Frequency
40 to 44	2
45 to 49	7
50 to 54	13
55 to 59	12
60 to 64	7
65 to 69	3

This frequency distribution shows that two presidents were between 40 and 44 years old when they were inaugurated, seven presidents were between the ages of 45 and 49, and so on.

To construct a histogram, we begin by drawing two axes as shown. Mark the beginning of each class at the bottom of the graph on the horizontal axis, including the value that would be the lower limit of the next class. In the example, that will be the numbers 40, 45, 50, 55, 60, 65, and 70. On the vertical axis, mark the frequencies. Make sure your axis goes at least to the highest frequency in the frequency distribution.



A bar is drawn above each class, and the height of the bar is determined by the frequency of that class. The first bar, from 40 to 45, should have a height of 2; the second bar should have a height of 7; and so on. Here is the histogram.



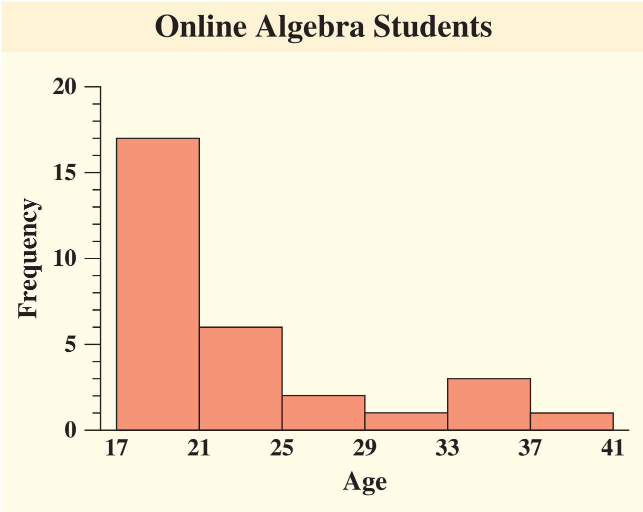
From this histogram, we can see that the bulk of the values are between 50 and 60, which is the center of the values.

EXAMPLE 5 Here is a frequency distribution showing the ages of 30 students enrolled in an online algebra class.

Age	Frequency
17 to 20	17
21 to 24	6
25 to 28	2
29 to 32	1
33 to 36	3
37 to 40	1

Draw a histogram for the frequency distribution.

SOLUTION On the horizontal axis, we begin the labels at 17 and increase by 4 until we reach 41. On the vertical axis, we must have labels that reach at least 17, which is the largest frequency.



Quick Check 5

Here is a frequency distribution showing the ages of 86 passengers on a cruise.

Ages	Frequency
25 to 34	3
35 to 44	7
45 to 54	6
55 to 64	15
65 to 74	35
75 to 84	20

Draw a histogram for the frequency distribution.

BUILDING YOUR STUDY STRATEGY

Study Groups, 6 Keeping in Touch It is important for group members to share phone numbers and e-mail addresses. This will allow you to contact other group members if you misplace the homework assignment for that day. You can also contact others in your group if you have to miss class. This allows you to find out what was covered in class and which problems were assigned for homework. The person you contact can also give you advice on certain problems.

Some members of study groups agree to call each other if they are having trouble with homework problems. Calling a group member for this purpose should be a last resort. Be sure that you have used all available resources (examples in the text, notes, etc.) and have given the problem your fullest effort. Otherwise, you may end up calling for help on all of the problems.

Exercises 1.6

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Vocabulary

1. Statistics that describe the typical value for a set of data are often called measures of ______.
2. Statistics that describe how varied a set of data is are often called measures of ______.
3. The ______ of a set of data is the sum of all of the values divided by the number of values.
4. The ______ of a set of data is the value in the center of the data once the values are arranged in ascending order.
5. The ______ of a set of data is the value that is repeated most often.
6. The ______ of a set of data is the average of its minimum and maximum values.
7. The ______ of a set of data is the difference between its maximum and minimum values.
8. A(n) ______ is a graph that can be used to show how a set of data is distributed.

Find the mean for the given values.

9. 63, 98, 21, 42, 71
10. 84, 37, 29, 46, 15, 65
11. 5, 17, 21, 35, 42, 59, 89, 106
12. 30, 70, 74, 82, 95, 113, 128, 140

Find the median for the given values.

13. 97, 76, 22, 103, 80, 45, 66
14. 87, 3, 20, 62, 55, 73, 101, 49, 75
15. 68, 47, 32, 90, 85, 40, 83, 39, 50, 77
16. 123, 304, 290, 175, 260, 209, 321, 275

Find the mode, if it exists, for the given values.

17. 70, 56, 63, 35, 56, 63, 36, 56, 19
18. 80, 50, 70, 80, 50, 70, 80, 40, 60
19. 7, 41, 32, 56, 41, 19, 8, 32, 25
20. 61, 47, 47, 17, 29, 16, 25, 92, 16
21. 5, 35, 89, 106, 42, 17, 59, 21
22. 88, 45, 6, 99, 32, 75, 16, 100, 42

For the given values, find the midrange and the range.

23. 70, 140, 87, 62, 196, 125, 155
24. 93, 47, 28, 80, 94, 60, 93
25. 406, 354, 509, 427, 516, 379
26. 165, 82, 97, 155, 79, 203, 121, 99

For Exercises 27–34, find the a) mean, b) median, c) mode, d) midrange, and e) range.

27. 22, 13, 16, 30, 32, 19, 24, 30, 21
28. 59, 41, 46, 62, 41, 50, 65
29. 48, 45, 63, 36, 50, 38, 73, 63
30. 98, 84, 44, 40, 50, 82, 43, 84, 46, 70

31. 63, 86, 76, 85, 59, 71, 34, 44, 30, 67, 44, 77, 50, 83, 76

32. 48, 91, 38, 101, 93, 66, 31, 57, 84, 47, 73, 41, 86, 90, 96, 86, 62

33. 196, 295, 213, 69, 371, 77, 253, 210, 298, 210, 426, 327, 270, 323, 262, 70, 459, 481, 278, 192

34. 257, 50, 23, 223, 125, 249, 197, 191, 99, 194, 239, 227, 192, 96, 50, 147, 259, 296

Find the mean for the given set of data.

35. IQ of 12 college students

95 82 104 119 118 126 82 96 116 85 90 90

36. Red Sox home runs (2001–2008)

2001	2002	2003	2004	2005	2006	2007	2008
198	177	238	222	199	192	166	173

37. Systolic blood pressure of seven 60-year-old men

133 112 142 154 102 139 149

38. Systolic blood pressure of nine 60-year-old women

119 160 121 92 109 95 114 112 122

39. Weight (in ounces) of 10 newborn baby girls

101 110 125 120 106 113 102 108 132 135

40. Starting salary for 5 bachelor's degrees

Chemical Engineering	Computer Science	Mathematics	Political Science	English
\$61,800	\$54,200	\$43,500	\$39,400	\$36,700

(*Source: PayScale.com*)

Find the median for the given set of data.

41. Number of Facebook friends for 6 college math instructors

243 18 21 152 93 125

42. Serum glucose level (mg/dL) of 8 people

90 91 94 122 113 142 59 92

43. Math test scores of 9 members of a study group

80 96 100 89 74 96 95 98 87

44. Pregnancy duration (days) for 11 women

267 255 263 261 265 273 264 267 268 275 273

45. Number of hours spent studying last week by 10 college students

25 12 17 3 20 20 16 34 1 9

46. Number of hours spent working last week by 10 college students

8 4 20 0 12 32 8 40 20 16

For exercises 47–52, find the a) mean, b) median, c) mode, d) midrange, and e) range.

47. Room rate at 10 Las Vegas hotels (Valentine's Day, 2010)

\$219 \$259 \$127 \$199 \$259 \$169
\$219 \$229 \$299 \$199

48. Touchdown passes thrown by Joe Montana by year (16 seasons)

1 15 19 17 26 28 27 8 31 18 26 26 0 2 13 16

49. Time (in seconds) of the 8 songs on Bruce Springsteen's "Born to Run"

289 191 180 390 271 270 198 574

50. Number of calories in 16 different brands of beer

188 166 163 165 149 209 135 150 96 145 170
124 158 110 314 94

51. Cell phone minutes used by 14 families last month

636 754 662 884 1346 659 1006 1357
1129 904 1747 1336 1234 388

52. Systolic blood pressure of thirteen 65-year-old smokers (mmHg)

110 118 137 127 134 163 129 102
102 136 150 130 113

Find the missing value x that satisfies the given condition for the set of values.

53. Mean: 75

80 86 100 81 30 57 90 x

54. Mean: 82.5

43 96 90 x 81 104 111 72 66 89

55. Median: 101.5

112 98 121 72 x 65

56. Median: 93

97 x 81 100 104 88 121 79

57. Range: 84; midrange: 52

66	25	94	37	x	85	42

58. Range: 47; midrange: 35.5

30	29	58	12	16	45	x	50

Construct a histogram for the given frequency distribution.

