INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 2.1]

LINES

Compiled by Christos Nikolaidis

O.	**Practice questions**

1. [Maximum mark: 8]   ***[without GDC]***

Consider the points $A(2,7)$, $B(5,11)$

(a) Find the gradient of the line (AB). [2]

(b) Write down the gradient of a perpendicular line to (AB). [1]

(c) Let M be the midpoint of the line segment $[AB]$. Find the coordinates of M. [1]

(d) Find the distance between A and B (i.e. the length AB). [2]

(e) Find the coordinates of the point C if B is the midpoint of the line segment $[AC]$. [2]

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2. [Maximum mark: 8]   *[without GDC]*

Consider the points $A(3,5)$, $B(a,1)$ and $C(7, -3)$ where $a \in R$. Find the values of a in each of the following cases:

(a)   If the gradient of the line (AB) is 2.   [2]

(b)   If the midpoint of the line segment [AB] is $M(7,3)$.   [2]

(c)   If B is the midpoint of the line segment [AC].   [1]

(d)   If the distance between A and B is 5.   [3]

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3. [Maximum mark: 9]   *[without GDC]*

Consider the line L given by $y = 2x + 4$

(a)   Write down

(i)   the gradient of the line   (ii) the y-intercept   (iii) the x-intercept   [3]

(b)   Draw the line on the diagram below.   [3]

(c)   Check if the points A(7,19) and B(8,20) lie on the line.   [3]

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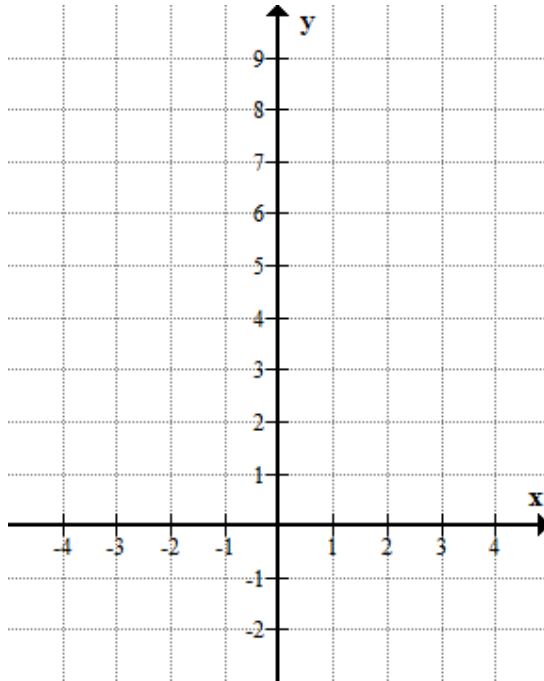
4. [Maximum mark: 7]   *[without GDC]*

Find the equation of the line passing through A(3,4) and B(5,7)

(a)   in the gradient-point form $y - y_1 = m(x - x_1)$ [3]

(b)   in the gradient-intercept form $y = mx + c$. [2]

(c)   in the form $ax + by = d$, where a, b, d are integers. [2]
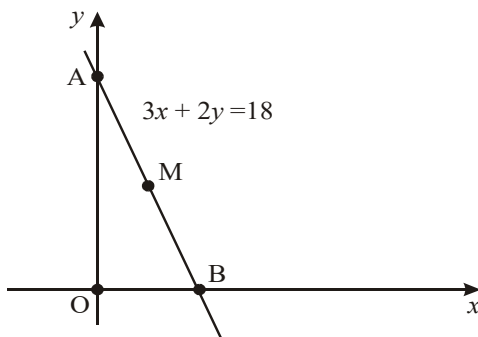
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5. [Maximum mark: 6]   *[without GDC]*

The diagram below shows the line with equation $3x + 2y = 18$. The points A and B are

the y and x-intercepts respectively. M is the midpoint of [AB].

Find the coordinates of (i) the point A;   (ii) the point B;   (iii) the point M.

