Transforming Linear Algebra Education with GeoGebra Applets
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Lines and Planes - Vector Forms

Line - Vector Form

Objectives:

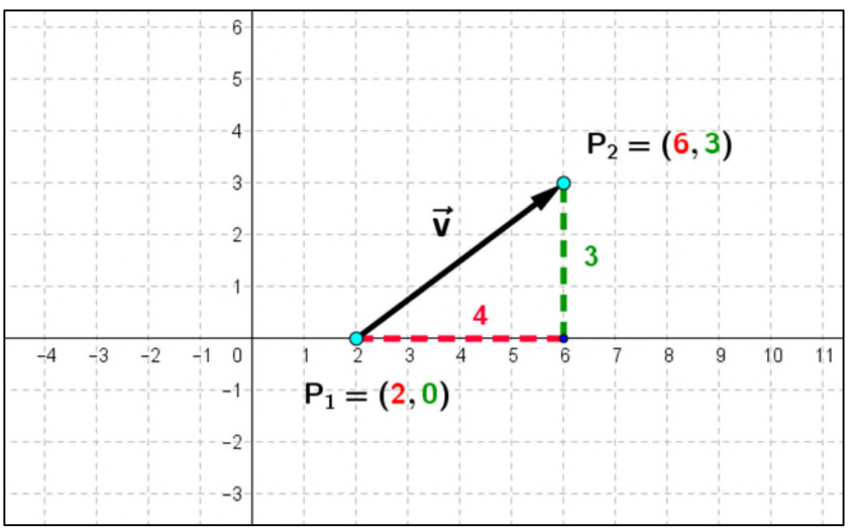
1. Convert a line in any given algebraic form into any other algebraic form.
2. Geometrically draw and label a line given in terms of its vector form.

Equations of a line in $\mathbb{R}^2$

Algebraic Forms: $\vec{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$ ***(called a coordinate vector) is the vector form of the point*** $\boldsymbol{p} = (p_1, p_2)$;

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ ***is the vector form of any point*** $\boldsymbol{x} = (x_1, x_2)$

General form	Vector form	Parametric form
$ax + by = c$	$\vec{x} = \vec{p} + t \cdot \vec{d}, \; -\infty < t < \infty$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + t \cdot \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$	$x_1 = p_1 + d_1 t$ $x_2 = p_2 + d_2 t$



Algebraically, state the vector

$\vec{\mathbf{v}} =$

Find the slope of the line $\boldsymbol{L}$ defined by the initial and terminal points of the vector $\vec{\mathbf{v}}$. $\boldsymbol{m} = \frac{\Delta y}{\Delta x} =$ _________

Find the General form $(ax + by = c)$ of the line $\boldsymbol{L}$.

Draw and label the line $\boldsymbol{L}$ on the graph.

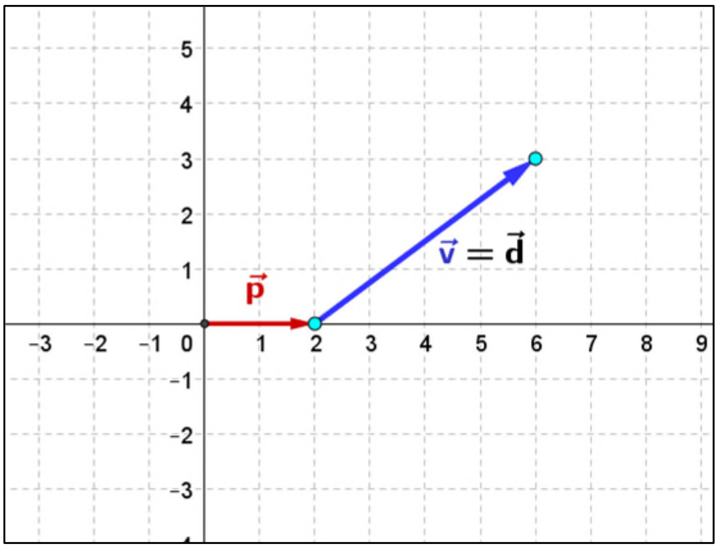
Consider the vector form of a line:

Vector form

$$\vec{x} = \vec{p} + t \cdot \vec{d}, \; -\infty < t < \infty$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + t \cdot \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