59. Average 2007 SAT math score for the 50 states and Washington, D.C.
(*Source: The College Board*)

Average Score	Frequency
460 to 479	2
480 to 499	6
500 to 519	14
520 to 539	6
540 to 559	6
560 to 579	8
580 to 599	5
600 to 619	4

60. Scores of 40 students on an algebra exam

Score	Frequency
40 to 49	1
50 to 59	3
60 to 69	4
70 to 79	7
80 to 89	14
90 to 99	11

61. High school graduation rates for the 50 states (2005)

Graduation Rate	Frequency
45.0% to 49.9%	1
50.0% to 54.9%	2
55.0% to 59.9%	2
60.0% to 64.9%	4
65.0% to 69.9%	7
70.0% to 74.9%	16
75.0% to 79.9%	13
80.0% to 84.9%	5

62. Daily caloric intake of 60 participants in a health study

Calories	Frequency
1400 to 1599	1
1600 to 1799	6
1800 to 1999	15
2000 to 2199	21
2200 to 2399	9
2400 to 2599	8

Complete the frequency distribution for the given data and use it to construct a histogram.

63. Number of heads in 1000 coin flips, repeated by 60 students

471	505	515	503	506	507	471	522	548	478	514	490
511	463	515	514	490	485	467	531	487	482	500	506
504	492	522	497	508	499	515	499	516	495	499	496
510	520	509	500	488	512	501	506	488	497	498	503
496	488	522	505	517	497	500	502	472	525	477	506

Number of Heads	Frequency
450 to 469	
470 to 489	
490 to 509	
510 to 529	
530 to 549	

64. Starting salaries of 50 Certified Public Accountants (CPAs)

$52,500	$57,700	$55,100	$60,400	$61,900
$57,700	$52,600	$57,200	$52,700	$59,200
$58,300	$58,800	$61,400	$56,600	$56,900
$60,300	$57,700	$55,400	$56,200	$59,300
$59,000	$58,300	$52,300	$56,000	$59,400
$60,100	$53,400	$54,700	$55,600	$62,800
$62,300	$55,000	$56,300	$57,200	$59,600
$56,200	$62,300	$55,900	$50,100	$64,500
$55,900	$56,600	$57,200	$59,600	$60,300
$55,900	$63,400	$62,100	$55,800	$63,100

Salary	Frequency
$50,000 to $52,499	
$52,500 to $54,999	
$55,000 to $57,499	
$57,500 to $59,999	
$60,000 to $62,499	
$62,500 to $64,999	

65. IQs of 36 college students

104	114	88	96	105	120	92	107	134	110	95	133
132	119	115	116	129	114	100	95	105	140	110	131
97	113	104	120	95	128	128	106	96	114	127	133

IQ	Frequency
85 to 94	
95 to 104	
105 to 114	
115 to 124	
125 to 134	
135 to 144	

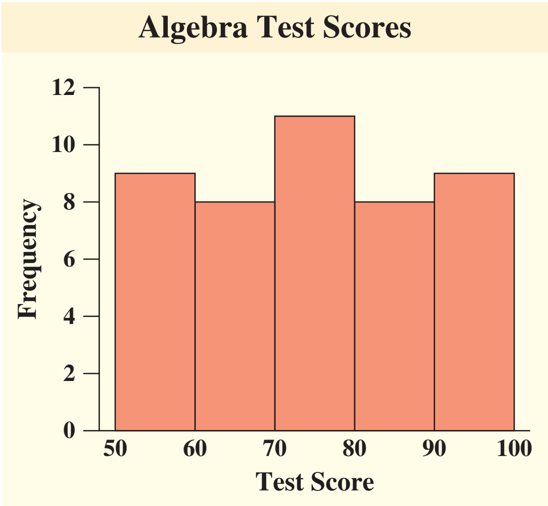
66. Blood glucose level (mg/dL) of 40 women participating in a clinical study

63	141	84	108	93	64	94	80	95	100
90	115	96	89	68	114	130	111	86	75
85	93	116	102	72	87	95	105	109	74
115	120	119	101	92	100	84	148	97	100

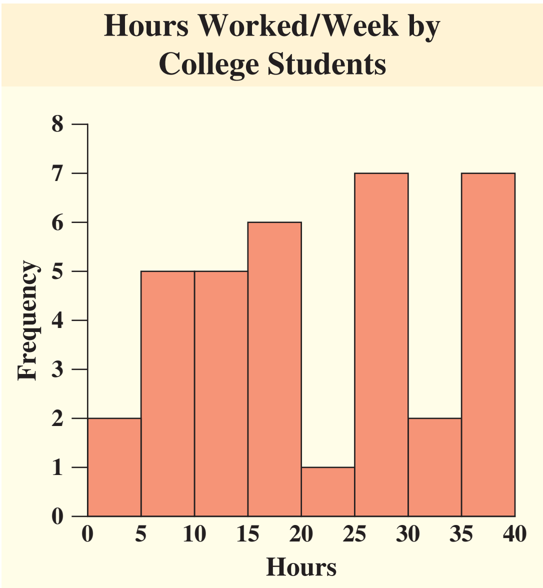
Blood Glucose Level	Frequency
60 to 74	
75 to 89	
90 to 104	
105 to 119	
120 to 134	
135 to 149	

Use the given histogram to create a frequency distribution.

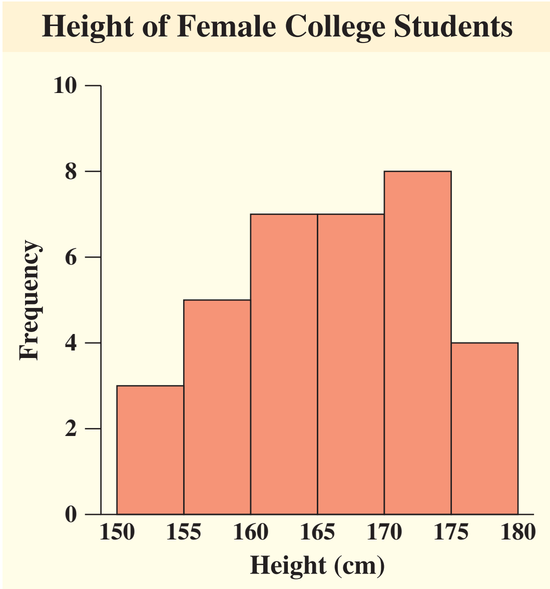
67.



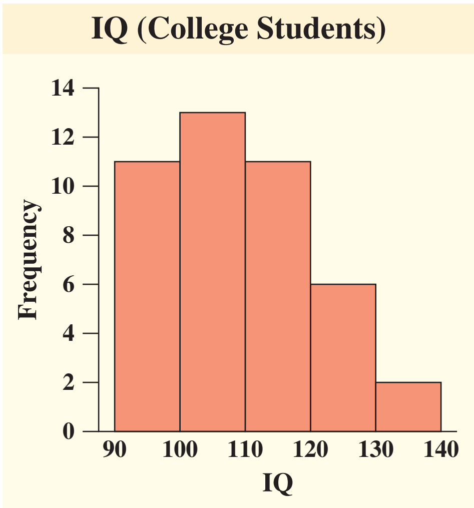
68.



69.



70.



1.7 Exponents and Order of Operations

OBJECTIVES

1. Simplify exponents.
2. Use the order of operations to simplify arithmetic expressions.

Exponents

Objective 1 Simplify exponents. The same number used repeatedly as a factor can be represented using **exponential notation**. For example, $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ can be written as 3^5.

Base, Exponent

For the expression 3^5, the number being multiplied (3) is called the **base**. The **exponent** (5) tells how many times the base is used as a factor.

We read 3^5 as *three raised to the fifth power* or simply *three to the fifth power.* When raising a base to the second power, we usually say that the base is being **squared**. When raising a base to the third power, we usually say that the base is being **cubed**. Exponents of 4 or higher do not have a special name.

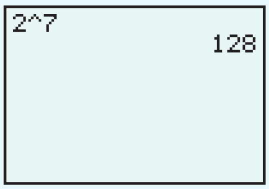
EXAMPLE 1 Simplify 2^7.

SOLUTION In this example, 2 is a factor seven times.

$$2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \quad \text{Write 2 as a factor seven times.}$$
$$= 128 \quad \text{Multiply.}$$

Quick Check 1
Simplify 4^3.

Using Your Calculator Many calculators have a key that can be used to simplify expressions with exponents. When using the TI-84, use the $\boxed{\wedge}$ key. Other calculators may have a key that is labeled $\boxed{y^x}$ or $\boxed{x^y}$. Here is the screen you should see when using the TI-84 to simplify the expression in Example 1.



EXAMPLE 2 Simplify $\left(\frac{2}{3}\right)^3$.

SOLUTION When the base is a fraction, the same rules apply. Use $\frac{2}{3}$ as a factor three times.

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \quad \text{Write as a product.}$$
$$= \frac{8}{27} \quad \text{Multiply.}$$

Quick Check 2
Simplify $\left(\frac{1}{8}\right)^4$.

