Nice pair − wise quadratic polynomials.

Two quadratic real polynomials is called a nice pair of quadratic polynomials if they fulfill the conditions below.

A: They have two intersection points.

B: Both polynomials have two roots and all four roots are distinct.

C: All the values associated with the three points below are integers:

1: Roots, 2: Intersection points, 3: Coefficients and the constant c.

A nice pair of quadratic polynomials is called a NPW pair.

Parametrized subsets of NPW pairs.

Theorem.

Let $(a, b, c) \in \mathbb{N}^3, a > b, c > 1, (q, k, s) \in \mathbb{Z}^3, q \neq 0, s \neq 0.$

Let $f(x)$ *and* $g(x)$ *be given by the following formulas:*

A:

$f(x) = sc^2(k + x)(-4\,a\,b\,q\,(c^2 - 1) + k + x)$

$g(x) = s(c^2 - 1)(-2abq\,(c^2 - 1) + k + x)(-2\,abq(c^2 - 1) - 2qc(a^2 - b^2) + k + x)$

B:

$f(x) = c^2 s\,(k + x)(-2\,q\,(a^2 - b^2)(c^2 - 1) + k + x)$

$g(x) = s\,(c^2 - 1)(-q(a^2 - b^2)(c^2 - 1) + k + x)(-q\,(a^2 - b^2)(c^2 - 1) - 4abqc + k + x)$

C.

$f(x) = x^2 - (a^2 - 2ab - b^2)^2$ *and* $g(x) = -x^2 + (a^2 + 2ab - b^2)^2$

D.

Let $m \in \mathbb{N}$, *let n be given by* $n = \sum_{i=0}^{m} \binom{2m+1}{2i+1} 4^{m-i} 3^i$ *and let*

$f(x) = x^2 - 1$ *and* $g(x) = -3x^2 + 3 \cdot n^2$

E.

Let $m \in \mathbb{N}$, *let n be given by* $n = \sum_{i=0}^{m} \binom{2m+1}{2i+1} 3^{m-i} 2^i$ *and let* [i]

$f(x) = x^2 - 1$ *and* $g(x) = -2x^2 + 2 \cdot n^2$

These parametrizations are subsets of the set of NPW pairs.

Proof.

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The proof of A is made with simple substitutions and the wellknown parametrization formula for Pythagorean triples. See the pictures in endnotes: [ii] [iii]

The proof of B is similar to the proof of A.

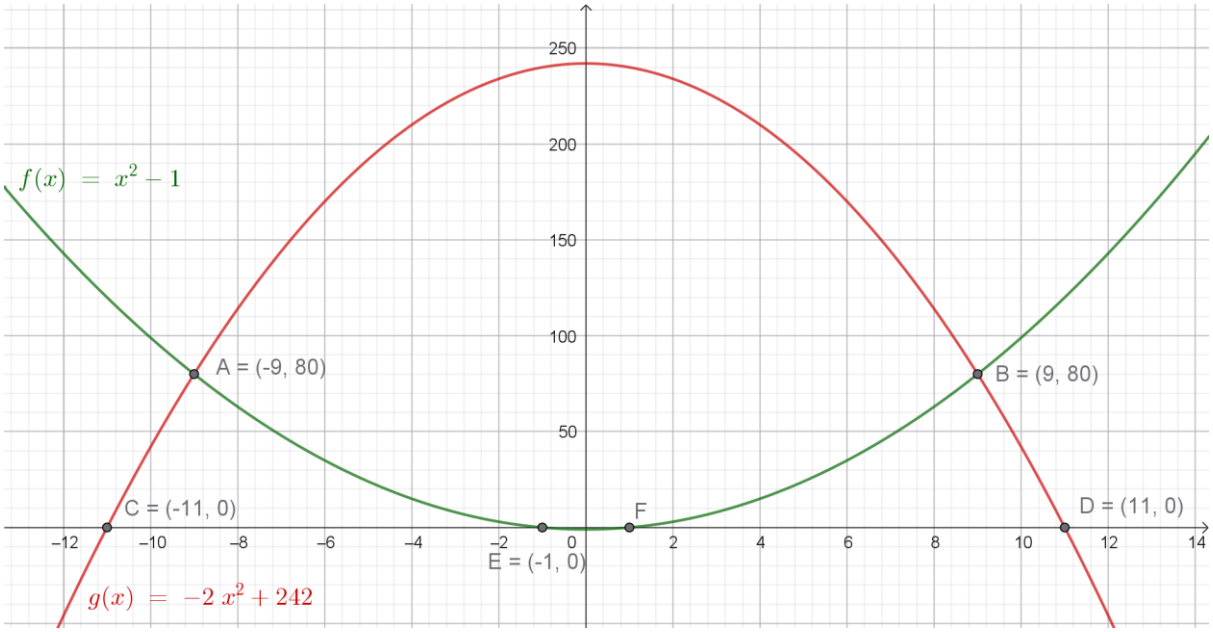
[iv] [v] [vi]

