VEKTORI I PRAVCI

Magdalena Štimac, 3.B

→ vektor → usmjerena dužina određena duljinom, smjerom, orijentacijom
→ isti vektori → sve 3 komponente jednake
→ suprotni vektori → isti smjer, suprotna orijentacija
→ jedinični vektor → $\vec{i}$ na x-osi $\quad \vec{e} = \frac{\vec{AB}}{|AB|}$
$\vec{j}$ na y-osi
→ kolinearnost vektora → $\vec{a} = k \cdot \vec{b}$
→ duljina vektora → $|\vec{a}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
→ skalarni umnožak → $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \varphi$ $\quad$ $\cos \varphi$ → kut između $\vec{a}$ i $\vec{b}$

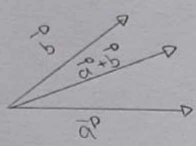
$\vec{a} = x_a \vec{i} + y_a \vec{i}$

$\vec{b} = x_B \vec{i} + y_B \vec{i}$

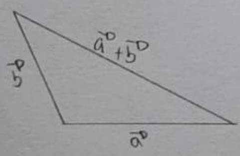
$\vec{a} \cdot \vec{b} = x_a x_b + y_a y_b$

→ zbrajanje vektora

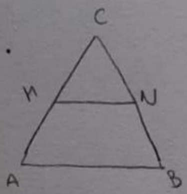
a) pravilo paralelograma



b) pravilo trokuta



1.



M, N → polovišta
Dokaži da vrijedi $\vec{AB} = 2\vec{MN}$

$\vec{AB} = \vec{AC} + \vec{CB}$

$= (\vec{AM} + \vec{MC}) + (\vec{CN} + \vec{NB})$

$= 2\vec{AM} + 2\vec{CN}$

$= 2(\vec{MC} + \vec{CN})$

$\vec{AB} = 2\vec{MN}$

2. $\vec{a} = (x-1)\vec{m} + \vec{n}$

$\vec{b} = 3\vec{m} + (x+1)\vec{n}$

$x = ?$, $\vec{a}$ i $\vec{b}$ kolinearni $\rightarrow \vec{a} = k \cdot \vec{b}$

$\vec{a} = k \cdot \vec{b}$

$(x-1)\vec{m} + \vec{n} = k(3\vec{m} + (x+1)\vec{n})$

$(x-1)\vec{m} + \vec{n} = 3k\vec{m} + k(x+1)\vec{n}$

$x - 1 = 3k \quad \rightarrow k = \frac{x-1}{3}$

$1 = k(x+1)$

$1 = \frac{x-1}{3}(x+1) \quad / \cdot 3$

$3 = (x-1)(x+1)$

$3 = x^2 - 1$

$x^2 = 4 \quad \rightarrow \quad x_1 = 2 \qquad x_2 = -2$

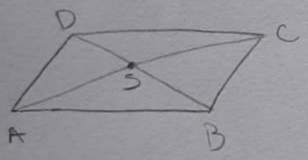
$k_1 = \frac{1}{3} \qquad k_2 = -1$

3. $A(2,-4)$

$B(1,1)$

$D(7,4)$

$C, S = ?$



$\vec{AD} = \vec{BC}$

$5\vec{i} + 8\vec{j} = (x_c - 1)\vec{i} + (y_c - 1)\vec{j}$

$5 = x_c - 1 \rightarrow x_c = 6 \qquad C(6,9)$

$8 = y_c - 1 \rightarrow y_c - 9$

$\vec{BS} = \vec{SD}$

$(x_s - 1)\vec{i} + (y_s - 1)\vec{j} = (7 - x_s)\vec{i} + (4 - y_s)\vec{j}$

$x_s - 1 = 7 - x_s \rightarrow x_s = 4$

$y_s - 1 = 4 - y_s \rightarrow y_s = \frac{5}{2} \qquad S\left(4, \frac{5}{2}\right)$