Consider the expression $(-3)^2$. The base is -3; so we multiply $(-3) \cdot (-3)$, and the result is positive 9. However, in the expression -2^4, the negative sign is not included in a set of parentheses with the 2. So the base of this expression is 2, not -2. We will use 2 as a factor 4 times and then take the opposite of the result: $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$.

Order of Operations

Objective 2 Use the order of operations to simplify arithmetic expressions. Suppose we were asked to simplify the expression $2 + 4 \cdot 3$. We could obtain two different results depending on whether we performed the addition or the multiplication first. Performing the addition first would give us $6 \cdot 3 = 18$, while performing the multiplication first would give us $2 + 12 = 14$. Only one result is correct. The **order of operations agreement** is a standard order in which arithmetic operations are performed, ensuring a single correct answer.

Order of Operations

1. **Remove grouping symbols.** Begin by simplifying all expressions within parentheses, brackets, and absolute value bars. Also perform any operations in the numerator or denominator of a fraction. This is done by following Steps 2–4, presented next.
2. **Perform any operations involving exponents.** After all grouping symbols have been removed from the expression, simplify any exponential expressions.
3. **Multiply and divide.** These two operations have equal priority. Perform multiplications or divisions in the order they appear from left to right.
4. **Add and subtract.** At this point, the only remaining operations should be additions and subtractions. Again, these operations are of equal priority, and we perform them in the order they appear from left to right. We also can use the strategy for totaling integers from Section 1.2.

Considering this information, when we simplify $2 + 4 \cdot 3$, the correct result is 14 because multiplication takes precedence over the addition.

$$\begin{aligned} 2 + 4 \cdot 3 &= 2 + 12 && \text{Multiply } 4 \cdot 3. \\ &= 14 && \text{Add.} \end{aligned}$$

EXAMPLE 3 Simplify $4 + 3 \cdot 5 - 2^6$.

SOLUTION In this example, the operation with the highest priority is 2^6, because simplifying exponents takes precedence over addition or multiplication. Note that the base is 2, not -2, because the negative sign is not grouped with 2 inside a set of parentheses.

$$\begin{aligned} 4 + 3 \cdot 5 - 2^6 &= 4 + 3 \cdot 5 - 64 && \text{Raise 2 to the 6th power.} \\ &= 4 + 15 - 64 && \text{Multiply } 3 \cdot 5. \\ &= -45 && \text{Simplify.} \end{aligned}$$

Quick Check 3
Simplify $-2 \cdot 7 + 11 - 3^4$.

EXAMPLE 4 Simplify $(-5)^2 - 4(-2)(6)$.

SOLUTION Begin by squaring negative 5, which equals 25.

$$\begin{aligned} (-5)^2 - 4(-2)(6) &= 25 - 4(-2)(6) && \text{Square } -5. \\ &= 25 - (-48) && \text{Multiply } 4(-2)(6). \\ &= 25 + 48 && \text{Write as a sum, eliminating the double signs.} \\ &= 73 \end{aligned}$$

Quick Check 4
Simplify $9^2 - 4(-2)(-10)$.

A WORD OF CAUTION When we square a negative number such as $(-5)^2$ in the previous example, the result is a positive number.

$$(-5)^2 = (-5)(-5) = 25$$

EXAMPLE 5 Simplify $8 \div 2 + 3(7 - 4 \cdot 5)$.

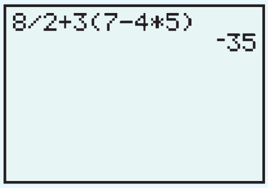
SOLUTION The first step is to simplify the expression inside the set of parentheses. Here the multiplication takes precedence over the subtraction. Once we have simplified the expression inside the parentheses, we proceed to multiply and divide. We finish by subtracting.

$$
\begin{aligned}
8 \div 2 + 3(7 - 4 \cdot 5) &= 8 \div 2 + 3(7 - 20) && \text{Multiply } 4 \cdot 5.\\
&= 8 \div 2 + 3(-13) && \text{Subtract } 7 - 20.\\
&= 4 + 3(-13) && \text{Divide } 8 \div 2.\\
&= 4 - 39 && \text{Multiply } 3(-13).\\
&= -35 && \text{Subtract.}
\end{aligned}
$$

Quick Check 5

Simplify $20 \div 5 \cdot 10(3 \cdot 6 - 9)$.

Using Your Calculator When using your calculator to simplify an expression using the order of operations, you may want to perform one operation at a time. However, if you are careful to enter all of the parentheses, you can enter the entire expression at one time. Here is how to simplify the expression in Example 5 using the TI-84.



Occasionally, an expression will have one set of grouping symbols inside another set, such as the expression $3[7 - 4(9 - 3)] + 5 \cdot 4$. This is called **nesting** grouping symbols. We begin by simplifying the innermost set of grouping symbols and work our way out from there.

EXAMPLE 6 Simplify $3[7 - 4(9 - 3)] + 5 \cdot 4$.

SOLUTION We begin by simplifying the expression inside the set of parentheses. Once we do that, we simplify the expression inside the square brackets.

$$
\begin{aligned}
&3[7 - 4(9 - 3)] + 5 \cdot 4 && \text{Subtract } 9 - 3 \text{ to simplify the expression inside}\\
&= 3[7 - 4 \cdot 6] + 5 \cdot 4 && \text{the parentheses. Then turn our attention to simplifying the expression inside the square brackets.}\\
&= 3[7 - 24] + 5 \cdot 4 && \text{Multiply } 4 \cdot 6.\\
&= 3[-17] + 5 \cdot 4 && \text{Subtract } 7 - 24.\\
&= -51 + 20 && \text{Multiply } 3[-17] \text{ and } 5 \cdot 4.\\
&= -31 && \text{Simplify.}
\end{aligned}
$$

Quick Check 6

Simplify $4[3^2 + 5(2 - 8)]$.

BUILDING YOUR STUDY STRATEGY

Study Groups, 7 **Productive Group Members** For any group or team to be effective, it must be made up of members who are committed to giving their full effort to success. Here are some pointers for being a productive study group member:

- **Arrive at each session fully prepared.** Make sure you have completed all required homework assignments. Bring a list of any questions you have.
- **Stay focused during study sessions.** Do not spend your time socializing.
- **Be open-minded.** During study sessions, you may be told that you are wrong. Keep in mind that the goal is to learn mathematics, not always to be correct initially.
- **Consider the feelings of others.** When a person has made a mistake, be supportive and encouraging.
- **Know when to speak and when to listen.** A study group works best when it is a collaborative body.

Exercises 1.7

Vocabulary

1. In exponential notation, the factor being multiplied repeatedly is called the ________.
2. In exponential notation, the exponent tells us how many times the ______ is repeated as a(n) ________.
3. When a base is ________, it is raised to the second power.
4. When a base is ________, it is raised to the third power.
5. Symbols such as ( ) and [ ] are called ________ symbols.
6. When simplifying arithmetic expressions, once all grouping symbols have been removed, we perform any operations involving ________.

Rewrite the given expression using exponential notation.

7. $2 \cdot 2 \cdot 2$
8. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
9. $(-2)(-2)(-2)(-2)$
10. $(-6)(-6)(-6)(-6)(-6)$
11. $-3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
12. $-8 \cdot 8 \cdot 8 \cdot 8$
13. *five to the third power*
14. *seven squared*

Simplify the given expression.

15. 3^4
16. 4^3
17. 7^5
18. 2^9
19. 10^6
20. 10^5
21. 1^{723}
22. 0^{2364}
23. $\left(\frac{3}{4}\right)^3$
24. $\left(\frac{1}{5}\right)^5$
25. 0.2^5
26. 0.5^4
27. $(-3)^4$
28. -3^4
29. -2^7
30. $(-2)^7$
31. $2^3 \cdot 5^2$
32. $9^2 \cdot 4^4$
33. $-6^2 \cdot (-2)^2$
34. $-3^2 \cdot (-8)^2$

Simplify the given expression.

