Lesson Plan: Rational Functions - Linear over Quadratic

Objective:
- Understand the definition and characteristics of rational functions, especially those with a linear numerator and quadratic denominator.
- Explore the concepts of vertical and horizontal asymptotes, domain, simplification, and the impact of coefficients on the function's graph.
- Apply these concepts to model and analyze real-world scenarios.

Time: 60 Minutes

Part 1: Understanding Rational Functions (10 minutes)
- Brief Lecture: Define rational functions and introduce key characteristics such as vertical and horizontal asymptotes, domain, and holes (removable discontinuities).
- Factual Questions: Pose questions to assess students' prior knowledge and introduce the day's learning objectives.

Part 2: Exploring Asymptotes and Domain (15 minutes)
- Interactive Discussion: Explain how to find vertical asymptotes and the domain of rational functions. Provide examples with different degrees of numerators and denominators.
- Activity: Students will identify vertical and horizontal asymptotes in given rational functions and discuss their findings.

Part 3: Mini-Investigation: Coefficients and Graph Behavior (20 minutes)
- Using an interactive applet or graphing software, students will manipulate the coefficients of a rational function (linear over quadratic) to observe changes in the graph.
  1. Explore how altering the quadratic denominator's coefficients affects the graph's shape and position.
  2. Investigate the impact of changing the linear numerator's coefficient on the graph's slope and intercepts.
  3. Identify vertical asymptotes and relate them to the denominator's factors, observing shifts when coefficients are tweaked.
  4. Examine the horizontal asymptote and the function's end behavior as x approaches infinity.
  5. Challenge: Find coefficients that create a hole (removable discontinuity) in the graph.

Part 4: Real-World Application and Discussion (10 minutes)
- Group Discussion: Based on the mini-investigation, discuss how rational functions can model real-world phenomena, like the concentration of a drug in the bloodstream over time.
- Conceptual and Debatable Questions: Engage in a discussion on the significance of asymptotes vs. intercepts, the practicality of rational functions in high school math, and the impact of technology on understanding these functions.

Part 5: Wrap-Up and Homework (5 minutes)
- Recap the key concepts learned and discuss how rational functions' understanding can be applied in various mathematical and real-world contexts.

- Assign homework focused on analyzing rational functions, identifying asymptotes, domain, and creating models of real-world scenarios using rational functions.

Materials Needed:
- Presentation slides/whiteboard for lectures.
- Access to graphing calculators or computer software for the mini-investigation.
- Handouts with rational function examples for asymptote and domain identification.

Assessment:
- Participation in discussions and mini-investigation activities.
- Accuracy in identifying asymptotes, domain, and simplifying rational functions in the assigned homework.
- Reflection on the application of rational functions to real-world scenarios.

Additional Notes:
- Encourage students to think critically about how changes in function coefficients affect the graph and to relate these changes to practical situations.
- Ensure all students are comfortable using the graphing technology required for the mini-investigation.