Limiting Cases in Geometry

Objective:
Students will explore notions of limits in geometry.

Connections to Previous Learning:
Student should be familiar with domain and range, with substitution and with the properties of inscribed angles

Connections to AP*:
AP Calculus Topic: Limits

Materials:
Student Activity pages

Teacher Notes:
In geometry, students should have opportunities to explore the concepts of domain and range in the context of geometric relationships.

Limiting Cases in Geometry

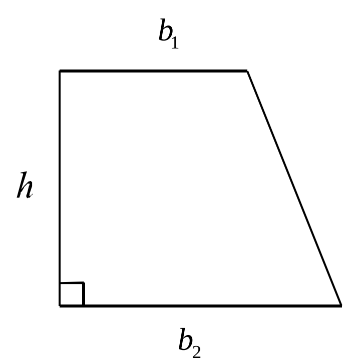
1. Consider a rectangle inscribed in a circle with a radius of *R*. What are the possible perimeters for the rectangle?

2. Consider a rectangle inscribed in a circle with a radius of *R*. What are the possible areas for the rectangle?

3. Consider a triangle inscribed in a semicircle with a radius of *R*. What are the possible perimeters for the triangle?

4. Consider a triangle inscribed in a semicircle with a radius of *R*. What are the possible areas for the triangle?

5. Given the area of the trapezoid is $A = \frac{1}{2}(b_1 + b_2) \cdot h$ complete the following



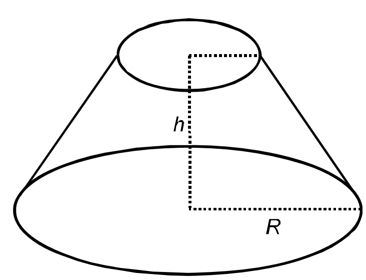
   a) Let $b_1 = b_2$. What figure results? Check the formula by substitution.

   b) Let $b_1 = 0$. What figure results? Check the formula by substitution.

   c) Let $b_2 = 0$. What figure results? Check the formula by substitution.

   d) Let $h = 0$. What figure results? Check the formula by substitution.

6. The lateral area of the frustrum of a cone is;
   $LA_{frustrum} = \pi(R + r)\sqrt{(R - r)^2 + h^2}$.
   Consider the following cases.

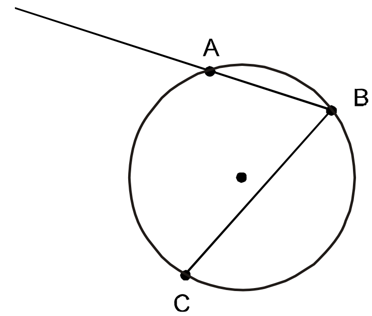


   a) Let *R* = *r*, describe the solid and the analogous lateral surface area and confirm the formula for this case.

   b) Let *r* = 0, describe the solid and the analogous lateral surface area and confirm the formula for this case.

   c) Let *h* = 0, describe this situation and the analogous lateral surface area and confirm the formula for this case.

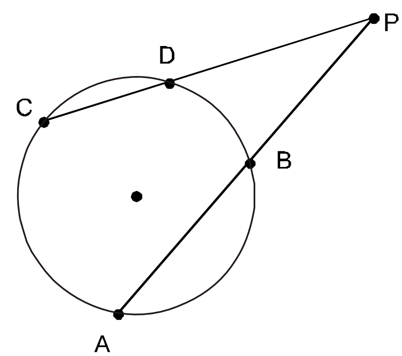
   d) Let *r* = *h* = 0, describe this situation and the analogous lateral surface area and confirm the formula for this case



7. The measure of an inscribed angle in a circle is half of the measure of the intercepted arc. Thus in the diagram is $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$, measure in degrees.

   a) Consider what happens when point *A* moves toward point *B*, what will happen when point *A* and *B* coincide? Fill in the blanks for the following statement. The measure of an angle formed by a chord and a _________________ is equal to _______________ the measure of the intercepted arc.

   b) Consider what happens when both points *A* and *C* move toward point *B*, what will happen when point *A* and *B* coincide? Can this situation make sense with respect to the theorem above? Explain.