35. $9 + 3 \cdot 4$
36. $8 - 2 \cdot 7$
37. $20 \div 5 \cdot 2^2$
38. $80 - 16 \div 2^3$
39. $8 \cdot 5 - 9 \cdot 7$
40. $8(5 - 9 \cdot 7)$
41. $3.2 + 2.8(-6.3)$
42. $(3.2 + 2.8) - 6.3$
43. $17.1 - 8.58 \div 3.9$
44. $4.7 \cdot 13.9 - 3.6^2$
45. $(-3)^2 - 4(-5)(-4)$
46. $(-6)^2 - 4(2)(7)$
47. $3^2 + 4^2$
48. $(3 + 4)^2$
49. $-4 - 5(7 - 3 \cdot 6)$
50. $3 - 6(5^2 - 4 + 2 \cdot 7)$
51. $\frac{1}{2} + \frac{1}{2} \cdot \frac{4}{7}$
52. $\frac{3}{5} \cdot \frac{2}{3} + \frac{15}{7} \cdot \frac{21}{20}$
53. $\frac{2}{9} \div \frac{5}{3} \cdot \frac{33}{50}$
54. $4 \cdot \frac{3}{8} - \left(\frac{7}{6}\right)^2$
55. $\frac{1}{9}\left(\frac{19}{28} - \frac{1}{4}\right)$
56. $\frac{8}{25} \div \left(\frac{4}{15} - \frac{1}{3} + \frac{3}{5}\right) \cdot \frac{7}{18}$
57. $\frac{2^2 - 9}{3^3 - 7}$
58. $\frac{3 + 5 \cdot 7 - 4 \cdot 2}{1 + 2 \cdot 17}$
59. $\frac{(3 + 5) \cdot 6 - 8}{1 + 3^2 + 2}$
60. $\frac{10^3 + 9^3}{1^3 + 12^3}$
61. $3 - [4(5 - 6 \cdot 7)]$
62. $18 + [9 - (4 - 5 \cdot 8)] \div 3^2$
63. $-4 \cdot 9 - |-7(3 + 5)| - 2^3$
64. $-6^2 + 2|(7 - 49) \div (2 \cdot 3)|$
65. $9 - |3^2 + 2^3 - 10 \cdot 9|$
66. $|-4^2 + 19| - |(-7)^2 + 5(10)|$
67. $-21(|4 - 3 \cdot 9| + |8(13 - 4)|)$
68. $(6 \cdot 5 + 40 \div 20)(|8^2 - 2 \cdot 17| - |10^2 - 2 \cdot 51|)$

Construct an "order-of-operations problem" of your own that involves at least four numbers and produces the given result. Answers will vary. Examples are shown.

69. 41
70. 0
71. −13
72. −19

The size of a computer's memory is measured by the number of bytes that it can store. The following table lists the number of bytes in commonly used storage units.

1 kilobyte (KB)	2^{10} bytes
1 megabyte (MB)	2^{20} bytes
1 gigabyte (GB)	2^{30} bytes
1 terabyte (TB)	2^{40} bytes

Calculate the number of bytes in each of the following.

73. 1 kilobyte

74. 1 megabyte

75. 1 gigabyte

76. 1 terabyte

Find the missing number.

77. $5^{?} = 125$

78. $6^{?} = 7776$

79. $?^{3} = 729$

80. $?^{4} = 2401$

81. $\left(\frac{3}{?}\right)^4 = \frac{81}{4096}$

82. $\left(\frac{2}{5}\right)^? = \frac{64}{15{,}625}$

Insert the arithmetic operation signs +, −, ·, and ÷ between the values to produce the desired results. (You may use parentheses as well.)

83. 3 5 9 8 = 14

84. 4 7 2 6 = 14

85. 3 7 9 2 = 33

86. 8 2 3 14 4 7 = 2

Writing in Mathematics

Answer in complete sentences.

87. Explain the difference between $(-2)^6$ and -2^6.

88. ***Newsletter*** Write a newsletter explaining how to use the order of operations to simplify an expression.

1.8 Introduction to Algebra

OBJECTIVES

1. Build variable expressions.
2. Evaluate algebraic expressions.
3. Use the commutative, associative, and distributive properties of real numbers.
4. Identify terms and their coefficients.
5. Simplify variable expressions.

Variables

The number of students absent from a particular English class changes from day to day, as does the closing price for a share of Microsoft stock and the daily high temperature in Providence, Rhode Island. Quantities that change, or vary, are often represented by variables. A **variable** is a letter or symbol that is used to represent a quantity that changes or that has an unknown value.

Variable Expressions

Objective 1 Build variable expressions. Suppose a buffet restaurant charges \$7 per person to eat. To determine the bill for a family to eat in the restaurant, we multiply the number of people by \$7. The number of people can change from family to family, so we can represent this quantity by a variable such as x. The bill for a family with x people can be written as $7 \cdot x$, or simply $7x$. This expression for the bill, $7x$, is known as a variable expression. A **variable expression** is a combination of one or more variables with numbers or arithmetic operations. When the operation is multiplication such as $7 \cdot x$, we often omit the multiplication dot. Here are other examples of variable expressions.

$$3a + 5 \qquad x^2 + 3x - 10 \qquad \frac{y + 5}{y - 3}$$

EXAMPLE 1 Write an algebraic expression for *the sum of a number and 17.*

SOLUTION Choose a variable to represent the unknown number. Let x represent the number. The expression is $x + 17$.

Other terms to look for that suggest addition are *plus, increased by, more than,* and *total*.

Quick Check 1

Write an algebraic expression for *9 more than a number*.

EXAMPLE 2 Write an algebraic expression for *five less than a number.*

SOLUTION Let x represent the number. The expression is $x - 5$.

Be careful with the order of subtraction when the expression *less than* is used. *Five less than a number* says that we need to subtract 5 from that number. A common error is to write the subtraction in the opposite order.

Other terms that suggest subtraction are *difference, minus,* and *decreased by.*

Quick Check 2

Write an algebraic expression for *a number decreased by 25.*

A WORD OF CAUTION *Five less than a number* is written as $x - 5$, not $5 - x$.

EXAMPLE 3 Write an algebraic expression for *the product of 3 and two different numbers.*

SOLUTION Because we are building an expression involving two different unknown numbers, we need to introduce two variables. Let x and y represent the two numbers. The expression is $3xy$.

Other terms that suggest multiplication are *times, multiplied by, of,* and *twice.*

Quick Check 3

Write an algebraic expression for *twice a number*.

EXAMPLE 4 Four friends decide to rent a fishing boat for the day. Assuming that all 4 friends decide to split the cost of renting the boat evenly, write a variable expression for the amount each friend will pay.

SOLUTION Let c represent the cost of the boat. The expression is $c \div 4$ or $\frac{c}{4}$.

Other terms that suggest division are *quotient, divided by,* and *ratio.*

Quick Check 4

Write an algebraic expression for *the quotient of a number and 20*.

Here are some phrases that translate to $x + 5$, $x - 10$, $8x$, and $\frac{x}{6}$:

$x + 5$	$x - 10$
The sum of a number and 5 A number plus 5 A number increased by 5 5 more than a number The total of a number and 5	10 less than a number The difference of a number and 10 A number minus 10 A number decreased by 10
$8x$	**$\frac{x}{6}$**
The product of 8 and a number 8 times a number 8 multiplied by a number	The quotient of a number and 6 A number divided by 6 The ratio of a number and 6

Evaluating Variable Expressions

Objective 2 Evaluate algebraic expressions. We often will have to evaluate variable expressions for particular values of variables. To do this, we substitute the appropriate numerical value for each variable and then simplify the resulting expression using the order of operations.

EXAMPLE 5 Evaluate $2x - 7$ for $x = 6$.

SOLUTION The first step in evaluating a variable expression is to rewrite the expression, replacing each variable with a set of parentheses. For example, rewrite $2x - 7$ as $2(\ ) - 7$. Then we can substitute the appropriate value for each variable and simplify.

$$2x - 7$$
$$2(6) - 7 \quad \text{Substitute 6 for } x.$$
$$= 12 - 7 \quad \text{Multiply.}$$
$$= 5 \quad \text{Subtract.}$$

The expression $2x - 7$ is equal to 5 for $x = 6$.

Quick Check 5
Evaluate $5x + 2$ for $x = 11$.

EXAMPLE 6 Evaluate $x^2 - 5x + 6$ for $x = -5$.

SOLUTION

$$x^2 - 5x + 6$$
$$(-5)^2 - 5(-5) + 6 \quad \text{Substitute } -5 \text{ for } x.$$
$$= 25 - 5(-5) + 6 \quad \text{Square } -5.$$
$$= 25 + 25 + 6 \quad \text{Multiply.}$$
$$= 56 \quad \text{Add.}$$

Quick Check 6
Evaluate $x^2 - 13x - 40$ for $x = -8$.

EXAMPLE 7 Evaluate $b^2 - 4ac$ for $a = 3$, $b = -2$, and $c = -10$.

SOLUTION

$$b^2 - 4ac$$
$$(-2)^2 - 4(3)(-10) \quad \text{Substitute 3 for } a, -2 \text{ for } b, \text{ and } -10 \text{ for } c.$$
$$= 4 - 4(3)(-10) \quad \text{Square negative 2.}$$
$$= 4 - (-120) \quad \text{Multiply.}$$
$$= 4 + 120 \quad \text{Rewrite without double signs.}$$
$$= 124 \quad \text{Add.}$$

Quick Check 7
Evaluate $b^2 - 4ac$ for $a = -1$, $b = 5$, and $c = 18$.

Properties of Real Numbers

Objective 3 Use the commutative, associative, and distributive properties of real numbers. Now we examine three properties of real numbers. The first is the **commutative property**, which is used for addition and multiplication.

Commutative Property

For all real numbers a and b,

$$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a.$$

This property states that changing the order of the numbers a sum or a product does not change the result. For example, $3 + 9 = 9 + 3$ and $7 \cdot 8 = 8 \cdot 7$. Note that this property does not work for subtraction or division. Changing the order of the numbers in subtraction or division generally changes the result.

