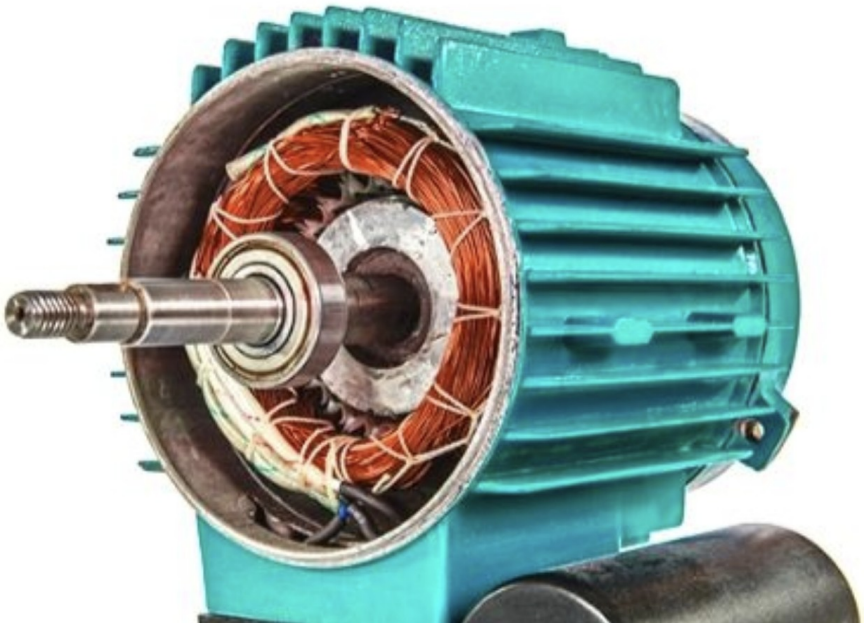
Quotient rule (AASL/HL)

Imagine you are monitoring the efficiency of a machine over time, where efficiency (E) is defined as the ratio of output (O) to input (I), so E = O / I.



Now, suppose you want to find out how this efficiency changes over time, which involves finding the derivative of E with respect to time (t). According to the quotient rule:

1. Compute the rate at which the output changes (dO/dt), which might be due to improvements in technology or variations in machine performance.
2. Compute the rate at which the input changes (dI/dt), which could change due to fluctuations in resource availability or changes in operational conditions.
3. The total rate of change of efficiency (dE/dt) is given by:
   - The rate of change in output (dO/dt) adjusted by the current level of input, minus
   - The rate of change in input (dI/dt) adjusted by the current level of output,
   - All divided by the square of the current input level to normalize the changes relative to the size of the input.

Mathematically, this is written as:

$$\frac{dE}{dt} = \frac{\frac{dO}{dt} \cdot I - O \cdot \frac{dI}{dt}}{I^2}$$

This formula explains how changes in output and input independently and jointly affect the overall efficiency of the machine, allowing you to understand the interdependencies and their impacts.