34.

$P(t) = P_0 e^{kt}$

$P(10) = 6P(0)$

$\cancel{P_0} e^{k(10)} = 6\cancel{P_0}$

$e^{k(10)} = 6$

$\ln e^{10k} = \ln 6$

$10k = \ln 6$

$k = \frac{1}{10} \ln(6)$

$k = 0{,}1791$

Al reemplazar k tenemos que

$P(t) = P_0 e^{0{,}1791 \cdot t}$

Para saber cuanto le toma a la población duplicarse hallamos el valor de t, donde $P(t) = 2P(0)$

$\cancel{P_0} e^{0{,}1791t} = 2\cancel{P_0}$

$e^{0{,}1791t} = 2$

$\ln e^{0{,}1791t} = \ln 2$

$t = \frac{1}{0{,}1791} \cdot \ln 2$

$t = 3{,}87$ horas

35.

$\frac{dN}{dt} = -k \cdot N$

$\frac{dN}{N} = -k\,dt$

$\int \frac{dN}{N} = \int -k\,dt$

$\ln|N| = -k \cdot t + c$

Para hallar la constante de integración asumimos que $t=0$

$\ln N_0 = -k(0) + c \Rightarrow \ln N_0 = c$

$\ln N = -kt + \ln N_0$

$e^{\ln N} = e^{-kt + N_0}$

$N = e^{-kt} \cdot e^{N_0}$

$N = N_0 e^{-kt}$

$k = 0,0001216$

$^{14}C_0\, e^{-kt}$

$\frac{1}{6}\, ^{14}C_0 = {}^{14}C_0\, e^{-kt}$

$\frac{1}{6} = e^{-0,0001216 \cdot t}$

$Ln(1/6) = Ln\, e^{-0,0001216 \cdot t}$

$Ln(1/6) = e^{-0,0001216 \cdot t}$

$\frac{Ln(1/6)}{-0,0001216} = t$ $\qquad 14734,8$ años $= t$

36. $C_k = {}^{14}C_0\, e^{-kt}$

$\frac{^{14}C_k}{^{14}C} = \frac{4,6 \times 10^{10}}{5,0 \times 10^{10}} = \frac{^{14}C_0}{^{14}C_0} e^{-kt}$

$\frac{4,6 \times 10^{10}}{5,0 \times 10^{10}} = e^{-kt}$

$Ln(4,6/5) = Ln\, e^{-kt}$

$Ln(4,6/5) = -kt$

$\frac{Ln(4,6/5)}{-k} = t$

$\frac{Ln(4,6/5)}{-0,0001216}$

$685,7$ años $= t$

R// A pesar de ser una reliquia no es de los tiempos de Cristo

37. $\frac{dA}{dt} = rA$

$r = \frac{R\%}{100} = r = \frac{8\%}{100} = 0,08$

$A(t) = A_0\, e^{rt}$

$= 5000\, e^{0,08t}$

$A(18) = 5000\, e^{0,08(18)} = 21,103$

↳ Acumulacion del ahorro tras 18 años

38.

$A(t) = 30\, e^{0,05t}$

$A(100) = 30\, e^{0,05(100)}$

$A(100) = 4452$ \$ $\Rightarrow$ total a pagar

39.

$\frac{dA}{dt} = -\lambda A$

$\frac{dA}{A} = \lambda\, dt$

$\int \frac{dA}{A} = \int -\lambda\, dt$

$\ln(A) = -\lambda t + c$

$e^{\ln A} = e^{-\lambda t + c} \rightarrow e^{-\lambda t}\, e^{c}$

$A(t) = A\, e^{-\lambda t}$

$A(t) = A_0\, e^{-\lambda t}$

si $t = 0$, $A(0)$

$A \cdot e^{-\lambda 0} = A_0$

$A = A_0$

Ahora suporemos $t = 0$

$A(1\ hora) = 45\ mg \times 50$

$A_0\, e^{-\lambda(1)} = 2250\ mg$

$A_0 = 2250\, e^{\lambda}$

$A_0 = 2250\, e^{\frac{\ln 2}{5}}$

$A_0 = 2585\ mg$

para encontrar λ =

A 5 horas $A_0/2$

$\cancel{A_0}\, e^{-\lambda 5} = \cancel{A_0}/2$

$\lambda = \frac{\ln 2}{5}$

R// Se le debe suministral al perro 2585 mg para sedarlo 1 hora

A2.

