Onbepaalde integraal

www.karelappeltans.be

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Contents

1 Rekenregel

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a) \text{ met } F' = f$$

We moeten dus op zoek naar een een primitieve functie $F(x)$ voor elke mogelijke functie $f(x)$. Deze zoektocht heet onbepaalde integralen oplossen. We noteren:

$$\int f(x)dx = F(x) + c$$

2 Basis integralen

Lijst met fundamentele (basis) integralen

$$\int 0dx = c$$

$$\int 1dx = x + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \; (n \in \mathbb{R}\backslash\{-1\})$$

$$\int \frac{1}{x} dx = ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = Arcsinx + c = -Arccosx + c$$

$$\int \frac{1}{1+x^2} dx = Arctanx + c$$

$$\int \frac{1}{k+x^2} dx = \frac{1}{\sqrt{k}} Arctan\left(\frac{x}{\sqrt{k}}\right) + c \;\; (k > 0)$$

$$\int \frac{1}{\sqrt{k-x^2}} dx = Arcsin\left(\frac{x}{\sqrt{k}}\right) + c \;\; (k > 0)$$

Figure 1: `https://www.geogebra.org/m/RMwPMmq7`

3 Splitsing

3.1 Voorbeelden

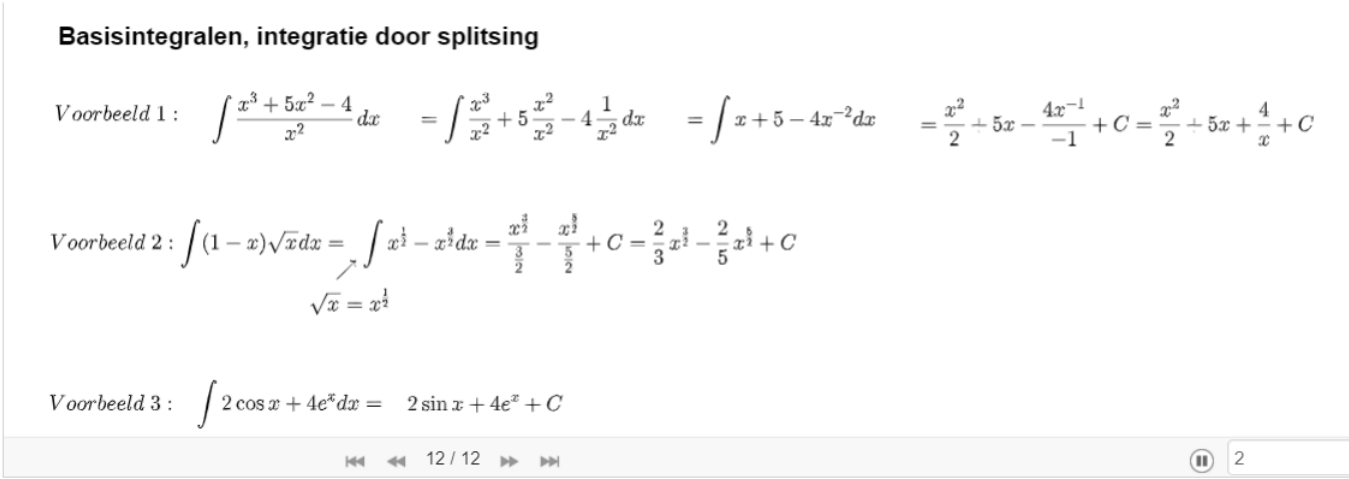


Figure 2: `https://www.geogebra.org/m/RMwPMmq7`

3.2 Oefeningen

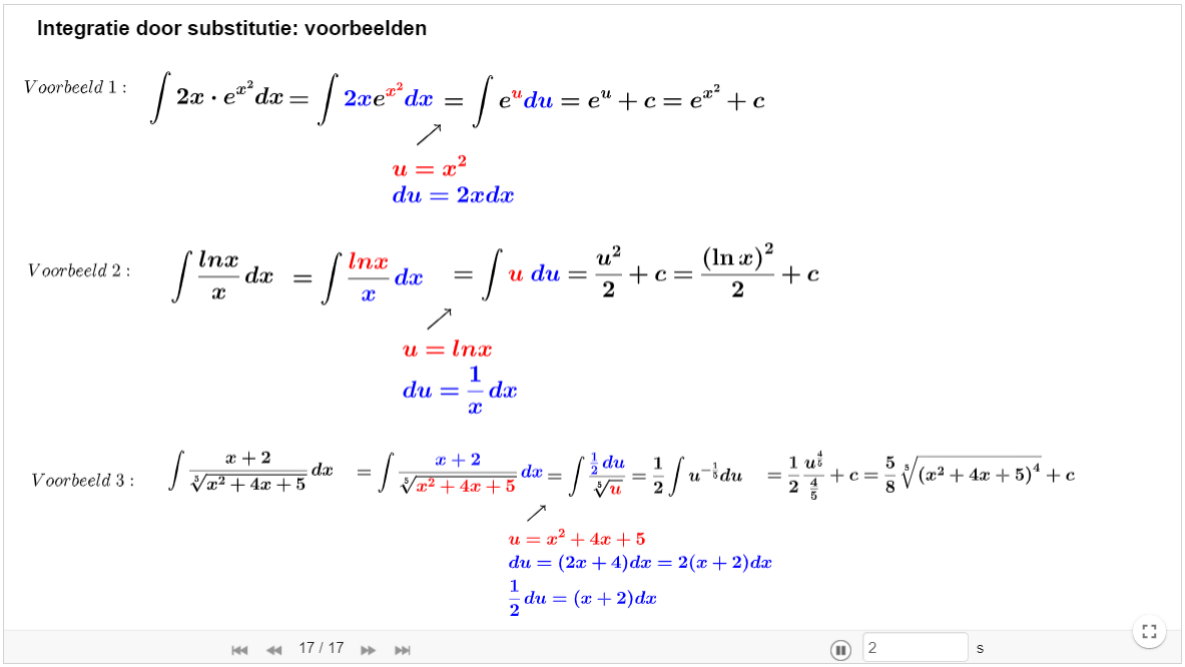
1. Los op:

1. $\int x^5 dx$
2. $\int x^{\frac{3}{4}} dx$
3. $\int \frac{1}{x^2} dx$
4. $\int 5dx$
5. $\int (x^{\frac{1}{2}} - 3x^{\frac{2}{3}} + 6)dx$
6. $\int \left(3\sqrt{x} - \frac{2}{x^3} + \frac{1}{x}\right) dx$
7. $\int \left(\frac{e^x}{2} + x\sqrt{x}\right) dx$
8. $\int \left(\sqrt{x^3} - \frac{1}{2\sqrt{x}} + \sqrt{2}\right) dx$
9. $\int \left(\frac{1}{3x} - \frac{3}{2x^2} + e^2 + \frac{\sqrt{x}}{2}\right) dx$
10. $\int \frac{x^2+2x+1}{x^2} dx$
11. $\int x^3 \left(2x + \frac{1}{x}\right) dx$
12. $\int \sqrt{x}(x^2 - 1)dx$
13. $\int x(2x+1)^2 dx$

2. los op:
   (a) $\int \cot^2 x dx$
   (b) $\int \frac{5^x}{3^{x-1}} dx$

4 Substitutie

4.1 Voorbeelden



Speciale substitutie

$$\int x\sqrt{x+3}dx$$

$$\left| \begin{array}{l} u = \sqrt{x+3} \quad \Leftrightarrow u^2 - 3 = x \\ du = \frac{1}{2\sqrt{x+3}} dx \quad \Leftrightarrow 2\sqrt{x+3}du = dx \Leftrightarrow 2udu = dx \end{array} \right.$$

$$\int (u^2-3) \cdot u \cdot 2udu = \int 2u^4 - 6u^2 du = \frac{2}{5}u^5 - 2u^3 + C = \frac{2}{5}\left(\sqrt{x+3}\right)^5 - 2\left(\sqrt{x+3}\right)^3 + C$$

$$\int \frac{x^2}{\sqrt{1-x}} dx$$

$$\left| \begin{array}{l} u = \sqrt{1-x} \Leftrightarrow x = 1 - u^2 \\ du = \frac{1}{2\sqrt{1-x}} dx \quad \Leftrightarrow 2udu = dx \end{array} \right.$$

$$\int \frac{(1-u^2)^2}{\cancel{x}} \cancel{2u}du = 2\int 1 - 2u^2 + u^4 du = 2\left(u - \frac{2}{3}u^3 + \frac{u^5}{5}\right) + C = 2\sqrt{1-x} - \frac{4}{3}(\sqrt{1-x})^3 + \frac{2(\sqrt{1-x})^5}{5} + C$$

