Sección 1.6

29.

$$2x \operatorname{sen} y \cos y \frac{dy}{dx} = 4x^2 + \operatorname{sen}^2 y$$

$$x\, 2\operatorname{sen} y \cos y \frac{dy}{dx} = 4x^2 + \operatorname{sen}^2 y$$

$$u = \operatorname{sen}^2 y$$

$$\frac{d}{dx}(u) = \frac{d}{dx}(\operatorname{sen}^2 y)$$

$$\frac{du}{dx} = 2\operatorname{sen} y \cos y \frac{dy}{dx}$$

$$x \frac{du}{dx} = 4x^2 + u$$

$$x \frac{du}{dx} - u = 4x^2$$

$$\frac{du}{dx} - \frac{1}{x} u = 4x$$

$$\frac{1}{x}\left(\frac{du}{dx} - \frac{1}{x} u\right) = \frac{1}{x} 4x$$

ecuación lineal primer orden

$$\rho(x)\ e^{\int p(x)\,dx}$$

$$\rho(x) = e^{\int -\frac{1}{x} dx}$$

$$\rho(x)\ e^{-\ln x}$$

$$\rho(x) = e^{\ln x^{-1}} = x^{-1}$$

$$\rho(x) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\frac{1}{x} u\right) = 4$$

$$\frac{d}{dx}\left(\frac{1}{x} u\right) = 4\,dx$$

$$\int d\left(\frac{1}{x} u\right) = \int 4\,dx$$

$$\frac{1}{x} u = 4x + c$$

$$u = 4x^2 + cx$$

$$\operatorname{sen}^2 y = 4x^2 + cx$$

Ahora probemos la idea de reducir el orden de una EDO de segundo orden

$$F(x, y, y', y'') = 0$$

$$F(x, y', y'') = 0$$

$$F(y, y', y'') = 0$$

para reducir el orden hacemos $u(x) = y'$

para reducir el orden hacemos $u(y) = y'$

30.

$(x+e^{y})\,y' = xe^{-y}-1$

$(x+e^{y})\dfrac{dy}{dx} + 1 - xe^{-y} = 0$

$(x+e^{y})\,dy + (1-xe^{-y})\,dx = 0$

$(1-xe^{-y})\,dx + (x+e^{y})\,dy = 0$

$M(x,y)\,dx + N(x,y)\,dy = 0 \quad \boxed{1}$

donde $M_y = xe^{-y} \quad \wedge \quad N_x = 1$

$\Rightarrow M_y \neq N_x$ edo $\boxed{1}$ no es exacta

Observamos que $\dfrac{N_x - M_y}{M} = \dfrac{1-xe^{-y}}{1-xe^{-y}} = 1$

$\Rightarrow$ existe el factor integrante $\rho(y) = e^{\int \frac{N_x - M_y}{M} dy}$

$\rho(y) = e^{\int 1\, dy}$

$\rho(y) = e^{y}$

$\Rightarrow (1-xe^{-y})\,dx + (x+e^{y})\,dy = 0$

$e^{y}(1-xe^{-y})\,dx + e^{y}(x+e^{y})\,dy = 0$

$(e^{y}-x)\,dx + (xe^{y}+e^{2y})\,dy = 0 \quad \boxed{2}$

$\rightarrow$ Observamos que $m_y = e^{y} \quad \wedge \quad n_x = e^{y}$

$m_y = n_x$ edo $\boxed{2}$ es exacta

Hay una función $f(x,y)$ tal que $f(x,y) = c$ es la familia monoparamétrica de soluciones de la edo $\boxed{2}$ donde

$\dfrac{\partial f}{\partial x} = m(x,y) \quad \wedge \quad \dfrac{\partial f}{\partial y} = n(x,y)$