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6. [Maximum mark: 5]   *[without GDC]*

    (a)   Find the equation of the line passing though the points $A(2,-5)$ and $B(2,8)$.   [2]

    (b)   Find the equation of the line passing though the points $C(6,5)$ and $D(-3,5)$.   [2]

    (c)   Find the point of intersection P between the lines L_1 and L_2 [1]

7. [Maximum mark: 6]

    (a)   Find the equation of the line which is parallel to x-axis and passes through A(2,3)   [2]

    (b)   Find the equation of the line which is parallel to y-axis and passes through A(2,3)   [2]

    (c)   Find the equation of the line passing though the origin and A(2,3)   [2]

8. [Maximum mark: 9] *[without GDC]*

Consider the line L on the diagram below
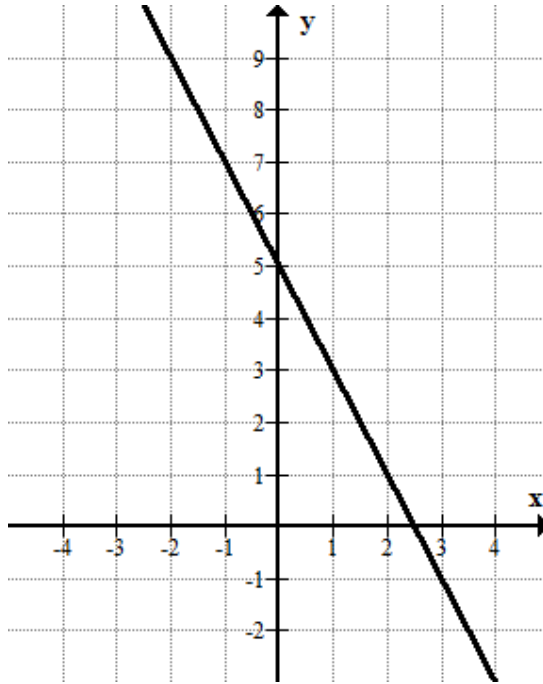
(a) Write down

(i) the gradient of the line   (ii) the y-intercept   (iii) the x-intercept   [3]

(b) Write down the equation of the line in the gradient-intercept form $y = mx + c$ [2]

(c) Given that P(1, y) and Q(x,1) lie on the line write down the values of x and y.   [2]

(d) Given that A(a,–5) and B(–5,b) lie on the line find the values of a and b.   [2]

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A. Exam style questions (SHORT)

9. [Maximum mark: 8]   *[without GDC]*

Consider the line L with equation $y + 2x = 3$. The line L_1 is parallel to L and passes through the point $(6, -4)$.

(a) Find the gradient of L_1. [1]

(b) Find the equation of L_1 in the form $y = mx + b$. [3]

(c) Find the x-coordinate of the point where line L_1 crosses the x-axis. [2]

(d) Draw the two lines on the diagram below. [2]

10. [Maximum mark: 9]   *[without GDC]*

Consider the points A(2,5) and B(3,8). The line L_1 passes through A and B.

(a) Find

(i) the gradient of the line L_1 (ii) the equation of the line L_1.   [3]

(b) Find the line L_2 which is perpendicular to L_1 and passes through the point A.   [2]

(c) Express both equations of L_1 and L_2 in the form $ax+by = c$, where a, b, c are

integers.   [2]

(d) Write down the solution of the two simultaneous equations found in (d) and make

a comment about the solution by sketching the two lines L_1, L_2 .   [2]

[it is not necessary to draw the lines on the Cartesian plane]

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11. [Maximum mark: 7]   *[without GDC]*

Consider the points A(–2,5) and B(4,9).

(a) Find the gradient of the line L passing through A and B. [2]

(b) Find the coordinates of the midpoint M between A and B. [1]

(c) Find the equation of the line which is perpendicular to L and passes through the point M. (i.e. the **perpendicular bisector** of the line segment [AB]) [2]

(d) Find the distance between the points A and M. [2]

12. [Maximum mark: 7]   *[without GDC]*

The points A(2,1) and B(6,3) are shown in the diagram below.

Let L, be the **perpendicular bisector** of the line segment [AB]

(a)   Find the equation of L.   [5]

(b)   Write down the y-intercept of L and draw an accurate line for L on the diagram

above.   .   [2]

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13*. [Maximum mark: 7]   *[with GDC]*

Find the **perpendicular bisector** of the line segment [AB] with A(8,13) and B(20,41), in the form $ax + by = d$, where a, b, d are positive integers.

14**. [Maximum mark: 6]   *[with / without GDC]*

Find the coordinates of a point P on L_1: $y = x + 1$ given that the distance between the

origin and P is 5.

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15**. [Maximum mark: 6]   *[without GDC]*

Find the coordinates of a point A on L_1: $y = x + 1$ and a point B on L_2: $y = 2x + 1$,

so that M(5,8) is the midpoint of the line segment [AB].