Figure 3: `https://www.geogebra.org/m/vvnmeyj8`

4.2 grenzen aanpassen bij substitutie

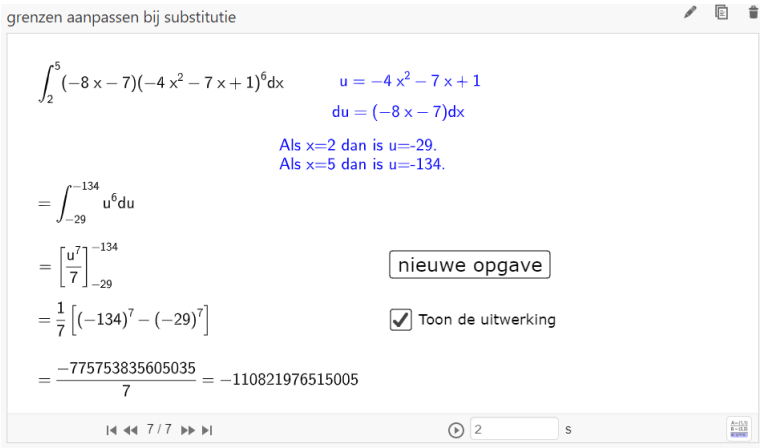


Figure 4: `https://www.geogebra.org/m/vvnmeyj8`

4.3 Oefeningen

1. Bereken volgende onbepaalde integralen

1. $\int e^{-4x}\,dx$
2. $\int_0^8 e^{-4x}\,dx$
3. $\int (x^3-2)\,(x^3-6x)^{207}\,dx$
4. $\int e^{\cos x}\sin x\,dx$
5. $\int_0^{\pi/2} e^{\cos x}\sin x\,dx$
6. $\int \frac{3x}{(x^2+1)^7}\,dx$
7. $\int \frac{12x^3}{3x^4+1}\,dx$
8. $\int (-3x+4)\,e^{-3x^2+8x}\,dx$
9. $\int_0^2 (-3x+4)\,e^{-3x^2+8x}\,dx$
10. $\int \frac{x+5}{x^2+1}\,dx$
11. $\int \frac{e^x}{e^x+1}\,dx$
12. $\int \cos x \sin^5 x\,dx$
13. $\int \frac{x}{\sqrt{x+1}}\,dx$
14. $\int_0^{15} \frac{x}{\sqrt{x+1}}\,dx$
15. $\int_0^3 \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$
16. $\int \frac{1}{\sqrt{x}\,(\sqrt{x}+1)}\,dx$
17. $\int_0^1 \frac{x+1}{x^2+1}\,dx$
18. $\int_0^1 xe^{-3x^2}\,dx$
19. $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$
20. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
21. $\int \frac{x^3}{(x^2+2)^2}\,dx$
22. $\int \frac{1}{x^2+9}\,dx$
23. $\int \frac{1}{a^2x^2+b^2}\,dx$
24. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$
25. $\int_0^{\pi/3} \sin 2x \sin x\,dx$
26. $\int 42\cos x \sin x\,(\sin x+1)^5\,dx$
27. $\int_e^{e^2} \frac{1}{x\ln\sqrt{x}}\,dx$
28. $\int \tan x\,dx$
29. $\int \sec x\,dx$
30. $\int \frac{x^2}{\sqrt{1-(x^3-1)^2}}\,dx$
31. $\int \frac{e^x}{e^{2x}+1}\,dx$
32. $\int \frac{1}{\sqrt{16-x^2}}\,dx$
33. $\int_4^5 \frac{1}{4x\ln x\,[\ln(\ln x)]^3}dx$
34. $\int_0^{\pi/3} \sin x\,(\cos x - \cos^2 x)\,dx$
35. $\int \sin^2 x\,dx$
36. $\int_{-1}^1 \sqrt{1-x^2}\,dx$
37. $2\int_{-r}^r \sqrt{r^2-x^2}\,dx$

2. Verklaar waarom de opp tussen x-as en de grafiek van de functie $f(x) = \frac{(\sqrt{x}-1)^2}{2\sqrt{x}}$ in het interval $[4, 9]$ en deze tussen de x-as en de grafiek van de functie $f(x) = x^2$ in het interval $[1, 2]$ aan elkaar gelijk zijn.

3. Over een continue functie $f : \mathbb{R} \to \mathbb{R}$ is gegeven dat $\int_0^1 f(x)dx = 8$, $\int_1^2 f(x)dx = 2$ en $\int_2^4 f(x)dx = 4$. Dan is $\int_0^2 f(2x)dx =$

4.4 oplossingen

1. Oplossingen oefening 1

1.) $-\frac{1}{4}e^{-4x} + C$ 2.) $\frac{1}{4} - \frac{1}{4}e^{-32}$ 3.) $\frac{1}{624}\left(x^3 - 6x\right)^{208} + C$ 4.) $-e^{\cos x} + C$ 5.) $e - 1$

6.) $-\frac{1}{4\left(x^2+1\right)^6} + C$ 7.) $\ln\left(3x^4 + 1\right) + C$ 8.) $\frac{1}{2}e^{-3x^2+8x} + C$ 9.) $\frac{1}{2}e^4 - \frac{1}{2}$

10.) $5\tan^{-1} x + \frac{1}{2}\ln\left(x^2+1\right) + C$ 11.) $\ln\left(e^x + 1\right) + C$ 12.) $\frac{1}{6}\sin^6 x + C$ 13.) $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$

14.) 36 15.) $\ln\left(e^3 + \frac{1}{e^3}\right) - \ln 2$ 16.) $2\ln\left(\sqrt{x}+1\right) + C$ 17.) $\frac{\pi}{4} + \frac{\ln 2}{2}$ 18.) $\frac{1}{6} - \frac{1}{6e^3}$

19.) $\frac{4}{3}\left(1+\sqrt{x}\right)\sqrt{1+\sqrt{x}} + C$ 20.) $-2\cos\sqrt{x} + C$ 21.) $\frac{1}{2}\ln\left(x^2+2\right) + \frac{1}{x^2+2} + C$ 22.) $\frac{1}{3}\tan^{-1}\frac{1}{3}x + C$

23.) $\frac{1}{ab}\tan^{-1}\left(\frac{a}{b}x\right) + C$ 24.) $\frac{1}{2}\left(\sin^{-1} x\right)^2 + C$ 25.) $\frac{\sqrt{3}}{4}$ 26.) $6(\sin x + 1)^7 - 7(\sin x + 1)^6 + C$

27.) $2\ln 2$ 28.) $\ln|\sec x| + C$ 29.) $\ln|\sec x + \tan x| + C$ 30.) $\frac{1}{3}\sin^{-1}\left(x^3 - 1\right) + C$

31.) $\tan^{-1}\left(e^x\right) + C$ 32.) $\sin^{-1}\left(\frac{x}{4}\right) + C$ 33.) $\frac{1}{8\ln^2(\ln 4)} - \frac{1}{8\ln^2(\ln 5)}$ 34.) $\frac{9}{64}$