The second property is the **associative property**, which also holds for addition and multiplication.

Associative Property

For all real numbers a, b, and c,

$$(a + b) + c = a + (b + c) \quad \text{and} \quad (ab)c = a(bc).$$

This property states that changing the grouping of the numbers in a sum or a product does not change the result. Notice that $(2 + 7) + 3$ is equal to $2 + (7 + 3)$.

$$\begin{aligned}&(2 + 7) + 3 \\ &= 9 + 3 \\ &= 12\end{aligned} \qquad \begin{aligned}&2 + (7 + 3) \\ &= 2 + 10 \\ &= 12\end{aligned}$$

Also notice that $5(12x)$ can be rewritten using the associative property as $(5 \cdot 12)x$, which is equal to $60x$.

The third property of real numbers is the **distributive property**.

Distributive Property

For all real numbers a, b, and c,

$$a(b + c) = ab + ac.$$

This property says that we can distribute the factor outside the parentheses to each number being added in the parentheses, perform the multiplications, and then add. (This property also holds true when the operation inside the parentheses is subtraction.) Consider the expression $3(5 + 4)$. The order of operations says that this is equal to $3 \cdot 9$, or 27. Here is how to simplify the expression using the distributive property.

$$\begin{aligned}3(5 + 4) &= 3 \cdot 5 + 3 \cdot 4 && \text{Apply the distributive property.} \\ &= 15 + 12 && \text{Multiply } 3 \cdot 5 \text{ and } 3 \cdot 4. \\ &= 27 && \text{Add.}\end{aligned}$$

Either way we get the same result.

EXAMPLE 8 Simplify $2(4 + 3x)$ using the distributive property.

SOLUTION

$$\begin{aligned}2(4 + 3x) &= 2 \cdot 4 + 2 \cdot 3x && \text{Distribute the 2.} \\ &= 8 + 6x && \text{Multiply. Recall from the associative property that } 2 \cdot 3x \text{ equals } 6x.\end{aligned}$$

Quick Check 8

Simplify $7(5x - 4)$ using the distributive property.

A WORD OF CAUTION The expression $8 + 6x$ is not equal to $14x$.

EXAMPLE 9 Simplify $7(2 + 3a - 4b)$ using the distributive property.

Quick Check 9

Simplify $12(x - 2y + 3z)$ using the distributive property.

SOLUTION When more than two terms are inside the parentheses, distribute the factor to each term. Also, it is a good idea to distribute the factor and multiply mentally.

$$7(2 + 3a - 4b) = 14 + 21a - 28b$$

EXAMPLE 10 Simplify $-4(2x - 5)$ using the distributive property.

SOLUTION When the factor outside the parentheses is negative, the negative number must be distributed to each term inside the parentheses. This will change the sign of each term inside the parentheses.

$$\begin{aligned} -4(2x - 5) &= (-4)\cdot 2x - (-4)\cdot 5 && \text{Distribute the } -4. \\ &= -8x - (-20) && \text{Multiply.} \\ &= -8x + 20 && \text{Rewrite without double signs.} \end{aligned}$$

Quick Check 10

Simplify $-6(4x + 11)$ using the distributive property.

Simplifying Variable Expressions

Objective 4 Identify terms and their coefficients. In an algebraic expression, a **term** is a number, a variable, or a product of a number and variables. Terms in an algebraic expression are separated by addition. The expression $7x - 5y + 3$, has three terms: $7x$, $-5y$, and 3. The numerical factor of a term is its **coefficient**. The coefficients of these terms are 7, −5, and 3.

EXAMPLE 11 For the expression $-5x + y + 3xy - 19$, determine the number of terms, list them, and state the coefficient for each term.

SOLUTION This expression has four terms: $-5x$, y, $3xy$, and −19.

What is the coefficient for the second term? Although y does not appear to have a coefficient, its coefficient is 1. This is because y is the same as $1 \cdot y$. So the four coefficients are −5, 1, 3, and −19.

Quick Check 11

For the expression $x^3 - x^2 + 23x - 59$, determine the number of terms, list them, and state the coefficient for each term.

Objective 5 Simplify variable expressions. Two terms that have the same variable factors with the same exponents, or that are both constants, are called **like terms**. Consider the expression $9x + 8y + 6x - 3y + 8z - 5$. There are two sets of like terms in this expression: $9x$ and $6x$ as well as $8y$ and $-3y$. There are no like terms for $8z$ because no other term has z as its sole variable factor. Similarly, there are no like terms for the constant term −5.

Combining Like Terms

When simplifying variable expressions, we can combine like terms into a single term with the same variable part by adding or subtracting the coefficients of the like terms.

EXAMPLE 12 Simplify $4x + 11x$ by combining like terms.

SOLUTION These two terms are like terms because they both have the same variable factors. We can simply add the two coefficients to produce the expression $15x$.

$$4x + 11x = 15x$$

Quick Check 12

Simplify $3x + 7y + y - 5x$ by combining like terms.

A general strategy for simplifying algebraic expressions is to begin by applying the distributive property. We can then combine any like terms.

EXAMPLE 13 Simplify $8(3x - 5) - 4x + 7$.

SOLUTION The first step is to use the distributive property by distributing the 8 to each term inside the parentheses. Then we will be able to combine like terms.

$$\begin{aligned} 8(3x - 5) - 4x + 7 &= 24x - 40 - 4x + 7 && \text{Distribute the 8.} \\ &= 20x - 33 && \text{Combine like terms.} \end{aligned}$$

Quick Check 13

Simplify $5(2x + 3) + 9x - 8$.

EXAMPLE 14 Simplify $5(9 - 7x) - 10(3x + 4)$.

SOLUTION We must make sure that we distribute the *negative* 10 into the second set of parentheses.

$$\begin{aligned} 5(9 - 7x) - 10(3x + 4) &= 45 - 35x - 30x - 40 && \text{Distribute the 5 to each term in the first set of parentheses and distribute the } -10 \text{ to each term in the second set of parentheses.} \\ &= -65x + 5 && \text{Combine like terms.} \end{aligned}$$

Usually we write the simplified result with variable terms preceding constant terms, but it also is correct to write $5 - 65x$ because of the commutative property.

Quick Check 14

Simplify $3(2x - 7) - 8(x - 9)$.

A WORD OF CAUTION If a factor in front of a set of parentheses is negative or has a subtraction sign in front of it, we must distribute the negative sign along with the factor.

BUILDING YOUR STUDY STRATEGY

Study Groups, 8 Dealing with Unproductive Group Members Occasionally, a group will have a member who is not productive or is even disruptive. You should not allow one person to prevent your group from being successful. If you have a group member who is not contributing to the group in a positive way, talk to that person one-on-one discreetly.

Try to find solutions that will be acceptable to the group and the group member in question. Seek advice from your instructor. Undoubtedly, your instructor has seen a similar situation and may have valuable advice for you.

Try to be professional about the situation rather than adversarial. Remember, you will continue to see this person in class each day and you want to stay on good terms.

Exercises 1.8

MyMathLab PRACTICE WATCH DOWNLOAD READ REVIEW

Vocabulary

1. A(n) __________ is a letter or symbol used to represent a quantity that changes or has an unknown value.
2. A combination of one or more variables with numbers and/or arithmetic operations is a(n) __________ expression.
3. To __________ a variable expression, substitute the appropriate numerical value for each variable and then simplify the resulting expression using the order of operations.
4. For any real numbers a and b, the __________ property states that $a + b = b + a$ and $a \cdot b = b \cdot a$.
5. For any real numbers a, b, and c, the __________ property states that $(a + b) + c = a + (b + c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
6. For any real numbers a, b, and c, the __________ property states that $a(b + c) = ab + ac$.
7. A(n) __________ is a number, a variable, or a product of numbers and variables.
8. A(n) __________ is the numerical factor of a term.
9. Two terms that have the same variable factors, or that are constant terms, are called __________.
10. To combine two like terms, add their __________.

Build a variable expression for the following phrases.

11. A number increased by 15
12. A number decreased by 33
13. Twenty-four less than a number
14. Forty-one more than a number
15. Three times a number
16. A number divided by 8
17. Nineteen more than twice a number
18. Seven less than 4 times a number
19. The sum of two different numbers
20. One number divided by another
21. Seven times the difference of two numbers
22. Half the sum of a number and 25
23. A college charges $325 per credit for tuition. Letting c represent the number of credits that a student is taking, build a variable expression for the student's tuition.
24. The admission charge for a particular amusement park is $54.95 per person. Letting p represent the number of people that attended the amusement park yesterday, build a variable expression for the total admission charges the park collected yesterday.
25. A professional baseball player is appearing at a baseball card convention. The promoter agreed to pay the player a flat fee of $25,000 plus $22 per autograph signed. Letting a represent the number of autographs signed, build a variable expression for the amount of money the player will be paid.
26. Jim Rockford, a private investigator from the 1970's TV show *The Rockford Files,* charged $200 per day plus expenses to take a case. Letting d represent the number of days Rockford worked on a case and assuming that he had $425 in expenses, build a variable expression for the amount of money he would charge for the case.