$$\frac{R}{A_r} = 1 \qquad A_r = R$$

$$A_r = \frac{1}{9}$$

$$A_r = 1\,(P_0 - P_0\, e^{-kt})$$

$$P = A_r$$

$$P_0\, e^{-\lambda t} = \frac{1}{9}\,(P_0 - P_0\, e^{-\lambda t})$$

$$9\, P_0\, e^{-kt} = P_0\,(1 - e^{-kt})$$

$$9\, e^{-kt} = 1 - e^{-kt}$$

$$10\, e^{-kt} = 1$$

$$e^{-kt} = 1/10$$

$$\ln e^{-kt} = \ln(1/10) \rightarrow -\ln 10$$

$$-kt = -\ln 10$$

$$t = \frac{-\ln 10}{k} \rightarrow k = \frac{\ln}{Tm}$$

$$t = \frac{\ln 10}{\frac{\ln 2}{Tm}} \rightarrow Tm\ P = 1{,}28 \times 10^9$$

$$t = \frac{\ln 10}{\ln 2} \cdot Tm$$

$$t = \frac{\ln 10}{\ln 2} \cdot 1{,}28 \times 10^9$$

$$t = 4{,}25 \times 10^9 \text{ años} = 4250 \text{ millones de años}$$

R// La roca tiene 4250 millones de años de edad

48. $\frac{{}^{238}U}{{}^{235}U} = \frac{{}^{238}U_0 \, e^{-k_{238} t}}{{}^{235}U_0 \, e^{-k_{235} t}} = 137,7$ $U^{238} = U^{235}$

$${}^{238}U = {}^{235}U$$

Entonces

$$e^{-k_{238} t + k_{235} t} = 137,7$$

$$e^{t(k_{235} - k_{238})} = 137,7$$

$$\ln e^{t(k_{235} - k_{238})} = \ln 137,7$$

$$t(k_{235} - k_{238}) = \ln(137,7)$$

$$t = \frac{\ln(137,7)}{k_{235} - k_{238}}$$

Al tener la vida media (Tm) de un materia radioactivo

$$K = \frac{\ln 2}{Tm}$$

$$t = \frac{\ln(137,7)}{\frac{\ln 2}{T_{m235}} - \frac{\ln}{T_{m238}}}$$

$$t = \frac{\ln(137,7)}{\ln 2} \cdot \frac{T_{m235} \cdot T_{m238}}{T_{m238} - T_{m235}}$$ Hice MCM

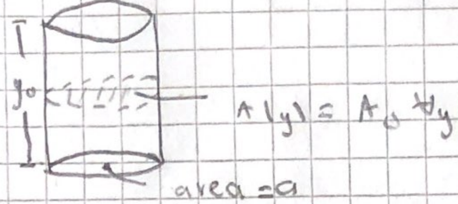
$$t = \frac{\ln(137,7)}{\ln 2} \cdot \frac{(7,10 \times 10^8)(4,51 \times 10^9)}{4,51 \times 10^9 - 7,10 \times 10^8}$$

$$t = \frac{\ln(137,7)}{\ln 2} \cdot \frac{(7,10)(4,51)(10^8)(10^9)}{10^8(45,1 - 7,10)}$$

$$t = 5,9874 \times 10^9 \approx 6000 \text{ millones}$$

R// La edad aproximada del universo es de 6000 millones de años

57



$A(y) \dfrac{dy}{dt} = -k\sqrt{y}$ ← ley Torricelli

$A_0 \dfrac{dy}{dt} = -k\sqrt{y}$

$\dfrac{dy}{\sqrt{y}} = -\dfrac{k}{A_0}\, dt$

$\int y^{-1/2}\, dy = \int B\, dt$

$2\sqrt{y} = Bt + C$ → 1. $y(0) = y_0$

$2\sqrt{y_0} = B(0) + C$

$2\sqrt{y_0} = C$

$2\sqrt{y} = Bt + 2\sqrt{y_0}$

$2\sqrt{y} = -\dfrac{2\sqrt{y_0}}{T} t + 2\sqrt{y_0}$

2. $y(T) = 0$

$2\sqrt{0} = BT + 2\sqrt{y_0}$

$-\dfrac{2\sqrt{y_0}}{t} = B$

$\sqrt{y} = \sqrt{y_0} - \sqrt{y_0} \cdot \dfrac{t}{T}$

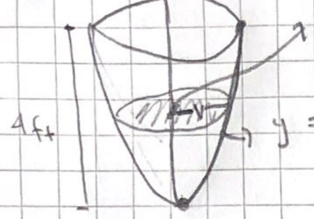
$\sqrt{y} = \sqrt{y_0}\left(1 - \dfrac{t}{T}\right)$

$y = y_0 \left(1 - \dfrac{t}{T}\right)^2$

$A_0 y = A_0\, y_0 \left(1 - \dfrac{t}{T}\right)^2$

Volumen → $V(t) = V_0 \left(1 - \dfrac{t}{T}\right)^2$

64. $t = 1\,ft - 1$



$A(y) = \pi \cdot r^2 = \pi x^2 = \pi [g(y)]^2$

$y = f(x)$ o $x = g(y)$

- $A(y) \dfrac{dy}{dt} = -k\sqrt{y}$

$A(y) \dfrac{dy}{dt} = -a\sqrt{2g}\sqrt{y}$

$\pi [g(y)]^2 \dfrac{dy}{dt} = -a\sqrt{2g}\sqrt{y}$; necesitamos que $v_t = \dfrac{dy}{dt}$ $-4 \dfrac{\text{pulgadas}}{\text{hora}}$