35.) $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ 36.) $\frac{\pi}{2}$ 37.) πr^2

2. uitgewerkte voorbeelden

Figure 5: `https://www.geogebra.org/m/vvnmeyj8`

5 Partiële integratie

5.1 Voorbeelden

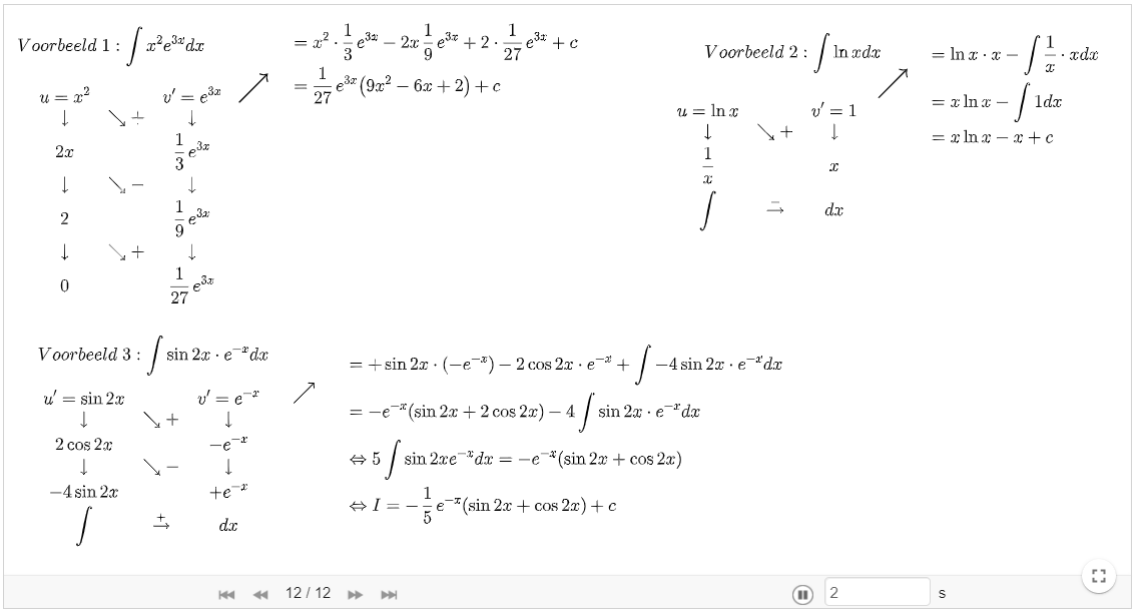


Figure 6: `https://www.geogebra.org/m/b4XpwQYH`

5.2 Oefeningen

1. Los op

1. $\int xe^{2x}\ dx$
2. $\int xe^{-3x}\ dx$
3. $\int_0^{\ln 2} xe^{-3x}\ dx$
4. $\int \cos^{-1} x\ dx$
5. $\int x2^x\ dx$
6. $\int x^2 2^x\ dx$
7. $\int x \cos x\ dx$
8. $\int x^2 \cos x\ dx$
9. $\int x \ln x\ dx$
10. $\int x^5 \ln x\ dx$
11. $\int x \sin 10x\ dx$
12. $\int_1^9 \frac{\ln x}{\sqrt{x}}\ dx$
13. $\int e^x \sin 2x\ dx$
14. $\int_0^{\pi/4} x \sin 2x\ dx$
15. $\int \frac{x^3}{(x^2+2)^2}\ dx$
16. $\int \frac{\ln x}{x^7}\ dx$
17. $\int e^{5x} \cos 3x\ dx$

5.3 Oplossingen

1. Oplossingen oef 1

1.) $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$ 2.) $-\frac{1}{9}e^{-3x} - \frac{1}{3}xe^{-3x} + C$ 3.) $\frac{7}{72} - \frac{1}{24}\ln 2$ 4.) $x\cos^{-1}x - \sqrt{1-x^2} + C$

5.) $\frac{2^x}{\ln 2}\left(x - \frac{1}{\ln 2}\right) + C$ 6.) $\frac{2^x}{\ln 2}\left(x^2 - \frac{2x}{\ln 2} + \frac{2}{\ln^2 2}\right) + C$ 7.) $x\sin x + \cos x + C$

8.) $x^2\sin x + 2x\cos x - 2\sin x + C$ 9.) $\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$ 10.) $\frac{1}{6}x^6\ln x - \frac{1}{36}x^6 + C$

11.) $\frac{1}{100}\sin 10x - \frac{1}{10}x\cos 10x + C$ 12.) $6\ln 9 - 8$ 13.) $\frac{1}{5}e^x\sin 2x - \frac{2}{5}e^x\cos 2x + C$ 14) $\frac{1}{4}$

15.) $\frac{1}{2}\ln\left(x^2+2\right) + \frac{1}{x^2+2} + C$ 16.) $-\frac{1}{36x^6} - \frac{1}{6x^6}\ln x + C$ 17.) $\frac{5}{34}\left(\cos 3x\right)e^{5x} + \frac{3}{34}\left(\sin 3x\right)e^{5x} + C$