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16**. [Maximum mark: 6]   *[without GDC]*

The following diagram shows the lines $x - 2y - 4 = 0$, $x + y = 5$ and the point P(1, 1).

A line is drawn from P to intersect with $x - 2y - 4 = 0$ at Q, and with $x + y = 5$ at R, so that P is the midpoint of [QR].

Find the **exact** coordinates of Q and of R.

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B. Exam style questions (LONG)

17*. [Maximum mark: 10]   *[without GDC]*

Let $A(3k, 4k)$ be a point of the line L with equation $y = \dfrac{4}{3}x$, where k is an integer.

(a) Verify that the point A lies on the line L.   [2]

(b) Find the possible values of k if the distance between the origin and A equals 10.   [4]

(c) Write down the coordinates of the two points on the line L whose distance from the origin is equal to 10.   [2]

(d) Demonstrate on the diagram below the result of question (c).   [2]

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18. [Maximum mark: 13]   *[with / without GDC]*

Consider the line L_1 with equation $y = 2x - 3$.

The line L_2 is **parallel** to L_1 and passes through the point A(1,9).

The line L_3 is **perpendicular** to L_1 and passes through the point A(1,9).

(a)   Find the equation (i)   of the line L_2.   (ii) of the line L_3.   [6]

The lines L_1 and L_3 intersect at point B

(b)   Find the coordinates of point B.   [2]

(d)   Find the distance between the points A and B.   [2]

(d)   Sketch a diagram and deduce the distance from the point A to the line L_1.   [3]

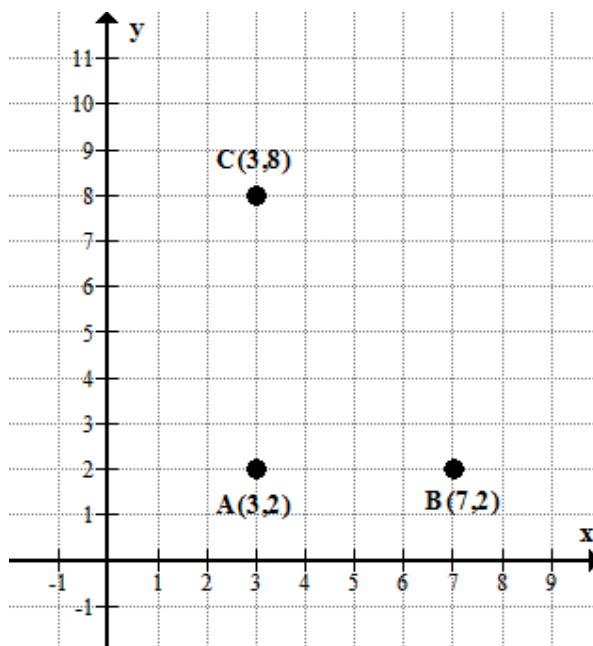*[it is not necessary to draw the lines on the Cartesian plane]*

19. [Maximum mark: 23]

The points A(3,2), B(7,2) and C(3,8) are shown in the diagram below.

(a) Find the equation of the perpendicular bisector of line segment [AB]. [2]

(b) Find the equation of the perpendicular bisector of line segment [AC]. [2]

(c) Write down the coordinates of the point of intersection P of the two bisectors and show that P is the midpoint of the line segment [BC]. [3]

(d) Find the areas of the triangles

    (i) ABC.   (ii) ABP   (iii) ACP [6]

(e) Find the equation of the perpendicular bisector L of the line segment [BC] in the form $ax + by + d = 0$ with $a, b, c \in Z.$ [5]

(f) Show that the line L does not pass through A. [2]

(g) Draw the three perpendicular bisectors of the sides of ABC on the diagram above. [3]

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20* [Maximum mark: 10] *[with / without GDC]*

The following three lines l_1, l_2, and l_3 are defined with equations

$$l_1 : x + y = 5, \qquad l_2 : x - 2y = 8, \qquad l_3 : x = -2$$

and are shown in the figure below.

(a) Find the coordinates of the common point A between the lines l_1 and l_2. [2]

(b) Write down the coordinates of

(i) the common point B between the lines l_1 and l_3

(ii) the common point C between the lines l_2 and l_3. [4]

(c) **Hence**, find the area of the triangle ABC. [4]

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