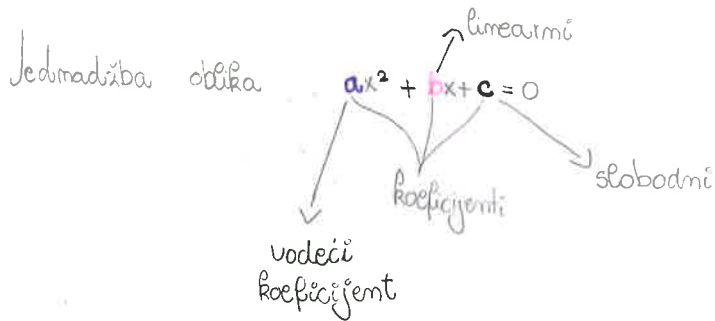


# KVADRATNE JEDNADŽBE

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$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1)  $4x^2 - 9 = 0$   
 $4x^2 = 9 \quad | :4$   
 $x^2 = \frac{9}{4} \quad | \sqrt{\quad}$   
 $x = \pm \frac{3}{2}$

3)  $4x^2 + 1 = 0$   
 $4x^2 = -1 \quad | :4$   
 $x^2 = -\frac{1}{4} \quad | \sqrt{\quad}$   
 $x = \pm \frac{1}{2} i$

2)  $7x^2 - 10x = 0$   
 $x(7x - 10) = 0$   
 $x_1 = 0$   
 $7x - 10 = 0$   
 $7x = 10$   
 $x_2 = \frac{10}{7}$

4)  $10x^2 + 3x - 4 = 0$

$$x_{1,2} = \frac{-3 \pm \sqrt{9 + 160}}{20}$$

$$x_{1,2} = \frac{-3 \pm 13}{20}$$

$$x_1 = \frac{10}{20} = \frac{1}{2}$$

$$x_2 = \frac{-16}{20} = -\frac{4}{5}$$

Kvadratna jednadžba sa zadanim rješenjima

$$a(x - x_1)(x - x_2) = 0$$

$$a \neq 0$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

1)  $4x^2 - x - 3 = 0$

$$x_{1,2} = \frac{1 \pm \sqrt{1 + 48}}{8}$$

$$x_{1,2} = \frac{1 \pm 7}{8}$$

$$x_1 = \frac{8}{8} = 1$$

$$x_2 = \frac{-6}{8} = -\frac{3}{4}$$

$$a(x - x_1)(x - x_2)$$

$$4(x - 1)\left(x + \frac{3}{4}\right) = (x - 1)(4x + 3)$$

7.

$$1) \frac{x^2-4}{8} - \frac{2x+3}{5} = 1 \quad | \cdot 40 \Rightarrow \text{mnożimo su } 40 \text{ kako bi se riješila razlomka}$$

$$5(x^2-4) - 8(2x+3) = 40 \Rightarrow \text{mnożimo suaki sa suakim}$$

$$5x^2 - 20 - 16x - 24 = 40$$

$$5x^2 - 16x - 84 = 0 \Rightarrow \text{formula}$$

$$x_{1,2} = \frac{16 \pm \sqrt{256 + 1680}}{10}$$

$$x_{1,2} = \frac{16 \pm 44}{10}$$

$$x_1 = \frac{16+44}{10}$$

$$x_2 = \frac{16-44}{10}$$

$$x_1 = \frac{60}{10}$$

$$x_2 = \frac{-28}{10}$$

$$x_1 = 6$$

$$x_2 = -\frac{14}{5}$$

14.

$$2) \frac{2}{5x-10} - \frac{1.6}{x^2-4} = \frac{x-1}{3x^2+6x} \Rightarrow \text{izlučivanje formule}$$

$$\frac{2}{5(x-2)} - \frac{1.6}{(x-2)(x+2)} = \frac{x-1}{3x(x+2)}$$

$$| \cdot 15x(x-2)(x+2) \neq 0 \Rightarrow \text{rješavamo se}$$

$$6x(x+2) - 24x = 5(x-2)(x-1)$$

$$x \neq 2$$

$$x \neq -2$$

$$6x(x+2) - 24x - 5(x^2 - x - 2x + 2) = 0$$

maksimumika i

postavljamo uvjete

$$6x^2 + 12 - 24x - 5(x^2 - 3x + 2) = 0$$

$$6x^2 + 12 - 24x - 5x^2 + 15x - 10 = 0$$

$$x^2 + 3x - 10 = 0 \Rightarrow \text{vietaove formule}$$

$$\begin{array}{c} / \quad \backslash \\ 5 \quad -2 \end{array}$$

$$x_1 = -5 \quad \checkmark$$

$$x_2 = 2 \quad \times$$

# DISKRIMINANTA

$$D = b^2 - 4ac$$

Primjer

$$2x^2 - 5x + 3 = 0$$

$$D = b^2 - 4ac$$

$$= (-5)^2 - 4 \cdot 2 \cdot 3 =$$

$$D = 1$$

$$D > 0$$

↳ postoje 3 različita rješenja kvadratne jednačine:

$D > 0$  - jednačina ima dva realna i različita rješenja

$D = 0$  - jednačina ima jedno dvostruko realno rješenje

$D < 0$  - jednačina ima kompleksno konjugirana rješenja

5.

$$1) \quad x^2 - bx + 4 = 0 \quad D = 0$$

$$(-b)^2 - 4 \cdot 1 \cdot 4 = 0$$

$$b^2 - 16 = 0$$

$$b^2 = 16 / \sqrt{\quad}$$

$$b = \pm 4$$

Za  $b = \pm 4$  jednačina ima jednaka rješenja

# VIETEOVE FORMULE

Ako su  $x_1$  i  $x_2$  rješenja kvadratne jednačine  $ax^2+bx+c=0$  tada vrijede vieteove formule

$$x_1 + x_2 = -\frac{b}{a}$$

$$x_1 \cdot x_2 = \frac{c}{a}$$

1.