$\dfrac{\partial f}{\partial x} = e^{y} - x \quad \wedge \quad \dfrac{\partial f}{\partial y} = xe^{y} + e^{2y}$

$$\int \frac{dv}{1+\sqrt{v}} = \int \frac{2(w-1)}{w}\, dw$$

$$= \int \frac{2w}{w} - \frac{2}{w}\, dw$$

$$= \int 2 - \frac{2}{w}\, dw$$

$$= 2w - 2\ln|w| + c$$

$$= 2(1+\sqrt{v}) - 2\ln|1+\sqrt{v}| + c$$

$$= 2 + 2\sqrt{v} - 2\ln|\sqrt{v}+1| + c$$

$$= 2\sqrt{v} - 2\ln(\sqrt{v}+1) + c_1$$

$w = 1+\sqrt{v} \Rightarrow w-1 = \sqrt{v}$

$dw = \frac{1}{2\sqrt{v}}\, dv$

$2\sqrt{v}\, dw = dv$

$2(w-1)\, dw = dv$

46. Resolver $xy'' + y' = 4x$; para reducir el orden sea $v(x) = y'$

$$\frac{d}{dx}(v(x)) = \frac{d}{dx}(y')$$

$$v'(x) = y''$$

Reemplazando las expresiones $y' = v$ y $y'' = v'$ en la e.d.o. tenemos

$$\Rightarrow xv'(x) + v(x) = 4x$$

$$x\frac{dv}{dx} + v = 4x$$

$$\frac{dv}{dx} + \frac{1}{x}v = 4$$

Ecuación lineal

$$\rho(x) = e^{\int p(x)\,dx} = e^{\int \frac{1}{x}\,dx}$$

$$\rho(x) = e^{\ln x} = x$$

$$x\left(\frac{dv}{dx} + \frac{1}{x}v\right) = 4x$$

$$\frac{d}{dx}(xv) = 4x$$

$$d(xv) = 4x\,dx$$

$$\int d(xv) = \int 4x\,dx$$

$$xv = 2x^2 + c_1$$

$$v = 2x + \frac{c_1}{x}$$

Integrar parcialmente respecto a y $\quad f(x,y) = \int \frac{\partial f}{\partial y}\, dy$

$$f(x,y) = \int xe^{y} + e^{2y}\, dy$$

$$f(x,y) = xe^{y} + \frac{1}{2}e^{2y} + g(x)$$

derivo parcialmente respecto a x $\quad \frac{\partial f}{\partial x} = e^{y} + 0 + g'(x)$

$$\Rightarrow \quad e^{y} - x = e^{y} + g'(x)$$

$$-x = g'(x)$$

$$\int -x\, dx = g(x)$$

$$-\frac{1}{2}x^{2} + c_1 = g(x)$$

Entonces la solucion a la edo exacta

$$f(x,y) = c$$

$$xe^{y} + \frac{1}{2}e^{2y} - \frac{1}{2}x^{2} + c_1 = c$$

$$xe^{y} + \frac{1}{2}e^{2y} - \frac{1}{2}x^{2} = c_2 \quad (\text{donde } c_2 = c - c_1)$$

$$2xe^{y} + e^{2y} - x^{2} = k \quad (\text{donde } k = 2c_2)$$

$$2\sqrt{x+y+1} - 2\ln(\sqrt{x+y+1} + 1) = x + c_1$$

$$x + c_1 = 2\sqrt{x+y+1} - 2\ln(\sqrt{x+y+1} + 1)$$

$$x = 2\sqrt{x+y+1} - 2\ln(\sqrt{x+y+1} + 1) + c$$

Solución implicita a la Edo.