2. Uitgewerkte oefeningen

Figure 7: `https://www.geogebra.org/m/b4XpwQYH`

6 Partieelbreuken

6.1 Voorbeelden

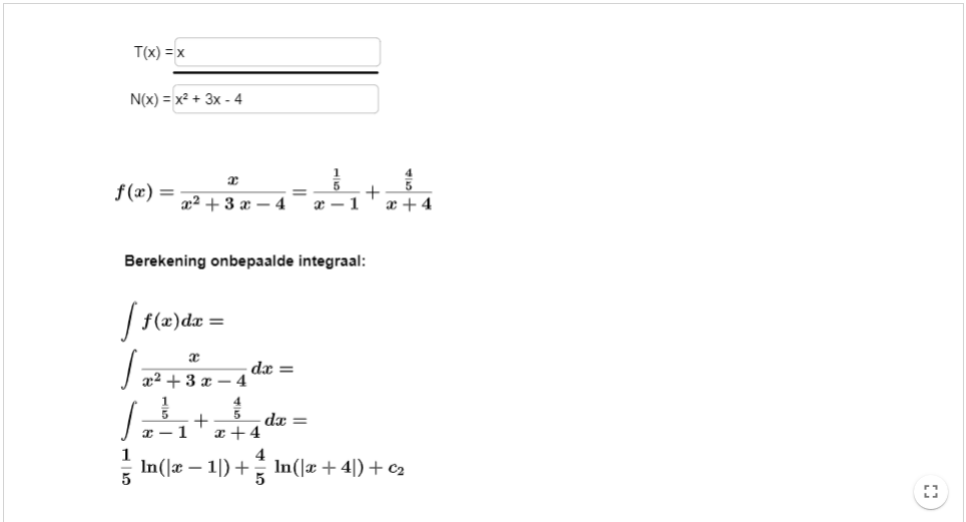


Figure 8: `https://www.geogebra.org/m/j5JTq5y8`

6.2 Oefeningen

1. Bereken:

1. $\int \frac{1}{x^2+3x}\,dx$

2. $\int \frac{x-5}{x^2-2x-8}\,dx$

3. $\int \frac{1}{x^2-a^2}\,dx$

4. $\int \frac{x-1}{x^2-4}\,dx$

5. $\int \frac{x-1}{x^2+4}\,dx$

6. $\int \frac{x^2}{x^2+2x-3}\,dx$

7. $\int \frac{2x^3-x^2-10x-4}{x^2-4}\,dx$

8. $\int \frac{5x-17}{x^2-6x+9}\,dx$

9. $\int \frac{2x^2+7x+3}{x^2+1}\,dx$

10. $\int \frac{2x^2-x+20}{(x-2)(x^2+9)}\,dx$

11. $\int \frac{x^4}{x^4-16}\,dx$

12. $\int \frac{2x+1}{x^2+1}\,dx$

13. $\int \frac{x^2+2}{x(x^2+6)}\,dx$

14. $\int \frac{-x+6}{(x+3)^2}\,dx$

15. $\int \frac{2x-3}{x^2+9}\,dx$

16. $\int \frac{x^2+2x-1}{x^3-x}\,dx$

6.3 Oplossingen

1. Oplossingen oef 1

1.) $\frac{1}{3}\ln|x| - \frac{1}{3}\ln|x+3| + C$ 2.) $\frac{7}{6}\ln|x+2| - \frac{1}{6}\ln|x-4| + C$ 3.) $\frac{1}{2a}\ln|x-a| - \frac{1}{2a}\ln|x+a| + C$

4.) $\frac{1}{4}\ln|x-2| + \frac{3}{4}\ln|x+2| + C$ 5.) $\frac{1}{2}\ln\left(x^2+4\right) - \frac{1}{2}\tan^{-1}\frac{1}{2}x + C$

6.) $x + \frac{1}{4}\ln|x-1| - \frac{9}{4}\ln|x+3| + C$ 7.) $x^2 - x - 3\ln|x-2| + \ln|x+2| + C$

8.) $5\ln|x-3| + \frac{2}{x-3} + C$ 9.) $2x + \frac{7}{2}\ln\left(x^2+1\right) + \tan^{-1}x + C$

10.) $2\ln|x-2| - \frac{1}{3}\tan^{-1}\frac{x}{3} + C$ 11.) $x + \frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| - \tan^{-1}\frac{x}{2} + C$

12.) $\tan^{-1}x + \ln\left(x^2+1\right) + C$ 13.) $\frac{1}{3}\ln\left|x^3+6x\right| + C$ 14.) $-\ln|x+3| - \frac{9}{x+3} + C$

15.) $\ln\left(x^2+9\right) - \tan^{-1}\frac{x}{3} + C$ 16.) $\ln|x| + \ln|x-1| - \ln|x+1| + C$

2. Uitgewerkte oefeningen

oefeningen partieelbreuken

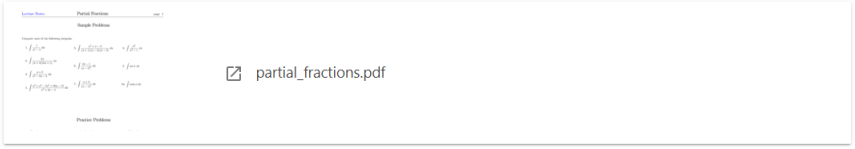


Figure 9: `https://www.geogebra.org/m/j5JTq5y8`

7 Alle technieken door elkaar

Alle integralen kan je stap voor stap laten berekenen via `https://www.integral-calculator.com/`

1–80 ■ Evaluate the integral.