On the graph, the vector $\vec{\mathbf{p}} = \begin{bmatrix} \mathbf{2} \\ \mathbf{0} \end{bmatrix}$ is as shown below:



Define the vector form of the line $\boldsymbol{L}$ defined with $\vec{\mathbf{v}} = \vec{\mathbf{d}}$ as its direction vector in the graph

Draw and label the vector sum

$\vec{x} = \vec{\mathbf{p}} + \vec{\mathbf{d}}$ **Note:** $\boldsymbol{t = 1}$ in the vector form where $-\infty < \boldsymbol{t} < \infty$

For $\boldsymbol{t} = -\mathbf{1}$, Draw and label the : $\vec{x} = \vec{p} + \boldsymbol{t} \cdot \vec{d}, \; -\infty < \boldsymbol{t} < \infty$ Algebraically, perform the calculation of $\vec{x}$

On the graph, draw and label the line $\boldsymbol{L}$ as it is defined in terms of its vector form.

For the line $\boldsymbol{L}$, using its vectors form write its parametric form

Parametric form

$$x_1 = p_1 + d_1 t$$
$$x_2 = p_2 + d_2 t$$

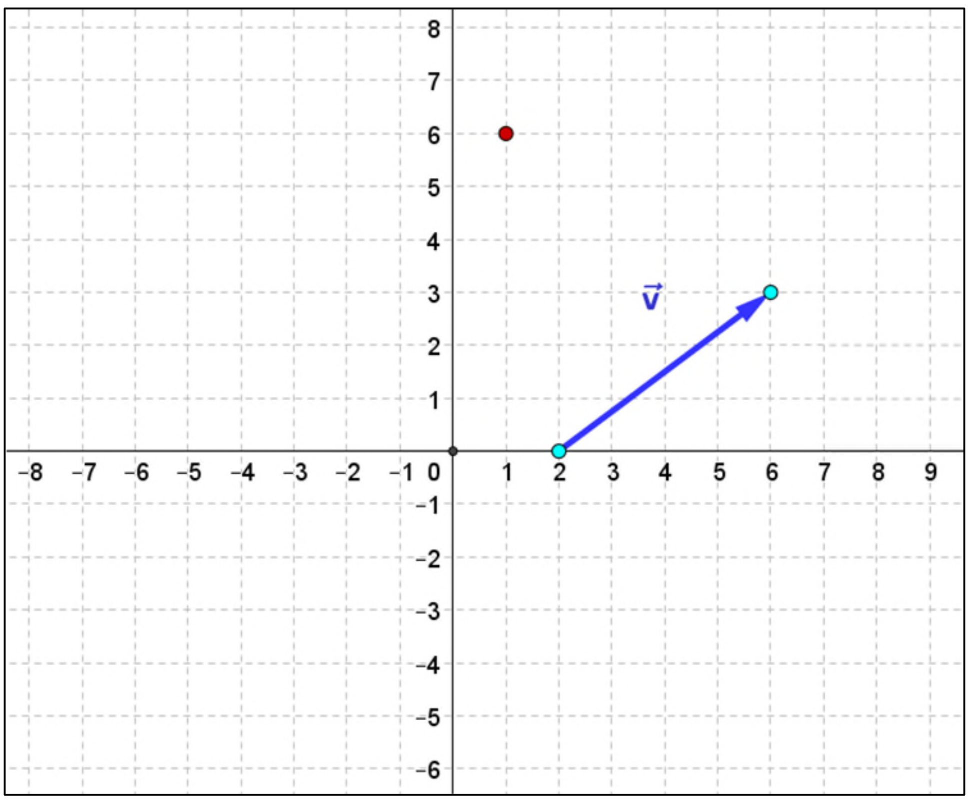
Find the slope of a line perpendicular to $\boldsymbol{L}$ written as $\boldsymbol{m}_{\perp} =$__________
(Look this up if you do not remember how to find it.)

Use the slope $\boldsymbol{m}_{\perp}$ to define the general form of the line $\boldsymbol{L}_{\perp}$ that contains the point (6, 3).

What can you say about the coefficients $\boldsymbol{a}, \boldsymbol{b}$ in the lines $\boldsymbol{L}$ *and* $\boldsymbol{L}_{\perp}$ when both lines are in the general form $\boldsymbol{ax} + \boldsymbol{by} = \boldsymbol{c}$?