$$v(x) = 2x + \frac{c_1}{x}$$

$$dy = \left(2x + \frac{c_1}{x}\right) dx$$

$$\int dy = \int 2x + \frac{c_1}{x}\, dx$$

$$y(x) = x^2 + c_1 \ln x + c_2$$

49. Resolver $yy'' + (y')^2 = y\,y'$ para reducir el orden se hace $v(y) = y'$

$$\frac{d}{dx}(v(y)) = \frac{d}{dx}(y')$$

$$\frac{dv}{dy}\frac{dy}{dx} = y''$$

$$\frac{dv}{dy} y' = y''$$

$$\frac{dv}{dy} v = y''$$

Reemplazando las expresiones $y'' = v\frac{dv}{dy}$ y $y' = v$ en la ed.o obtenemos

$$\Rightarrow y\,v\frac{dv}{dy} + (v)^2 = y\,v$$

dividiendo por $v \cdot y$; $v = y' > 0$, $y > 0$ por condiciones dadas por el problema

$$\frac{dv}{dy} + \frac{1}{y} v = 1$$

Ecuacion lineal

$$\frac{dv}{dy} + P(y) \cdot v = P(y)$$

$$\rho(y) = e^{\int P(y)\,dy}$$

$$\rho(y) = e^{\int \frac{1}{y} dy}$$

$$\rho(y) = e^{\ln y} = y$$

$$y\left(\frac{dv}{dy} + \frac{1}{y} v\right) = y$$

$$\frac{d}{dy}(vy) = y$$

$$d(vy) = y\,dy$$

$$\int d(vy) = \int y\,dy$$

$$uy = \frac{1}{2}y^2 + c_1$$

$$u(y) = \frac{1}{2}y + \frac{c_1}{y}$$

$$y' = \frac{1}{2}y + \frac{c_1}{y}$$

$$\frac{dy}{dx} = \frac{1}{2}y + \frac{c_1}{y} \Rightarrow dy = \left(\frac{1}{2}y + \frac{c_1}{y}\right)dx$$

$$dy = \frac{y^2 + c_2}{2y}\,dx$$

$$\frac{2y}{y^2 + c_2}\,dy = dx$$

$$\int \frac{2y}{y^2 + c_2}\,dy = \int dx$$

$$\ln(y^2 + c_2) = x + c_3$$

$$e^{\ln(y^2 + c_2)} = e^{x + c_3} = e^x \cdot e^{c_3}$$

$$y^2 + c_2 = Be^x \qquad \text{donde } A = -c_2$$

$$y^2 = A + Be^x \qquad B = e^{c_3}$$

$$y = +\sqrt{A + Be^x} \qquad \text{asumimos que } y > 0$$

50.

Resolver $y'' = (x+y')^2$

primero realizamos un cambio de variable

$$U(x) = x + \frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = 1 + \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{du}{dx} - 1 = y''$$

Así que $\quad y'' = (x+y')^2$

$$\frac{du}{dx} - 1 = u^2$$

$$\frac{du}{dx} = 1 + u^2$$

$$du = (1+u^2)\,dx$$

$$\frac{du}{1+u^2} = dx$$

$$\int \frac{du}{1+u^2} = \int dx$$

$$\arctan u = x + c_1$$

$$\Rightarrow U(x) = \tan(x+c_1)$$

$$x + \frac{dy}{dx} = \tan(x+c_1)$$

$$\frac{dy}{dx} = \tan(x+c_1) - x$$

$$dy = (\tan(x+c_1) - x)\,dx$$

$$\int dy = \int \tan(x+c_1) - x \; dx$$

$$y(x) = -\ln|\cos(x+c_1)| - \frac{x^2}{2} + c_2$$

$$y(x) = \ln\left|\frac{1}{\cos(x+c_1)}\right| - \frac{1}{2}x^2 + c_2$$

$$y(x) = \ln|\sec(x+c_1)| - \frac{1}{2}x^2 + c_2$$

$$\int \tan(x+c_1)\,dx$$

$$\int \frac{\operatorname{sen}(x+c_1)}{\cos(x+c_1)}\,dx$$

$$w = \cos(x+c_1)$$

$$dw = -\operatorname{sen}(x+c_1)\,dx$$

$$\int -\frac{dw}{w}$$

$$-\ln|w| + c_2$$

$$-\ln|\cos(x+c_1)| + c_2$$