Write the given expression using words.

27. $x - 9$
28. $x + 16$
29. $7x$
30. $6x + 5$
31. $8x - 10$
32. $2x - 7$

Evaluate the following algebraic expressions under the given conditions.

33. $8x + 31$ for $x = 6$
34. $27 - 4x$ for $x = 9$
35. $7a - 13b$ for $a = 8$ and $b = 11$
36. $9m + 10n$ for $m = 15$ and $n = -5$
37. $2(3x + 8)$ for $x = 5$
38. $6x + 16$ for $x = 5$
39. $x^2 + 9x + 18$ for $x = 5$
40. $a^2 - 7a - 30$ for $a = 3$
41. $y^2 + 4y - 17$ for $y = -3$

42. $x^2 - 5x - 9$ for $x = -4$

43. $m^2 - 9$ for $m = 3$

44. $5 - x^2$ for $x = -2$

45. $b^2 - 4ac$ for $a = 2$, $b = 5$, and $c = -3$

46. $b^2 - 4ac$ for $a = -5$, $b = 7$, and $c = -4$

47. $b^2 - 4ac$ for $a = -1$, $b = -2$, and $c = 10$

48. $b^2 - 4ac$ for $a = 15$, $b = -6$, and $c = 9$

49. $5(x + h) - 17$ for $x = -3$ and $h = 0.01$

50. $-3(x + h) + 4$ for $x = -6$ and $h = 0.001$

51. $(x + h)^2 - 5(x + h) - 14$ for $x = -3$ and $h = 0.1$

52. $(x + h)^2 + 4(x + h) - 28$ for $x = 6$ and $h = 1$

53. The commutative property works for addition but does not work in general for subtraction.

a) Give an example of two numbers a and b such that $a - b \neq b - a$.

b) Can you find two numbers a and b such that $a - b = b - a$?

54. The commutative property works for multiplication but does not work in general for division.

a) Give an example of two numbers a and b such that $\frac{a}{b} \neq \frac{b}{a}$.

b) Can you find two numbers a and b such that $\frac{a}{b} = \frac{b}{a}$?

Simplify where possible.

55. $3(x - 9)$

56. $4(2x + 5)$

57. $5(3 - 7x)$

58. $12(5x - 7)$

59. $-2(5x + 7)$

60. $-3(8x + 3)$

61. $-6(9x - 5)$

62. $-4(-10x + 1)$

63. $7x + 9x$

64. $14x + 3x$

65. $3x - 8x$

66. $22a - 15a$

67. $3x - 7 + 4x + 11$

68. $15x + 32 - 19x - 57$

69. $5x - 3y - 7x - 19y$

70. $15m - 11n - 6m + 22n$

71. $3(2x - 5) + 4x + 7$

72. $8 - 19k + 6(3k - 5)$

73. $2(4x - 9) - 11$

74. $3(2a + 11b) - 13a$

75. $6y - 5(3y - 17)$

76. $5 - 9(3 - 4x)$

77. $3(4z - 7) + 9(2z + 3)$

78. $-6(5x - 9) - 7(13 - 12x)$

79. $-2(5a + 4b - 13c) + 3b$

80. $-7(-2x + 3y - 17z) - 5(3x - 11)$

For the following expressions:
a) Determine the number of terms.
b) Write down each term.
c) Write down the coefficient for each term.

Make sure you simplify each expression before answering.

81. $5x^3 + 3x^2 - 7x - 15$

82. $-3a^2 - 7a - 10$

83. $3x - 17$

84. $9x^4 - 10x^3 + 13x^2 - 17x + 329$

85. $5(3x^2 - 7x + 11) - 6x$

86. $4(3a - 5b - 7c - 11) - 2(6b - 5c)$

87. $5(-7a - 3b + 5c) - 3(6b - 9c - 23)$

88. $2(-4x + 7y) - (3x + 9y)$

Writing in Mathematics

Answer in complete sentences.

89. Give an example of a real-world situation that can be described by the variable expression $60x$. Explain why the expression fits your situation.

90. Give an example of a real-world situation that can be described by the variable expression $20x + 35$. Explain why the expression fits your situation.

91. ***Solutions Manual*** Write a solutions manual page for the following problem.

Simplify $4(2x - 5) - 3(3x - 8) - 7x$.

CHAPTER 1 SUMMARY

Section 1.1 Integers, Opposites, and Absolute Value

Inequalities, pp. 2–4

The number a is greater than b $(a > b)$ if a is to the right of the number b on the number line.
The number a is less than b $(a < b)$ if a is to the left of b on the number line.

Place the correct sign, $<$ or $>$, between the two integers: 7 ______ 3
$7 > 3$ because 7 is to the right of 3 on the number line.

Opposites, pp. 3–4

Two numbers are opposites if they are on different sides of 0 on the number line and are the same distance from 0.

The opposite of 4: -4, The opposite of -12: 12

Absolute Values, pp. 4–5

The absolute value of a number a, denoted $|a|$, is the distance between a and 0 on the number line.

$|-15| = 15, |9| = 9$

Section 1.2 Operations with Integers

Adding a Positive Integer and a Negative Integer, pp. 6–8

Find the difference between the absolute values of the two integers.
The sign of the result is the same as the sign of the number that has the largest absolute value.

Add: $-13 + 8$

$$|-13| = 13, |8| = 8$$
$$13 - 8 = 5$$
$$-13 + 8 = -5$$

Adding Two Integers with the Same Sign, p. 8

Add the absolute values of the two integers.
The sign of the result is the same as the sign of the two integers.

Add: $-25 + (-18)$

$$|-25| = 25, |-18| = 18$$
$$25 + 18 = 43$$
$$-25 + (-18) = -43$$

General Strategy for Adding/Subtracting Integers, p. 9

- Simplify double signs.
- Determine whether each integer is contributing positively or negatively to the total.
- Add all integers that are contributing positively to the total. Add all integers that are contributing negatively to the total.
- Finish by finding the sum of these two totals.

Simplify: $9 + (-17) - 16 + 4 - (-11)$

$$9 + (-17) - 16 + 4 - (-11)$$
$$= 9 - 17 - 16 + 4 + 11$$
$$= 24 - 33$$
$$= -9$$

Section 1.3 Fractions

Factor Set, p. 15

The collection of all factors of a natural number is called its factor set.

Find the factor set of 12.

$$12 = 1 \cdot 12,\ 2 \cdot 6,\ 3 \cdot 4$$
Factor set: $\{1, 2, 3, 4, 6, 12\}$

Prime Numbers, pp. 15–16

A natural number is prime if it is greater than 1 and its only two factors are 1 and itself.

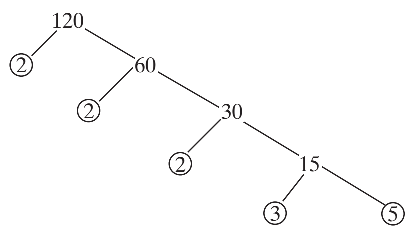
Is 39 prime?

No, because $3 \cdot 13 = 39$.

Prime Factorization, p. 16

To find the prime factorization of a natural number, rewrite the number as a product of prime factors.

Find the prime factorization of 120.



$$120 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

Lowest Terms, pp. 16–17

A fraction is in lowest terms if its numerator and denominator do not have any common factors other than 1.

Write in lowest terms: $\frac{12}{42}$

$$\frac{12}{42} = \frac{2 \cdot 2 \cdot 3}{2 \cdot 3 \cdot 7} = \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 7} = \frac{2}{7}$$

Rewriting an Improper Fraction as a Mixed Number, pp. 17–18

To rewrite an improper fraction as a mixed number, divide the numerator by the denominator.
The quotient is the whole number part of the mixed number, and the remainder is the numerator of the fraction part.

Express as a mixed number: $\frac{53}{8}$

$$\begin{array}{r} 6 \\ 8\overline{)53} \\ \underline{-48} \\ 5 \end{array}$$

$$\frac{53}{8} = 6\frac{5}{8}$$

Rewriting a Mixed Number as an Improper Fraction, p. 18

To change a mixed number to an improper fraction, multiply the whole number part of the mixed number by the denominator of the fraction part.
Add the numerator to this product to find the numerator of the improper fraction.

Express as an improper fraction: $7\frac{2}{9}$

$$7 \cdot 9 = 63$$
$$63 + 2 = 65$$
$$7\frac{2}{9} = \frac{65}{9}$$

Section 1.4 Operations with Fractions

Multiplying Fractions, pp. 20–21

To multiply two fractions, divide out any factors that are common to a numerator and a denominator.
Multiply the two numerators and multiply the two denominators.

Multiply: $\frac{3}{10} \cdot \frac{8}{9}$

$$\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}} \cdot \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{4}{15}$$

Dividing Fractions, pp. 21–22

To divide a fraction by another fraction, invert the divisor and multiply the resulting fractions.