4. $\vec{a} = 3\vec{i} + 4\vec{j}$

$\vec{b} = -5\vec{i} + 2\vec{j}$

$\measuredangle(\vec{a}+\vec{b}, \vec{a}-\vec{b})$

$\vec{p} = \vec{a} + \vec{b}$

$\vec{p} = -2\vec{i} + 6\vec{j} \to \sqrt{(-2)^2 + 6^2} = 2\sqrt{10}$

$\vec{q} = \vec{a} - \vec{b}$

$\vec{q} = 8\vec{i} - 2\vec{j} \to \sqrt{8^2 + (-2)^2} = 2\sqrt{17}$

$\vec{p} \cdot \vec{q} = |\vec{p}| \cdot |\vec{q}| \cdot \cos\varphi$

$$\cos\varphi = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| \cdot |\vec{q}|}$$

$$\cos\varphi = \frac{((-2)\cdot 8) + (6 \cdot (-2))}{2\sqrt{10} \cdot 2\sqrt{17}}$$

$$\cos\varphi = \frac{-16 - 12}{4\sqrt{170}}$$

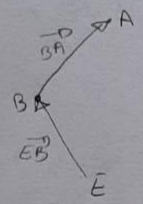
$\cos\varphi = -0{,}5369 \to \varphi = 122°28'16''$

5. $\vec{v} = \frac{2}{3}\vec{i} + \frac{8}{3}\vec{j}$

B(-4,2)

A(-1,5)

E(-3,-3)



$\vec{EB} = -\vec{i} + 5\vec{j}$

$\vec{BA} = 3\vec{i} + 3\vec{j}$

$\vec{v} = a \cdot \vec{EB} + b \cdot \vec{BA}$

$a, b = ?$

$\frac{2}{3}\vec{i} + \frac{8}{3}\vec{j} = a(-\vec{i} + 5\vec{j}) + b(3\vec{i} + 3\vec{j})$

$\frac{2}{3} = -a + 3b \;/\cdot(-1)$

$\frac{8}{3} = 5a + 3b$

$-\frac{2}{3} = a - 3b$

$\frac{8}{3} = 5a + 3b \;/+$

$-\frac{2}{3} + \frac{8}{3} = 6a$

$2 = 6a \to a = \frac{1}{3}$

$\frac{8}{3} = \frac{5}{3} + 3b$

$\frac{8}{3} - \frac{5}{3} = 3b$

$1 = 3b$

$b = \frac{1}{3}$

6. $\left.\begin{matrix} A(-5,-3) \\ B(5,-4) \end{matrix}\right\} \vec{a} = 10\vec{i} - \vec{j} \rightarrow \sqrt{10^2 + (-1)^2} = \sqrt{101}$

$|\vec{v}| = \sqrt{909}$

$\vec{v} = ?$, $\vec{v}$ i $\vec{a}$ kolinearni

$\vec{v} = k \cdot \vec{a}$

$\vec{v_1} = 3(10\vec{i} - \vec{j})$

$\vec{v_1} = 30\vec{i} - 3\vec{j}$

$\vec{v_2} = -3(10\vec{i} - \vec{j})$

$\vec{v_2} = -30\vec{i} + 3\vec{j}$

$\vec{v} = k \cdot \vec{a}$

$|\vec{v}| = |k| \cdot |\vec{a}|$

$\sqrt{909} = |k| \cdot \sqrt{101}$

$|k| = 3$

$k_1 = 3 \quad k_2 = -3$

pravac

eksplicitni oblik

$y = kx + l$ k-koeficijent smjera
$l \rightarrow$ odsječak na y-osi

implicitni oblik

$ax + by + c = 0$

segmentni oblik

$\frac{x}{m} + \frac{y}{n} = 1$ $m \rightarrow$ odsječak na x-osi
$n \rightarrow$ odsječak na y-osi