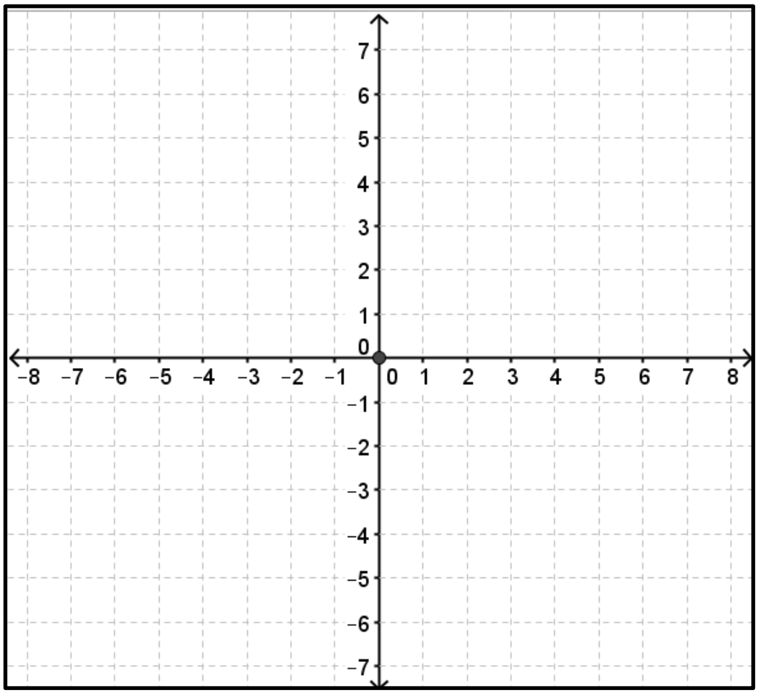
Using what you know about vectors and trigonometry, find the distance from the point **Q = (1, 6)** to the line ***L*.**

Draw and label a geometric illustration of your solution corresponding to your algebraic solution.

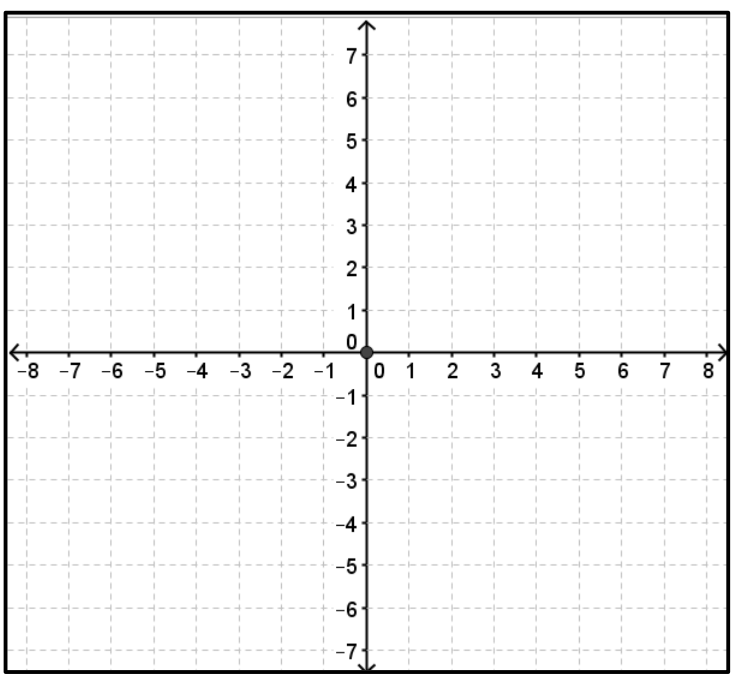


<u>Homework:</u> Show all your work. **Draw and label a geometric illustration of your solution corresponding to your algebraic solution** for the following problems.