Divide: $\frac{35}{12} \div \frac{21}{20}$

$$\frac{35}{12} \div \frac{21}{20} = \frac{35}{12} \cdot \frac{20}{21}$$
$$= \frac{\overset{5}{\cancel{35}}}{\underset{3}{\cancel{12}}} \cdot \frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{21}}}$$
$$= \frac{25}{9}$$

Adding and Subtracting Fractions with the Same Denominator, p. 22

To add or subtract fractions that have the same denominator, add or subtract the numerators, placing the result over the common denominator.

Add: $\frac{7}{8} + \frac{5}{8}$

$$\frac{7}{8} + \frac{5}{8} = \frac{12}{8}$$
$$= \frac{3}{2}$$

Least Common Multiple, pp. 22–23

The least common multiple (LCM) of two natural numbers is the smallest number that both numbers divide into evenly.
To find the LCM of two numbers:

- Find the prime factorization of each number.
- Find the common factors of the two numbers.
- Multiply the common factors by the remaining factors of the two numbers.

Find the LCM of 36 and 54.

$$36 = 2 \cdot 2 \cdot 3 \cdot 3$$
$$54 = 2 \cdot 3 \cdot 3 \cdot 3$$
$$\text{LCM: } 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 108$$

Adding and Subtracting Fractions with Different Denominators, pp. 23–24

- Find the LCM of the denominators.
- Rewrite each fraction as an equivalent fraction whose denominator is the LCM of the original denominators.
- Add or subtract the numerators, placing the result over the common denominator.

Add: $\frac{3}{8} + \frac{7}{12}$

LCM: 24

$$\frac{3}{8} + \frac{7}{12} = \frac{3}{8}\cdot\frac{3}{3} + \frac{7}{12}\cdot\frac{2}{2} = \frac{9}{24} + \frac{14}{24} = \frac{23}{24}$$

Section 1.5 Decimals and Percents

Addition/Subtraction with Decimals, p. 27

Align the decimal points and add or subtract as you would with integers.

Add: 9.62 + 4.583

$$\begin{array}{r} \overset{1}{9}.\overset{1}{6}20 \\ +\ 4.583 \\ \hline 14.203 \end{array}$$

Multiplication with Decimals, p. 27

Multiply two decimal numbers as you would integers. The number of decimal places in the product is the total number of decimal places in the two factors.

Multiply: $3.47 \cdot 5.2$

$$\begin{array}{r} 347 \\ \times\ 52 \\ \hline 694 \\ 17350 \\ \hline 18044 \end{array}$$

$3.47 \cdot 5.2 = 18.044$

Division with Decimals, p. 28

Move the decimal point in the divisor to the right so that it becomes an integer. Move the decimal point in the dividend to the right by the same number of spaces. The decimal point in the answer will be aligned with this new location of the decimal point in the dividend.

Divide: $9.568 \div 2.3$

$9.568 \div 2.3 \rightarrow 95.68 \div 23$

$$\begin{array}{r} 4.16 \\ 23\overline{)95.68} \\ -92 \\ \hline 36 \\ -23 \\ \hline 138 \\ -138 \\ \hline 0 \end{array}$$

Rewriting Fractions as Decimals, p. 28

To rewrite any fraction as a decimal, divide its numerator by its denominator.

Rewrite as a decimal: $\frac{8}{25}$

$$\frac{8}{25} = 8 \div 25 = 0.32$$

Rewriting Decimals as Fractions, pp. 28–29

Write the decimal as a whole number in the numerator. The denominator of the fraction can be found by determining the place value of the last decimal place.

Rewrite as a fraction: 0.74

$$0.74 = \frac{74}{100} = \frac{37}{50}$$

Section 1.6 Basic Statistics

Mean, p. 33

The mean of a set of data is the arithmetic average of the values.

Find the mean of 7, 12, 16, 25, 40.

$$\text{Mean} = \frac{7 + 12 + 16 + 25 + 40}{5} = \frac{100}{5} = 20$$

Median, pp. 33–34

To find the median for a set of data, begin by writing the values in ascending order.
If there is an odd number of values, the single value in the middle of the set of data is the median.
If there is an even number of values, the median is the average of the two center values.

7, 12, 16, 25, 40
Median = 16

13, 18, 20, 33, 40, 59

$$\text{Median} = \frac{20 + 33}{2} = \frac{53}{2} = 26.5$$

Mode, p. 34

The mode of a set of data is the value that is repeated most often.

12, 7, 19, 3, 7, 11, 16, 12, 7, 15
Mode = 7

Midrange, p. 34

The midrange of a set of data is the average of the set's minimum value and maximum value.

13, 18, 20, 33, 40, 59

$$\text{Midrange} = \frac{13 + 59}{2} = \frac{72}{2} = 36$$

Range, pp. 34–35

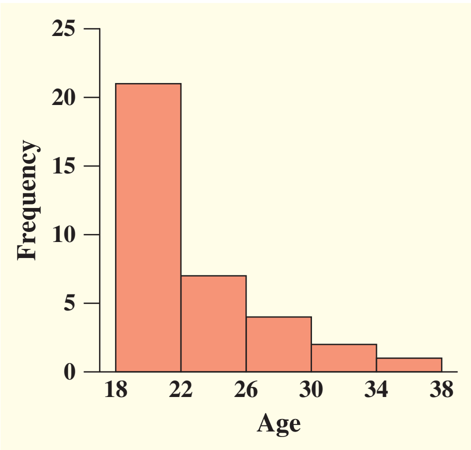
To find the range of a set of values, subtract the minimum value in the set from the maximum value in the set.

13, 18, 20, 33, 40, 59
Range = 59 − 13 = 46

Histogram, pp. 35–37

A histogram is a graph that can be used to show how a set of data is distributed, giving an idea of where the data values are centered as well as how they are dispersed.

Age	Frequency
18 to 21	21
22 to 25	7
26 to 29	4
30 to 33	2
34 to 37	1



Section 1.7 Exponents and Order of Operations

Exponents, p. 43

Using the number x repeatedly as a factor y times can be represented using exponential notation as x^y. The base is x, and the exponent is y.

Simplify: 4^3

$$4^3 = 4 \cdot 4 \cdot 4 = 64$$

Order of Operations, pp. 44–45

The order of operations is a standard order in which arithmetic operations are performed, ensuring a single correct answer.

1. Remove grouping symbols.
2. Perform any operations involving exponents.
3. Perform any multiplication or division from left to right.
4. Perform any addition or subtraction from left to right.

Simplify: $8 \div (3 \cdot 5 - 19) - 7^2$

$$\begin{aligned} 8 \div (3 \cdot 5 - 19) - 7^2 &= 8 \div (15 - 19) - 7^2 \\ &= 8 \div (-4) - 7^2 \\ &= 8 \div (-4) - 49 \\ &= -2 - 49 \\ &= -51 \end{aligned}$$

Section 1.8 Introduction to Algebra

Evaluating Variable Expressions, p. 49

To evaluate a variable expression for a particular value of a variable, substitute the value for the variable and simplify the resulting expression using the order of operations.

Evaluate $x^2 - 5x - 32$ for $x = -8$.

$$\begin{aligned} &(-8)^2 - 5(-8) - 32 \\ &= 64 - 5(-8) - 32 \\ &= 64 + 40 - 32 \\ &= 72 \end{aligned}$$

Distributive Property, pp. 50–51

For all real numbers *a, b,* and *c*, $a(b + c) = ab + ac$.

Simplify: $7(3x - 2)$

$$\begin{aligned} 7(3x - 2) &= 7 \cdot 3x - 7 \cdot 2 \\ &= 21x - 14 \end{aligned}$$

Combining Like Terms, pp. 51–52

When simplifying variable expressions, we can combine like terms into a single term by adding or subtracting the coefficients of the like terms.

Simplify: $5x - 9 - 2x - 18$

$$5x - 9 - 2x - 18 = 3x - 27$$

SUMMARY OF CHAPTER 1 STUDY STRATEGIES

Creating a study group is one of the best ways to help you learn mathematics and improve your performance on quizzes and exams.

- Sometimes a concept will be easier for you to understand if one of your peers explains it to you. One of your group members may have some insight into a particular topic, and that person's explanation may be just the trick to turn on the "lightbulb" in your head.
- When you explain a certain topic to a fellow student or show a student how to solve a particular problem, you increase your chances of retaining this knowledge.
- You also will find it helpful to have a support group—students who can support one another when times are difficult.