Examples to E. [vii]

Example 1.

$$m = 1, n = \binom{3}{1}3^{1-0}2^0 + \binom{3}{3}3^{1-1}2^1 = 9 + 2 = 11$$

$$f(x) = x^2 - 1 \text{ and } g(x) = -2x^2 + 2 \cdot 11^2 = -2x^2 + 242$$



Example 2.

$$m = 2, n = \binom{5}{1}3^{2-0}2^0 + \binom{5}{3}3^{2-1}2^1 + \binom{5}{5}3^{2-2}2^2 = 45 + 60 + 4 = 109$$

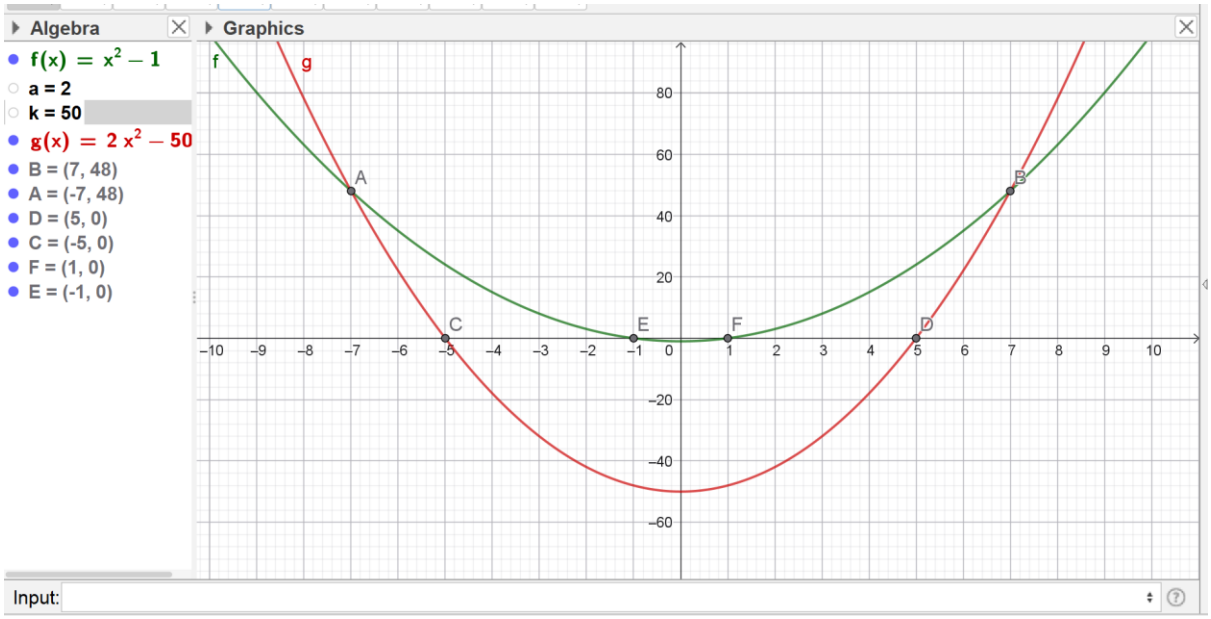
$$f(x) = x^2 - 1 \text{ and } g(x) = -2x^2 + 2 \cdot 109^2 = -2x^2 + 23762$$

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Other examples of NPW.

$f(x) = x^2 - 1, g(x) = 2x^2 - 50$.[viii] *Exercise: Derive* $g(x)$ *from a or b. Is it possible?*



Examples to A.

Some examples.