1)  $2x^2 - 3x + 1 = 0$

$$\begin{aligned} x_1 + x_2 &= -\frac{b}{a} \\ &= -\frac{-3}{2} \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} x_1 \cdot x_2 &= \frac{c}{a} \\ &= \frac{1}{2} \end{aligned}$$

2)  $3x^2 + x - 2 = 0$

$$\begin{aligned} x_1 + x_2 &= -\frac{b}{a} \\ &= -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} x_1 \cdot x_2 &= \frac{c}{a} \\ &= \frac{-2}{3} \end{aligned}$$

2.

1)  $x^2 - 5x + 6 = 0$

$$x_1 + x_2 = \frac{5}{1} = 5$$

$$x_1 \cdot x_2 = \frac{6}{1} = 6$$

2)  $x^2 - x - 6 = 0$

$$x_1 + x_2 = \frac{1}{1} = 1$$

$$x_1 \cdot x_2 = -\frac{6}{1} = -6$$

3.

$3x^2 - 4x - 1 = 0$

$$x_1 + x_2 = \frac{4}{3}$$

$$x_1 \cdot x_2 = -\frac{1}{3}$$

1)

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = \left(\frac{4}{3}\right)^2 - 2 \cdot \left(-\frac{1}{3}\right) = \frac{16}{9} + \frac{2}{3} = \frac{22}{9}$$

2)

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2 + x_1}{x_1 \cdot x_2} = \frac{\frac{4}{3}}{-\frac{1}{3}} = -4$$

13.

$$(2px - 1)^2 = p(p - 2x)$$

$$4p^2x^2 - 4px + 1 = p^2 - 2px$$

$$4p^2x^2 - 4px + 1 - p^2 + 2px = 0$$

$$4p^2x^2 - 2px + (1 - p^2) = 0$$

17.

$$1) \quad 2x^2 + 7x - 15 = 0$$

$$x_{1,2} = \frac{-7 \pm \sqrt{49 + 120}}{4}$$

$$= \frac{-7 \pm 13}{4}$$

$$x_1 = \frac{-7 - 13}{4}$$

$$x_1 = \frac{-20}{4} = -5$$

$$x_2 = \frac{-7 + 13}{4}$$

$$= \frac{6}{4} = \frac{3}{2}$$

$$\alpha(x - x_1)(x - x_2) = \lambda \left(x - \frac{3}{2}\right)(x + 5) = (2x - 3)(x + 5)$$

18.

$$1) \quad (x^2 - 2x)^2 - 2(x^2 - 2x) - 3 = 0 \quad t = x^2 - 2x$$

$$t^2 - 2t - 3 = 0$$

$$t_{1,2} = \frac{2 \pm \sqrt{4 + 12}}{2}$$

$$= \frac{2 \pm 4}{2}$$

$$t_1 = \frac{2 + 4}{2}$$

$$t_1 = 3$$

$$t_2 = \frac{2 - 4}{2}$$

$$t_2 = -1$$

$$(t - t_1)(t - t_2) = 0$$

$$(t - 3)(t + 1) = 0$$

$$(x^2 - 2x - 3)(x^2 - 2x + 1) = 0$$

1.

$$1) x^4 + 3x^2 + 2 = 0$$

$$t = x^2$$

$$t^2 + 3t + 2 = 0$$

$$t_{1,2} = \frac{-3 \pm \sqrt{9-8}}{2}$$

$$t_{1,2} = \frac{-3 \pm 1}{2}$$

$$t_1 = -1$$

$$t_2 = -2$$

$$t = x^2$$

$$x^2 = -1 / \sqrt{\quad}$$

$$x_{1,2} = \pm i$$

$$x^2 = -2 / \sqrt{\quad}$$

$$x_{3,4} = \pm \sqrt{2} i$$

8.

1)

$$\begin{cases} x^2 + y^2 = 25 \\ 3x - y = 15 \end{cases} \Rightarrow y = 3x - 15$$

$$\Rightarrow y = 3 \cdot 4 - 15$$

$$y_1 = -3$$

$$y = 3 \cdot 5 - 15$$

$$y_2 = 0$$

$$x^2 + (3x - 15)^2 = 25$$

$$x^2 + 9x^2 - 90x + 225 - 25 = 0$$

$$10x^2 - 90x + 200 = 0 \quad | : 10$$

$$x^2 - 9x + 20 = 0$$

$$\Rightarrow x_{1,2} = \frac{9 \pm \sqrt{81-80}}{2}$$

$$x_{1,2} = \frac{9 \pm 1}{2}$$

$$x_1 = 4$$

$$x_2 = 5$$

13.

2)

$$\sqrt{3x-8} + 2 = x$$

$$\rightarrow \sqrt{3 \cdot 4 - 8} = 4 - 2$$

$$\sqrt{3 \cdot 3 - 8} = 4 - 3$$

$$\sqrt{3x-8} = x - 2 \quad |^2$$

$$2 = 2 \quad \checkmark$$

$$1 = 1 \quad \checkmark$$

$$3x - 8 = x^2 - 4x + 4$$

$$x_{1,2} = \frac{7 \pm \sqrt{49-48}}{2}$$

$$3x - 8 - x^2 + 4x - 4 = 0$$

$$x_{1,2} = \frac{7 \pm 1}{2}$$

$$-x^2 + 7x - 12 = 0 \quad | : (-1)$$

$$x_1 = 4 \quad \checkmark$$

$$x^2 - 7x + 12 = 0$$

$$x_2 = 3 \quad \checkmark$$