CHAPTER 1 REVIEW

Write the appropriate symbol, < or >, between the following integers. **[1.1]**

1. -10 ___ -7 **2.** 3 ___ -12

Find the following absolute values. **[1.1]**

3. $|8|$ **4.** $|-13|$

Find the opposite of the following integers. **[1.1]**

5. -6 **6.** 12

Add or subtract. **[1.2]**

7. $9 + (-16)$ **8.** $-10 + 7$

9. $-22 - 19$ **10.** $4 - (-23)$

Simplify. **[1.2]**

11. $3 - 16 - 24$ **12.** $8 - (-19) - 7$

Multiply or divide. **[1.2]**

13. $9(-6)$ **14.** $-144 \div (-9)$

Write the factor set for the following numbers. **[1.3]**

15. 42

16. 108

Write the prime factorization of the following numbers. (If prime, state this.) **[1.3]**

17. 32 **18.** 60

Simplify the following fractions to lowest terms. **[1.3]**

19. $\frac{24}{42}$ **20.** $\frac{9}{72}$

21. $\frac{40}{234}$ **22.** $\frac{98}{7}$

Rewrite the following mixed numbers as improper fractions. **[1.3]**

23. $5\frac{2}{3}$ **24.** $12\frac{7}{25}$

Rewrite the following improper fractions as mixed numbers. **[1.3]**

25. $\frac{38}{10}$ **26.** $\frac{55}{6}$

Multiply or divide. **[1.4]**

27. $\frac{9}{16} \cdot \frac{28}{75}$ **28.** $5\frac{1}{2} \cdot \frac{14}{33}$

29. $\frac{8}{15} \div \frac{20}{21}$ **30.** $3\frac{1}{5} \div 7\frac{1}{2}$

Add or subtract. **[1.4]**

31. $\frac{5}{8} + \frac{7}{12}$ **32.** $\frac{13}{20} + \frac{5}{6}$

33. $\frac{11}{18} - \frac{4}{9}$ **34.** $\frac{7}{36} - \frac{13}{42}$

Simplify. **[1.4]**

35. $\frac{30}{49} \cdot \frac{35}{66}$ **36.** $\frac{7}{16} - \frac{11}{48}$

37. $\frac{9}{40} \div \frac{39}{35}$ **38.** $\frac{7}{20} + \frac{1}{15}$

Simplify the following decimal expressions. **[1.5]**

39. $8.7 + 3.92$

40. $24.308 - 15.49$

41. $8.4 \cdot 3.6$

42. $40.92 \div 4.65$

Rewrite the following fractions as decimal numbers. **[1.5]**

43. $\frac{8}{25}$ **44.** $\frac{15}{16}$

Rewrite the following decimal numbers as fractions in lowest terms. **[1.5]**

45. 0.75 **46.** 0.28

Rewrite as a percent. **[1.5]**

47. $\frac{2}{5}$ **48.** $\frac{7}{25}$

49. 0.9 **50.** 0.45

Rewrite as a fraction. **[1.5]**

51. 30% **52.** 55%

Rewrite as a decimal. **[1.5]**

53. 90% **54.** 4%

Worked-out solutions to Review Exercises marked with ◯ can be found on page AN-3.

55. Jeff bought a book at a yard sale for \$23. If he had \$40 prior to buying the book, how much money did Jeff have after buying the book? [1.2]

56. Gray had \$78 in his checking account prior to writing a \$125 check to the bookstore for books and supplies. What is the new balance of his account? [1.2]

57. Three investors plan to start a new company. If start-up costs are \$37,800, how much will each person have to invest? [1.2]

58. If one recipe calls for $1\frac{1}{2}$ cups of flour and a second recipe calls for $2\frac{2}{3}$ cups of flour, how much flour is needed to make both recipes? [1.4]

59. Find the mean. Height (inches) of basketball players: 77, 71, 83, 80, 73, 74, 73, 81 [1.6]

60. Find the median. Pulse of 40-year-old men during exercise: 113, 127, 133, 128, 108, 126, 117, 129, 128, 121 [1.6]

61. Find the mode, midrange, and range. Exam scores of study group members: 96, 88, 95, 79, 88, 99 [1.6]

62. Construct a histogram for the given frequency distribution. [1.6]

25-Year-Old Females Systolic Blood Pressure (mmHg)	Frequency
80 to 89	2
90 to 99	8
100 to 109	12
110 to 119	19
120 to 129	6
130 to 139	3

***Simplify the given expression.* [1.7]**

63. 4^3

64. $\left(\frac{2}{5}\right)^3$

65. -2^6

66. $3^5 \cdot 5^2$

***Simplify the given expression.* [1.7]**

67. $5 + 8 \cdot 4$

68. $25 - 15 \div 5$

69. $3 + 13 \cdot 5 - 20$

70. $(3 + 13) \cdot 5 - 20$

71. $54 - 27 \div 3^2$

72. $\frac{3}{4} + \frac{1}{4} \cdot \frac{12}{25}$

***Build a variable expression for the following phrases.* [1.8]**

73. The sum of a number and 14

74. A number decreased by 20

75. Eight less than twice a number

76. Nine more than 6 times a number

77. A coffeehouse charges \$3.55 for a cup of coffee. Letting c represent the number of cups of coffee a coffeehouse sells on a particular day, build a variable expression for the revenue from coffee sales. [1.8]

78. A rental company rents moving vans for \$20 plus \$0.15 per mile. Letting m represent the number of miles, build a variable expression for the cost to rent a moving van from this rental company. [1.8]

***Evaluate the following algebraic expressions under the given conditions.* [1.8]**

79. $3x + 17$ for $x = 9$

80. $9 - 8x$ for $x = -2$

81. $10a - 4b$ for $a = 2$ and $b = -9$

82. $(8x - 9)(2x - 11)$ for $x = 4$

83. $x^2 - 7x - 30$ for $x = -3$

84. $b^2 - 4ac$ for $a = -1, b = -8,$ and $c = 5$

***Simplify.* [1.8]**

85. $5(x + 7)$

86. $6x + 21x$

87. $8x - 25 - 3x + 17$

88. $8y - 6(4y - 21)$

89. $15 - 23k + 7(4k - 9)$

90. $-8(2x + 25) - (103 - 19x)$

For the following expressions:
a) Determine the number of terms.
b) Write down each term.
***c) Write down the coefficient for each term.* [1.8]**

91. $x^3 - 4x^2 - 10x + 41$

92. $-x^2 + 5x - 30$

CHAPTER 1 TEST

For Extra Help
CHAPTER Test Prep VIDEOS

Step-by-step test solutions are found on the Chapter Test Prep Videos available via the Video Resources on DVD, in MyMathLab, and on YouTube (search "WoodburyElemIntAlg" and click on "Channels").

Write the appropriate symbol, < or >, between the following integers.

1. -15 ___ -18

Find the following absolute value.

2. $|-17|$

Simplify.

3. $7 + (-13)$

4. $-7(-9)$

5. Write the factor set for 45.

6. Write the prime factorization of 108. (If the number is prime, state this.)

7. Simplify $\frac{60}{84}$ to lowest terms.

8. Rewrite $\frac{67}{18}$ as a mixed number.

Simplify.

9. $\frac{11}{63} \cdot \frac{15}{44}$

10. $\frac{2}{9} \div \frac{8}{21}$

11. $\frac{3}{5} + \frac{11}{12}$

12. $\frac{5}{24} - \frac{4}{9}$

13. Simplify 8.05(2.27).

14. Rewrite 0.36 as a fraction. Your answer should be in lowest terms.

15. Eleanor bought 13 computers for \$499 each. What was the total cost for the 13 computers?

16. After Lindsay made a deposit of \$407.83 in her checking account, the balance was \$1203.34. What was the balance before the deposit?

17. Rewrite 72% as a fraction.

18. Rewrite 6% as a decimal.

19. Find the mean and median for the given set of values.

43 46 71 95 85 27 37 8 44 26 34 85 91 79 89 20

Simplify the given expression.

20. $16 - 8 \cdot 5$

21. $-9 + 4 \cdot 13 - 6 \cdot 3$

22. $\frac{4^2 + 3^2}{3 + 4 \cdot 13}$

23. Build a variable expression for the expression *seven less than four times a number.*

24. A landscaper charges \$50 plus \$20 per hour for yard maintenance. Letting h represent the number of hours spent working on a particular yard, build a variable expression for the landscaper's charge.

Evaluate the following algebraic expressions under the given conditions.

25. $16 - 5x$ for $x = -9$

26. $x^2 + 6x - 17$ for $x = -8$

Simplify.

27. $5(2x - 13)$

28. $7y - 8(2y - 30)$

Mathematicians in History

Srinivasa Ramanujan was a self-taught Indian mathe-matician, viewed by many to be one of the greatest mathematical geniuses in history. As a student, he became so totally immersed in his work with mathematics that he ignored his other subjects and failed his college exams. Ramanujan's life is chronicled in the biography *The Man Who Knew Infinity: A Life of the Genius Ramanujan* by Robert Kanigel.

Write a one-page summary (*or* make a poster) of the life of Srinivasa Ramanujan and his accomplishments.

Interesting issues:

- Where and when was Srinivasa Ramanujan born?
- At age 16, Ramanujan borrowed a mathematics book that strongly influenced his life as a mathematician. What was the name of the book?
- Ramanujan got married on July 14, 1909. The marriage was arranged by his mother. How old was his bride at the time?
- What jobs did Ramanujan hold in India?
- Which renowned mathematician invited Ramanujan to England in 1914?
- Ramanujan's health in England was poor. What was the cause of his poor health?
- What is the significance of the taxicab number 1729?
- What were the circumstances that led to Ramanujan's death? How old was Ramanujan when he died?