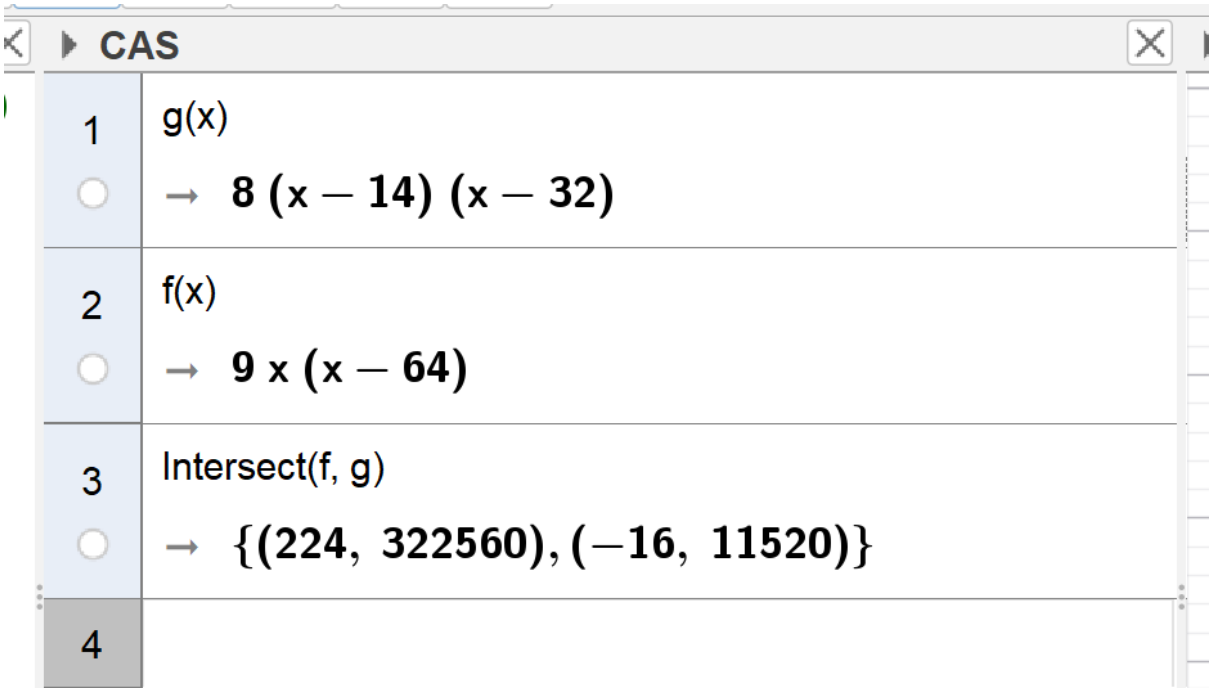
$f(x) = 4x \cdot (-24 + x)$	$g(x) = 3 \cdot (-12 + x)(0 + x)$
$f(x) = 9x \cdot (-64 + x)$	$g(x) = 8 \cdot (-32 + x)(-14 + x)$
$f(x) = 16x \cdot (-120 + x)$	$g(x) = 15 \cdot (-60 + x)(-36 + x)$
$f(x) = 25x \cdot (-192 + x)$	$g(x) = 24 \cdot (-96 + x)(-66 + x)$
$f(x) = 36x \cdot (-280 + x)$	$g(x) = 35 \cdot (-140 + x)(-104 + x)$
$f(x) = 49x \cdot (-384 + x)$	$g(x) = 48 \cdot (-192 + x)(-150 + x)$
$f(x) = 64x \cdot (-504 + x)$	$g(x) = 63 \cdot (-252 + x)(-204 + x)$
$f(x) = 81x \cdot (-640 + x)$	$g(x) = 80 \cdot (-320 + x)(-266 + x)$
$f(x) = 100x \cdot (-792 + x)$	$g(x) = 99 \cdot (-396 + x)(-336 + x)$

f(x)= 4x·(72 +x)	g(x)= 3(36 + x)(16 + x)
f(x)= 9x·(192 +x)	g(x)= 8(96 + x)(66 + x)
f(x)= 16x·(360 +x)	g(x)= 15(180 + x)(140 + x)
f(x)= 25x·(576 +x)	g(x)= 24(288 + x)(238 + x)
f(x)= 36x·(840 +x)	g(x)= 35(420 + x)(360 + x)
f(x)= 49x·(1152 +x)	g(x)= 48(576 + x)(506 + x)
f(x)= 64x·(1512 +x)	g(x)= 63(756 + x)(676 + x)