1. $\int \frac{2x+5}{x-3}\,dx$
2. $\int e^{x+e^x}\,dx$
3. $\int \sin^2 x \cos^3 x\,dx$
4. $\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx$
5. $\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx$
6. $\int_1^2 x^3 \ln x\,dx$
7. $\int \frac{\sqrt{x-2}}{x+2}\,dx$
8. $\int \frac{x}{(x+2)^2}\,dx$
9. $\int \ln(1+x^2)\,dx$
10. $\int \frac{\sqrt{1+\ln x}}{x \ln x}\,dx$
11. $\int_0^1 (1+\sqrt{x})^8\,dx$
12. $\int_0^{\pi/4} \tan^3 x \sec^4 x\,dx$
13. $\int \frac{x}{x^2-2x+2}\,dx$
14. $\int x \sin^{-1} x\,dx$
15. $\int \frac{\sqrt{9-x^2}}{x}\,dx$
16. $\int \frac{x}{x^2+3x+2}\,dx$
17. $\int x^2 \cosh x\,dx$
18. $\int \frac{x^3+x+1}{x^4+2x^2+4x}\,dx$
19. $\int \frac{\cos x}{1+\sin^2 x}\,dx$
20. $\int \cos\sqrt{x}\,dx$
21. $\int_0^1 \cos \pi x \tan \pi x\,dx$
22. $\int \frac{e^{2x}}{1+e^x}\,dx$
23. $\int e^{3x} \cos 5x\,dx$
24. $\int \cos 3x \cos 5x\,dx$
25. $\int \frac{dx}{x^3+x^2+x+1}$
26. $\int x^2 \ln(1+x)\,dx$
27. $\int x^5 e^{-x^3}\,dx$
28. $\int \tan^2 4x\,dx$
29. $\int \frac{1}{\sqrt{9x^2+12x-5}}\,dx$
30. $\int x^2 \tan^{-1} x\,dx$
31. $\int \sqrt[3]{x}(1-\sqrt{x})\,dx$
32. $\int \frac{dx}{e^x - e^{-x}}$
33. $\int \frac{x}{x^4+2x^2+10}\,dx$
34. $\int \frac{1}{x+\sqrt[3]{x}}\,dx$
35. $\int \sin^2 x \cos^4 x\,dx$
36. $\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$
37. $\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx$
38. $\int \frac{1+\cos x}{\sin x}\,dx$
39. $\int \frac{e^x}{e^{2x}-1}\,dx$
40. $\int \frac{1}{x^3-8}\,dx$
41. $\int_{-1}^1 x^5 \cosh x\,dx$
42. $\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\sin x \cos x}\,dx$
43. $\int_{-3}^3 |x^3+x^2-2x|\,dx$
44. $\int_0^{\pi/4} \cos^5\theta\,d\theta$
45. $\int \cot x \ln(\sin x)\,dx$
46. $\int \frac{1+e^x}{1-e^x}\,dx$
47. $\int \frac{x}{(x^2+1)(x^2+4)}\,dx$
48. $\int \frac{dx}{4-5\sin x}$
49. $\int x\sqrt[3]{x+c}\,dx$
50. $\int e^{\sqrt[3]{x}}\,dx$
51. $\int \frac{1}{x+4+4\sqrt{x+1}}\,dx$
52. $\int \frac{x^3+1}{x^3-x^2}\,dx$
53. $\int (x^2+4x-3)\sin 2x\,dx$
54. $\int \sin x \cos(\cos x)\,dx$
55. $\int \frac{x}{\sqrt{16-x^4}}\,dx$
56. $\int \frac{x^3}{(x+1)^{10}}\,dx$
57. $\int \cot^3 2x \csc^3 2x\,dx$
58. $\int (x+\sin x)^2\,dx$
59. $\int \frac{e^{\arctan x}}{1+x^2}\,dx$
60. $\int \frac{dx}{x(x^4+1)}$
61. $\int t^3 e^{-2t}\,dt$
62. $\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt$
63. $\int \sin x \sin 2x \sin 3x\,dx$
64. $\int_1^3 |\ln(x/2)|\,dx$
65. $\int \sqrt{\frac{1+x}{1-x}}\,dx$
66. $\int \frac{x \ln x}{\sqrt{x^2-1}}\,dx$
67. $\int \frac{x+a}{x^2+a^2}\,dx$
68. $\int \sqrt{1+x-x^2}\,dx$
69. $\int \frac{x^4}{x^{10}+16}\,dx$
70. $\int \frac{x+2}{x^2+x+2}\,dx$
71. $\int x \sec x \tan x\,dx$
72. $\int \frac{x}{x^4-a^4}\,dx$
73. $\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$
74. $\int \frac{1}{1+2e^x-e^{-x}}\,dx$
75. $\int \frac{\arctan\sqrt{x}}{\sqrt{x}}\,dx$
76. $\int \frac{\ln(x+1)}{x^2}\,dx$
77. $\int \frac{1}{e^{3x}-e^x}\,dx$
78. $\int \frac{1+\cos^2 x}{1-\cos^2 x}\,dx$
79. $\int \frac{dx}{x\sqrt{2x-25}}$
80. $\int \frac{\sin 2x}{\sqrt{9-\cos^4 x}}\,dx$