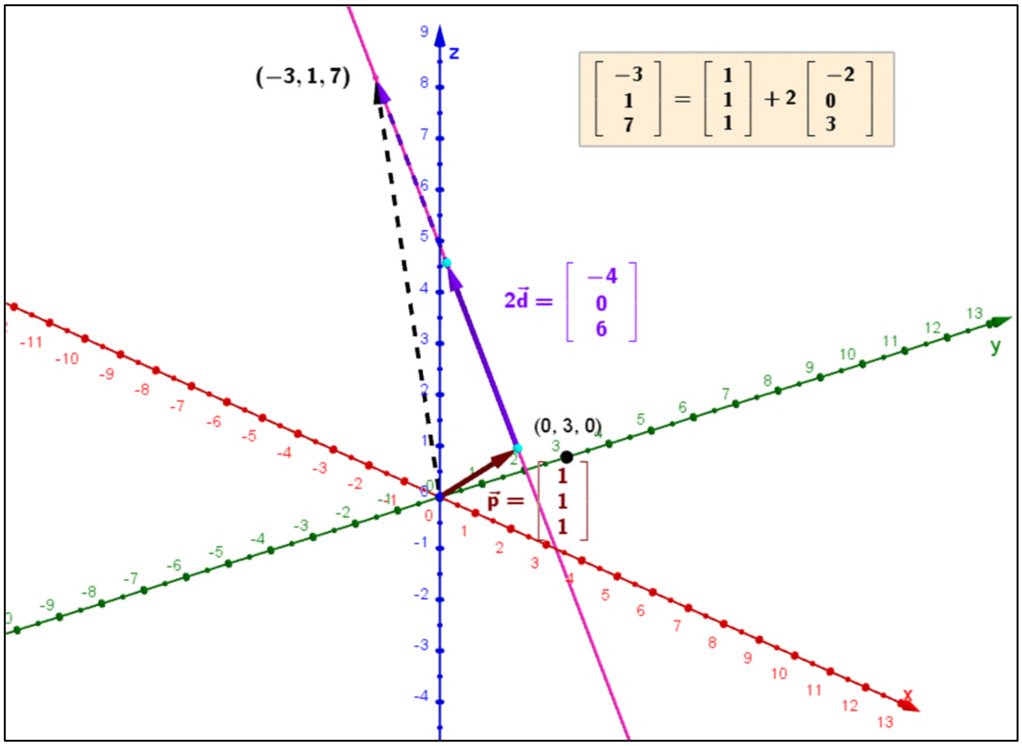
a. Find the distance (exact & approximate) from point **Q = (3, 4)** to line $\boldsymbol{L}$: $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \boldsymbol{t}\begin{bmatrix} \mathbf{2} \\ \mathbf{1} \end{bmatrix}, -\infty < \boldsymbol{t} < \infty$



b. Find the distance (exact & approximate) from point **Q = (-2, 5)** to line $\boldsymbol{L}$: $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{2} \\ \mathbf{1} \end{bmatrix} + \boldsymbol{t}\begin{bmatrix} \mathbf{3} \\ \mathbf{4} \end{bmatrix}, -\infty < \boldsymbol{t} < \infty$



c. Find the distance from the point **Q = (0, 3, 0)** to the line ***L***: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}, -\infty < t < \infty$



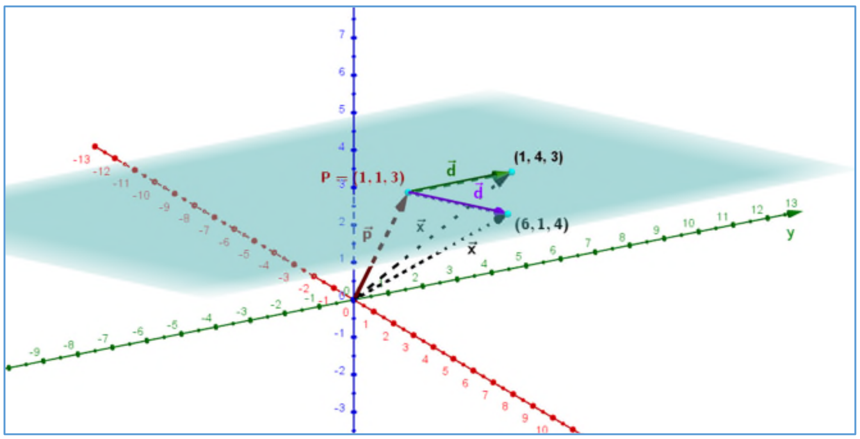
Plane – Vector Form

Objectives:

Understand how two vector forms of lines can define a plane .
Geometrically, draw and label a plane defined in terms of its vector form.