$\pi (g(y))^2 \left(\dfrac{-1}{108000}\right) = -\pi r_0^2 \cdot \sqrt{2 \cdot 32} \cdot \sqrt{y}$ $= \dfrac{-1}{10800}$ pie/seg

Podemos evaluar esta EDO para el tiempo t_0, tal que $x = 1$ y $y = 4$

$\pi [g(4)]^2 \left(\dfrac{-1}{10800}\right) = -\pi r_0^2 \sqrt{64}\sqrt{4}$

$(1)^2 \dfrac{1}{10800} = 16 r_0^2$

$\dfrac{1}{172800} = r_0^2$

$\dfrac{1}{\sqrt{172800}} = r_0 \Rightarrow r_0 = \dfrac{1}{240\sqrt{3}}$

$r_0 = \dfrac{\sqrt{3}}{720}$

$r_0 = \dfrac{\sqrt{3}}{720}$ (304,8 mm)

$r_0 = 0,733$ mm $\Rightarrow$ radio para el orificio circular del fondo

Una vez encontramos el radio del orificio, podemos usar de nuevo la ecuación diferencial

$$\pi(g(y))^2 \frac{dy}{dt} = -a\sqrt{2g}\sqrt{y}$$

$$a = \pi r_0^2 = \pi\left(\frac{1}{172800}\right) \text{pies}^2 \quad ; \quad \frac{dy}{dt} = \frac{1}{10800} \frac{\text{pies}}{\text{seg}}$$

Podemos establecer la relación $x = g(y)$ o $y = f(x)$

$$\pi[x]^2\left(-\frac{1}{10800}\right) = -\pi\frac{1}{172800}\sqrt{64}\sqrt{y}$$

$$\frac{172800\, x^2}{108000 \cdot \sqrt{64}} = \sqrt{y}$$

$$2x^2 = \sqrt{y}$$

$$y = 4x^4 \Rightarrow \text{curva } y = f(x)$$

6) $\frac{dT}{dt} = k(A - T(t))$ $A \to$ temperatura medio ambiente

$$\frac{dT}{A-T} = k\,dt$$

$$\int \frac{dT}{A-T} = \int k\,dt \qquad U = A - T$$

$$du = -dT$$

$$-\ln|A-T| = kt + C$$

$$\ln|A-T| = -kt + B \qquad \text{Suponemos que } T(0) = T_0$$

$$\ln|A-T_0| = -k(0) + B \quad \Big\} \to \text{suponiendo que } T(0) = T_0$$

$$\ln|A-T_0| = B$$

$$\to \ln|A - T_0|$$

$$\ln|A-T| - \ln|A-T_0|$$

$$\ln\left|\frac{A-T}{A-T_0}\right| = -kt$$

$$\ln\left(\frac{A-T}{A-T_0}\right) = -kt$$

$$e^{\ln\left(\frac{A-T}{A-T_0}\right)} = e^{-kt}$$

$$\frac{A-T}{A-T_0} = e^{-kt}$$

$$A-T = (A-T_0)e^{-kt} - A$$

$$-T = (A-T_0)e^{-kt} - A$$

$$T(t) = A-(A\ T_0)\ e^{-kt}$$ La temperatura del cuerpo en el instante t $(T_0 < A)$

Primero $A = 70°F$ $\quad T(t) = 70 - (70 - T_0)\ e^{-kt}$

Segundo tomamos $T(0) = 98{,}6°F$ con la cual

$$T(t) = (70 - 98{,}6)\ e^{-kt}$$

$$T(t) = 70 + 28{,}6\ e^{-kt}$$

$\rightarrow T(t_0) = 80°F \quad 80 = 70 + 28{,}6^{-kt_0} \quad 10 = 28{,}6e^{-k \cdot t_0}$ (1) t_0 = tiempo de deceso

$\rightarrow T(t_{0+1}) = 75°F \quad 70 = 70 + 28{,}6\ e^{-k(t_0+1)} \quad 5 = 28{,}6\ e^{-kt_0} \cdot e^{-k}$

$$\frac{10}{5} = \frac{28{,}6\ e^{-kt_0}}{28{,}6\ e^{-kt_0}\ e^{-k}}$$

$$2 = e^{k}$$

$$\ln 2 = \ln e^{k}$$

$$\ln 2 = k$$

Reemplazo $\ln 2 = k$ en 1

$$10 = 28{,}6\ e^{-\ln 2\ t_0}$$

$$\frac{10}{28{,}6} = e^{-\ln 2\ t_0}$$

$$\ln(10/28{,}6) = \ln\left(e^{-\ln 2 \cdot t_0}\right)$$

$$\ln(10/28{,}6) = -\ln 2 \cdot t_0$$

$$\frac{\ln(10 / 28{,}16)}{-\ln 2} = t_0$$

1,51 horas $= t_0$ → La victima lleva 1,51 horas fallecida

1 hora = 60 m

1,51 horas = B

$$B = \frac{(60 \text{ minutos})(1{,}51 \text{ horas})}{1 \text{ hora}}$$

B = 91 minutos

R// La victima fallecio sobre las 10:29 am.