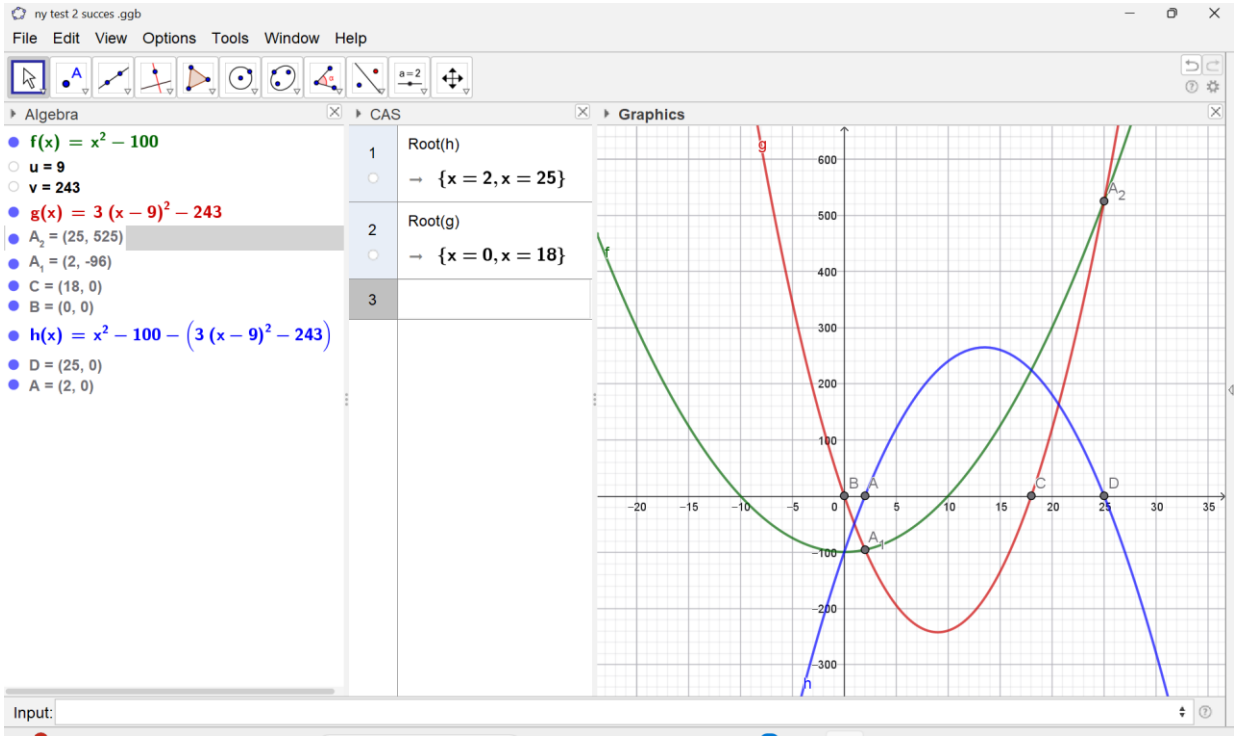
$f(x) = 9x \cdot (-64 + x)$	$g(x) = 8 \cdot (-32 + x)(-14 + x)$