Mat104 Fall 2002, Integration Problems From Old Exams

Compute the following integrals

(1) $\displaystyle\int \frac{\sin^5 x}{\cos x}\,dx$

(2) $\displaystyle\int \frac{dx}{(4+x^2)^{5/2}}$

(3) $\displaystyle\int \sin(\sqrt{1+x})\,dx$

(4) $\displaystyle\int \arctan(x)\,dx$

(5) $\displaystyle\int \cos^4 x\,dx$

(6) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{4-\sin^2 x}\,dx$

(7) $\displaystyle\int \frac{\ln(1+\ln x)}{x}\,dx$

(8) $\displaystyle\int x^2 \arctan x\,dx$

(9) $\displaystyle\int_{-1}^{2} \frac{dx}{(4+2x+x^2)^{5/2}}$

(10) $\displaystyle\int x\sin(x^2)e^{x^2}\,dx$

(11) $\displaystyle\int \frac{dx}{\sqrt{x^2+25}}$

(12) $\displaystyle\int \frac{2+x}{\sqrt[3]{x+2}+x}\,dx$

(13) $\displaystyle\int \frac{3x^2}{x^2+x-2}\,dx$

(14) $\displaystyle\int \frac{\cos\sqrt[3]{x}}{\sqrt[3]{x}}\,dx$

(15) $\displaystyle\int \frac{dx}{\sqrt{x^2+2x}}$

(16) $\displaystyle\int \frac{x^2+3x-3}{(x+1)(x^2+6x+10)}\,dx$

(17) $\displaystyle\int \frac{dx}{x\sqrt{1-x^2}}$

(18) $\displaystyle\int x^3 e^{x^2}\,dx$

(19) $\displaystyle\int x^2 \ln x\,dx$

(20) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

(21) $\displaystyle\int \tan^4\theta\,d\theta$

(22) $\displaystyle\int \frac{x+1}{x^2+4x+13}\,dx$

(23) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sin^2 x+5\sin x+6}\,dx$

(24) $\displaystyle\int \frac{e^{x/2}}{1+e^x}\,dx$

(25) $\displaystyle\int \frac{2x^2+5x+10}{x^3+2x^2+10x}\,dx$

(26) $\displaystyle\int (x-2)\sqrt{9-x^2}\,dx$

(27) $\displaystyle\int_1^{\sqrt{e}} \frac{\arcsin(\ln x)}{x}\,dx$

(28) $\displaystyle\int_0^1 xe^{-x}\,dx$

(29) $\displaystyle\int (\ln x)^2\,dx$

(30) $\displaystyle\int \frac{\sin x}{\sqrt{1+\cos x}}\,dx$

(31) $\displaystyle\int \frac{x^2}{x^6-1}\,dx$

(Hint: Try a substitution first.)

(32) $\displaystyle\int \sin^5 x\cos^2 x\,dx$

(33) $\displaystyle\int \frac{1+e^x}{1-e^x}\,dx$

(34) $\displaystyle\int_1^e \sin(\ln x)\,dx$

(35) $\displaystyle\int e^{\sqrt{x}}\,dx$

(36) $\displaystyle\int \frac{dx}{(4-x^2)^{3/2}}$

(37) $\displaystyle\int_2^3 \frac{e^{1/x}}{x^2}\,dx$

(38) $\displaystyle\int \frac{\sqrt{x^2-4}}{x^3}\,dx$

(39) $\displaystyle\int \frac{x-1}{x^3+x}\,dx$

(40) $\displaystyle\int \frac{dx}{x^2\sqrt{x^2+4}}$

(41) $\displaystyle\int \sin(\sqrt{x})\,dx$

(42) $\displaystyle\int \frac{dx}{x(1-x)^2}$

(43) $\displaystyle\int \frac{(x-5)(\sqrt{x-1}+3)}{\sqrt{x-1}+2}\,dx$

(44) $\displaystyle\int (2x+3)\ln x\,dx$

(45) $\displaystyle\int \frac{\sqrt{9+x^2}}{x^2}\,dx$

(46) $\displaystyle\int \frac{x}{(x^2+1)(x+1)}\,dx$

(47) $\displaystyle\int_0^1 (e^x+1)^{20}e^x\,dx$

(48) $\displaystyle\int \frac{x^2}{x^2+4x+5}\,dx$

(49) $\displaystyle\int \frac{x+1}{x^2+2x+3}\,dx$

Mat104 Fall 2002, Integration Problems From Old Exams

Warning: Many of these integrals can be done several different ways. If you choose a different method than I did, your answer may look quite different from the answer given here. The two different-looking answers may simply differ by a constant or perhaps they can be seen to be the same through the clever use of identities. If you believe that your answer is correct, but it does not match the one given here, consult your instructor! If you find errors, please let me know (jmjohnso@math.princeton.edu).

