A série de Fourier de $f(x) = \begin{cases} 0, se -2 < x \leq 0 \\ x-5, se\ 0 < x < 2 \end{cases}$ considerando que $f(x+4) = f(x)$, corresponde a:

$$2p = 4 \rightarrow p = 2$$

$$a_0 = \frac{1}{2}\int_0^2 (x-5)dx = \frac{1}{2}\left(\frac{x^2}{2} - 5x\right)\Big|_0^2 = \frac{1}{2}\big((2-10)-0\big) = -\frac{8}{2} = -4$$

$$a_n = \frac{1}{2}\int_0^2 (x-5)\cos\left(\frac{n\pi x}{2}\right)dx = ?$$

$$u = x-5 \rightarrow \frac{du}{dx} = 1 \rightarrow du = dx$$

$$dv = \cos\left(\frac{n\pi x}{2}\right)dx \rightarrow v = \int \cos\left(\frac{n\pi x}{2}\right)dx = \frac{2}{n\pi} sen\left(\frac{n\pi x}{2}\right)$$

Voltando:

$$a_n = \frac{1}{2}\int_0^2 (x-5)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{1}{2}\left((x-5)\frac{2}{n\pi} sen\left(\frac{n\pi x}{2}\right)\Big|_0^2 - \int_0^2 \frac{2}{n\pi} sen\left(\frac{n\pi x}{2}\right)dx\right) =$$

$$= \frac{1}{2}\left(0 + \frac{4}{(n\pi)^2}\cos\left(\frac{n\pi x}{2}\right)\Big|_0^2\right) = \frac{1}{2}\left(\frac{4}{(n\pi)^2}(\cos(n\pi) - 1)\right) = \frac{2}{(n\pi)^2}((-1)^n - 1)$$

$$b_n = \frac{1}{2}\int_0^2 (x-5)\,\mathrm{sen}\left(\frac{n\pi x}{2}\right)dx = ?$$

$$u = x-5 \rightarrow \frac{du}{dx} = 1 \rightarrow du = dx$$

$$dv = \mathrm{sen}\left(\frac{n\pi x}{2}\right)dx \rightarrow v = \int \mathrm{sen}\left(\frac{n\pi x}{2}\right)dx = -\frac{2}{n\pi} cos\left(\frac{n\pi x}{2}\right)$$

Voltando:

$$b_n = \frac{1}{2}\int_0^2 (x-5)\,\mathrm{sen}\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{1}{2}\left((x-5)\left(-\frac{2}{n\pi} cos\left(\frac{n\pi x}{2}\right)\Big|_0^2 - \int_0^2 -\frac{2}{n\pi} cos\left(\frac{n\pi x}{2}\right)dx\right) =$$

$$= \frac{1}{2}\left(-3\left(-\frac{2}{n\pi}\cos(n\pi) - \left(-5\left(-\frac{2}{n\pi}\right)\right) + \frac{4}{(n\pi)^2}\,\mathrm{sen}\left(\frac{n\pi x}{2}\right)\Big|_0^2\right)$$

$$= \frac{1}{2}\left(\frac{6}{n\pi}\cos(n\pi) - \frac{10}{n\pi}\right) = = \frac{3}{n\pi}(-1)^n - \frac{5}{n\pi} = \frac{3(-1)^n - 5}{n\pi}$$

$$\rightarrow SFf(x) \cong -2 + \sum_{n}^{\infty} \frac{2}{(n\pi)^2}((-1)^n - 1)\cos\left(\frac{n\pi x}{2}\right) + \frac{3(-1)^n - 5}{n\pi} sen\left(\frac{n\pi x}{2}\right)$$

https://www.geogebra.org/m/redkzqf9