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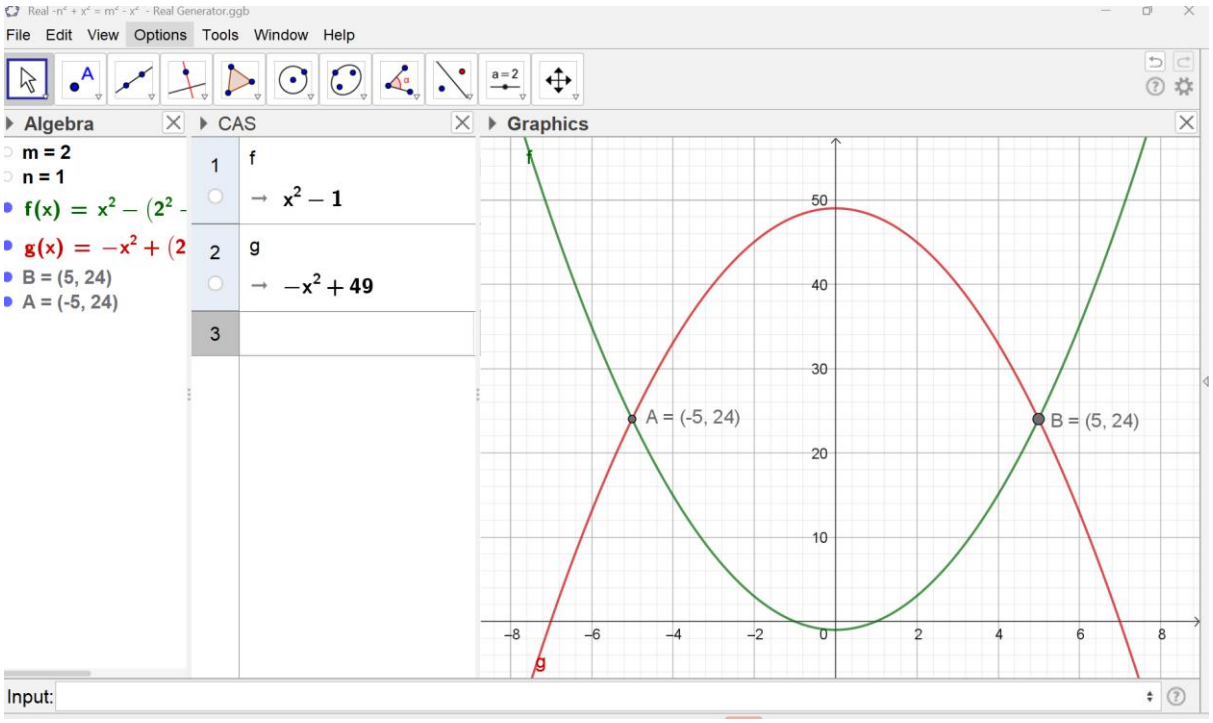
An example outside the range of the parametrizations?



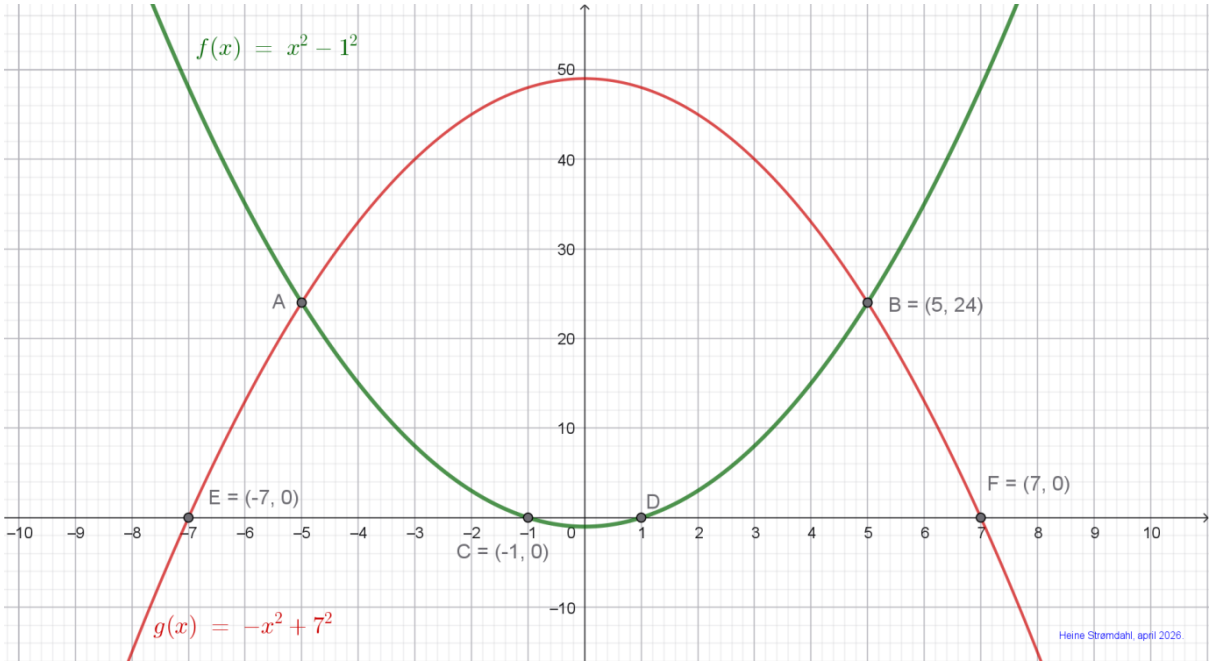
Examples to C.

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Same ↓



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