(1) $\cos^2 x - \dfrac{\cos^4 x}{4} + \ln|\sec x| + C$

(2) $\dfrac{1}{16}\dfrac{x}{\sqrt{4+x^2}} - \dfrac{1}{48}\cdot\dfrac{x^3}{(\sqrt{4+x^2})^3} + C$

(3) $-2\sqrt{1+x}\cos\sqrt{1+x} + 2\sin\sqrt{1+x} + C$

(4) $x\arctan x - \ln\sqrt{1+x^2} + C$

(5) $\dfrac{3x}{8} + \dfrac{\sin 2x}{4} + \dfrac{\sin 4x}{32} + C$ or, equivalently, $\dfrac{3x}{8} + \dfrac{\cos^3 x \sin x}{4} + \dfrac{3}{8}\cos x \sin x + C$

(6) $\dfrac{1}{4}\ln 3$

(7) $\ln(1+\ln x) + (\ln x)\ln(1+\ln x) - \ln x + C$

(8) $\dfrac{x^3\arctan x}{3} - \dfrac{x^2}{6} + \dfrac{\ln\sqrt{1+x^2}}{3} + C$

(9) $\dfrac{1}{8\sqrt{3}}$

(10) $\dfrac{e^{x^2}}{4}(\sin x^2 - \cos x^2) + C$

(11) $\ln|x + \sqrt{x^2+25}| + C$

(12) $u^3 - 3u + \dfrac{3}{4}\ln|u-1| + \dfrac{21}{8}\ln|u^2+u+2| + \dfrac{39}{4\sqrt{7}}\arctan\left(\dfrac{2u+1}{\sqrt{7}}\right) + C$ where $u = \sqrt[3]{x+2}$

(13) $3x - 4\ln|x+2| + \ln|x-1| + C$

(14) $3\sqrt[3]{x}\sin\sqrt[3]{x} + 3\cos\sqrt[3]{x} + C$

(15) Assume that $x > 0$ for simplicity. In that case, the answer is $\ln|x+1+\sqrt{x^2+2x}| + C$

(16) $\ln(x^2+6x+10) + \arctan(x+3) - \ln|x+1| + C$

(17) $-\ln\left|\dfrac{1}{x} + \dfrac{\sqrt{1-x^2}}{x}\right| + C$ (if we use $-\ln|\csc\theta + \cot\theta|$ as our antiderivative for $\csc\theta$.) Alternatively, we might use $\ln|\csc\theta - \cot\theta|$ as an antiderivative for $\csc\theta$ and this would give $\ln\left|\dfrac{1}{x} - \dfrac{\sqrt{1-x^2}}{x}\right| + C$ instead.

(18) $\frac{1}{2}(x^2e^{x^2} - e^{x^2}) + C.$

(19) $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

(20) $\frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2} + C$

(21) $\frac{\tan^3 \theta}{3} - \tan\theta + \theta + C$

(22) $\ln\sqrt{x^2+4x+13} - \frac{\arctan((x+2)/3)}{3} + C$

(23) $\ln(9/8)$

(24) $2\arctan(e^{x/2}) + C$

(25) $\ln|x| + \ln\sqrt{x^2+2x+10} + \frac{2}{3}\arctan\left(\frac{x+1}{3}\right) + C$

(26) $-\frac{(\sqrt{9-x^2})^3}{3} - 9\arcsin\left(\frac{x}{3}\right) - x\sqrt{9-x^2} + C$

(27) $\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$

(28) $1 - \frac{2}{e}$

(29) $x(\ln x)^2 - 2x\ln x + 2x + C$

(30) $-2\sqrt{1+\cos x} + C$

(31) $\frac{1}{6}\ln\left|\frac{x^3-1}{x^3+1}\right| + C$

(32) $\frac{2}{5}\cos^5 x - \frac{\cos^7 x}{7} - \frac{\cos^3 x}{3} + C$

(33) $x - 2\ln|1-e^x| + C$

(34) $\frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2}$

(35) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$

(36) $\frac{x}{4\sqrt{4-x^2}} + C$

(37) $\sqrt{e} - \sqrt[3]{e}$

(38) $\frac{\operatorname{arcsec}(x/2)}{4} - \frac{\sqrt{x^2-4}}{2x^2} + C$

(39) $-\ln|x| + \ln\sqrt{x^2+1} + \arctan x + C$

(40) $-\dfrac{1}{4}\cdot\dfrac{\sqrt{x^2+4}}{x} + C$

(41) $-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$

(42) $\ln|x| - \ln|x-1| - \dfrac{1}{x-1} + C$

(43) $\dfrac{(\sqrt{x-1})^4}{2} + \dfrac{2(\sqrt{x-1})^3}{3} - 6(\sqrt{x-1})^2 + C$

which can be simplified to $\dfrac{(x-1)^2}{2} + \dfrac{2}{3}(x-1)^{3/2} - 6(x-1) + C$

or, simplifying further, $\dfrac{x^2}{2} - 7x + \dfrac{2}{3}(x-1)^{3/2} + C$.

(44) $(x^2+3x)\ln x - \dfrac{x^2}{2} - 3x + C$

(45) $-\dfrac{\sqrt{9+x^2}}{x} + \ln\left|\dfrac{\sqrt{9+x^2}}{3} + \dfrac{x}{3}\right| + C$

(46) $\dfrac{1}{4}\ln(x^2+1) + \dfrac{1}{2}\arctan x - \dfrac{1}{2}\ln|x+1| + C$

(47) $\dfrac{(1+e)^{21}}{21} - \dfrac{2^{21}}{21}$

(48) $x - 2\ln(x^2+4x+5) + 3\arctan(x+2) + C$

(49) $\dfrac{1}{2}\ln(x^2+2x+3) + C$