8. Theorem; when two secants intersect in the exterior of a circle, the product of the lengths of one secant segment and its external segment equals the product of the length of the other secant segment and its external segment. Thus in the given diagram the equation (1) $PB \cdot PA = PD \cdot PC$ is true.

   a) Consider what happens when $\angle CPA$ increases by moving $\overleftrightarrow{CP}$. What will happen when points *C* and *D* coincide? Explain.

   b) Let *C* be the point where *C* and *D* coincide. Write a new equation using equation (1) for this case.

   c) Consider what happens when $\angle CPA$ increases by moving $\overleftrightarrow{CP}$ and $\overleftrightarrow{AP}$. What will happen when *C* coincides with *D* and *A* coincides with *B*?

   d) Let *C* be the point where *C* and *D* coincide. Let *A* be the point where *A* and *B* coincide. Write a new equation from equation (1) for this case.

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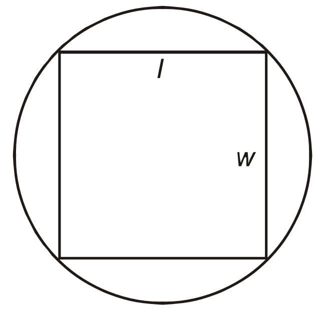
Answers:



1. $4R < P \le 4\sqrt{2} \cdot R$. Note as w approaches 0, P approaches $4R$, the largest perimeter occurs when $w = l$.

2. $0 < A \le 2R^2$. Note as w approaches 0 the Area approaches 0, the largest area occurs when $w = l$.

3. $4R < P \le (2 + 2\sqrt{2}) \cdot R$

4. $0 < A < R^2$

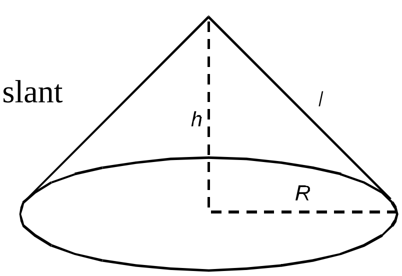
5. a) The resulting figure is a rectangle.
   $A = \frac{1}{2}(b_2 + b_2) \cdot h$ implies $A = b_2 \cdot h$

   b) The resulting figure is a right triangle with base b_2 and height h
   $A = \frac{1}{2}(0 + b_2) \cdot h$ implies $A = \frac{1}{2} b_2 \cdot h$

   c) The resulting figure is a right triangle with base b_1 and height h
   $A = \frac{1}{2}(b_1 + 0) \cdot h$ implies $A = \frac{1}{2} b_1 \cdot h$

   d) The resulting figure is a segment. $A = 0$

6. a) A cylinders lateral area; $LA = \pi(2r)h$



   b) A cones lateral area; $LA = \pi(R)\sqrt{(R)^2 + h^2}$ and since the slant height of the cone is l = $\sqrt{(R)^2 + h^2}$ the $LA = \pi(R) \cdot l$

   c) Concentric circles with the area bounded between them;
   $LA = \pi(R + r)\sqrt{(R - r)^2}$ and $LA = \pi R^2 - \pi r^2$

   d) The area of a circle; $LA = \pi(R)\sqrt{(R)^2} = \pi R^2$

7. a) When A and B coincide the segment will be tangent to the circle. …tangent…half

   b) $\overline{AB}$ and $\overline{BC}$ would both be tangent at the point B. In this case a tangent line would be formed at B. The line forms a straight angle at B (180°) which is half of the entire circle.

8. a) $\overleftrightarrow{CP}$ will be tangent to the circle.

   b) $PB \cdot PA = PC^2$

   c) $\overleftrightarrow{CP}$ and $\overleftrightarrow{AP}$ are both tangent to the circle

   d) $PA^2 = PC^2$