$\rightarrow$ kut između 2 pravca $\rightarrow \varphi$

$$tg\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right|$$

$\rightarrow$ udaljenost točke od pravca $\rightarrow d$

$$d = \left|\frac{Ax + By + C}{\sqrt{A^2 + B^2}}\right|$$

udaljenost 2 točke:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

1. $mx + y + 3 = 0 \rightarrow y = -mx - 3 \rightarrow k = -m$

$x - 2y - 1 = 0 \rightarrow y = \frac{1}{2}x - \frac{1}{2} \rightarrow k = \frac{1}{2}$

$\varphi = 45°$

$m = ?$

$$tg45° = \left|\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m}\right|$$

$$1 = \left|\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m}\right|$$

1. slučaj

$$\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m} = -1 \quad / \cdot 1 - \frac{1}{2}m$$

$m \neq 2$

$$-m - \frac{1}{2} = -1 + \frac{1}{2}m$$

$$-\frac{3}{2}m = -\frac{1}{2} \quad /:\left(-\frac{3}{2}\right)$$

$$m = \frac{1}{3}$$

2. slučaj

$$\frac{-m - \frac{1}{2}}{1 - \frac{1}{2}m} = 1 \quad / \cdot 1 - \frac{1}{2}m$$

$$-m - \frac{1}{2} = 1 - \frac{1}{2}m$$

$$-\frac{1}{2}m = \frac{3}{2} \quad /:\left(-\frac{1}{2}\right)$$

$$m = 3$$

2. Odredi udaljenost između pravaca:

$x-2y+1=0$

$x-2y+11=0$

$\to x=3$

$3-2y+1=0 \quad T(3,2)$

$-2y=-4$

$y=2$

$d=\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$

$d=\frac{|1\cdot3+(-2)\cdot2+11|}{\sqrt{1^2+(-2)^2}}$

$d=\frac{|10|}{\sqrt5}=2\sqrt5$

3. Za koji se najmanji kut pravac $y=x-3$ mora zakrenuti oko svoje nul-točke kako bi prošao točkom $T(4,4)$?

nul-točka S:

$0=x-3$

$x=3 \quad S(3,0)$

$y=x-3 \to k=1$

$y=4x-12 \to k=4$

$tg\,\varphi=\left|\frac{k_2-k_1}{1+k_1k_2}\right|$

$tg\,\varphi=\left|\frac{4-1}{1+4}\right|=0{,}6$

$\varphi=30°57'50''$

4. Kolika je površina kvadrata kojem 2 stranice pripadaju pravcima:

$3x+2y-5=0 \to 2y=-3x+5 \to y=-\frac{3}{2}x+\frac{5}{2}$

$3x+2y+8=0 \to 2y=-3x-8 \to y=-\frac{3}{2}x-4$

$\to x=2$

$6+2y-5=0$

$2y=-1$

$y=-\frac{1}{2}$

$d=\left|\frac{3\cdot2+2\cdot(-\frac{1}{2})+8}{\sqrt{3^2+2^2}}\right|=\sqrt{13}$

$P=d^2$

$P=(\sqrt{13})^2$

$P=13$

5. Dvije stranice pravokutnika leže na pravcima $x+2y-3=0$ i $2x-y+3=0$. Jedan vrh je točka $A(8,5)$. Kolika je površina trokuta?

$A(8,5)$

1. $x+2y-3=0$
2. $2x-y+3=0$

udalj. A od 1. pravca

$$d=\left|\frac{Ax+By+C}{\sqrt{A^2+B^2}}\right|$$

$$d=\left|\frac{1\cdot 8+2\cdot 5+(-3)}{\sqrt{1^2+2^2}}\right|=\left|\frac{15}{\sqrt{5}}\right|=\underbrace{3\sqrt{5}}_{a}$$

udalj. A od 2. pravca

$$d=\left|\frac{2\cdot 8+(-1)\cdot 5+3}{\sqrt{2^2+(-1)^2}}\right|=\left|\frac{14}{\sqrt{5}}\right|=\underbrace{\frac{14\sqrt{5}}{5}}_{b}$$

$P=a\cdot b$

$P=3\sqrt{5}\cdot\frac{14\sqrt{5}}{5}$

$P=42$