General form	Vector form	Parametric form
$ax + by + cz = d$	$\vec{x} = \vec{p} + s \cdot \vec{u} + t \cdot \vec{v}, \ -\infty < s, t < \infty$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + s \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + t \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$	$x_1 = p_1 + s \cdot u_1 + t \cdot v_1$ $x_2 = p_2 + s \cdot u_2 + t \cdot v_2$ $x_3 = p_3 + s \cdot u_3 + t \cdot v_3$

Given the direction vectors $\vec{d}$ and $\vec{d}$ whose initial point is **P = (1, 1, 3)** as shown below.



State the vectors $\vec{d}$, $\vec{d}$, and $\vec{p}$.

$\vec{d} =$

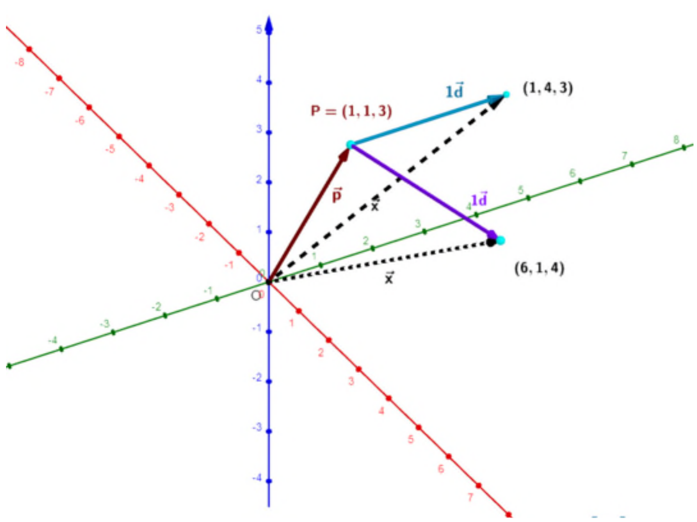
$\vec{d} =$

$\vec{p} =$

State the ***vector form of the line*** defined by $\vec{p}$ and $\vec{d}$. (Don't forget to state the range of the parameter for the line.)

Draw this line on the graph below

Label the line with coordinate numbers (… -3, -2, -1, 0, 1, 2, 3…}, where **…**
-2(means -2$\vec{d}$), **-1**(means -1$\vec{d}$), **0** (means 0$\vec{d}$), **1** (means 1$\vec{d}$), **2** (means 2$\vec{d}$), …

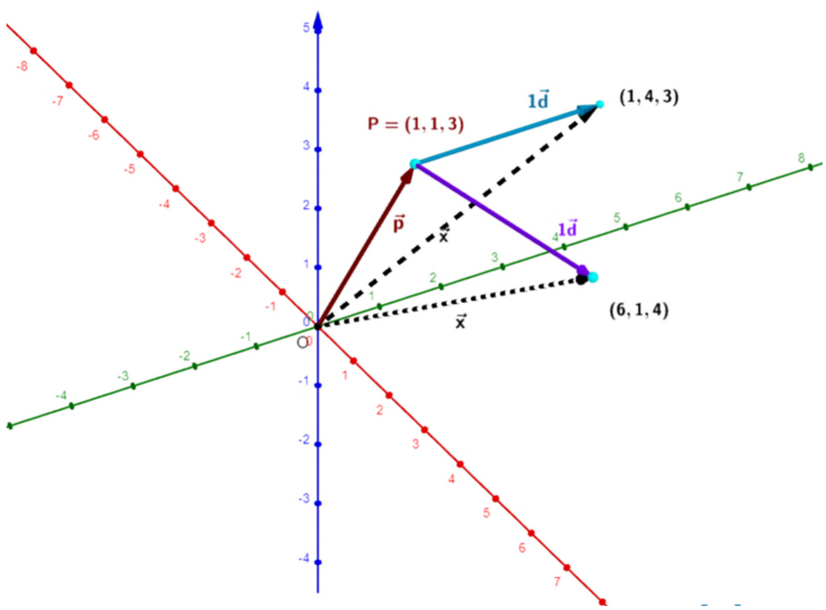


State the ***vector form of the line*** defined by $\vec{p}$ and $\vec{d}$. (Don't forget to state the range of the parameter for the line.)

Draw this line on the graph

Label this line with coordinate numbers (… -3, -2, -1, 0, 1, 2, 3…}, where **…**
-2(means -2$\vec{d}$), **-1**(means -1$\vec{d}$), **0** (means 0$\vec{d}$ **1** (means 1$\vec{d}$), **2** (means 2$\vec{d}$), …

Also, draw the previous line on the graph (labeling its coordinates)

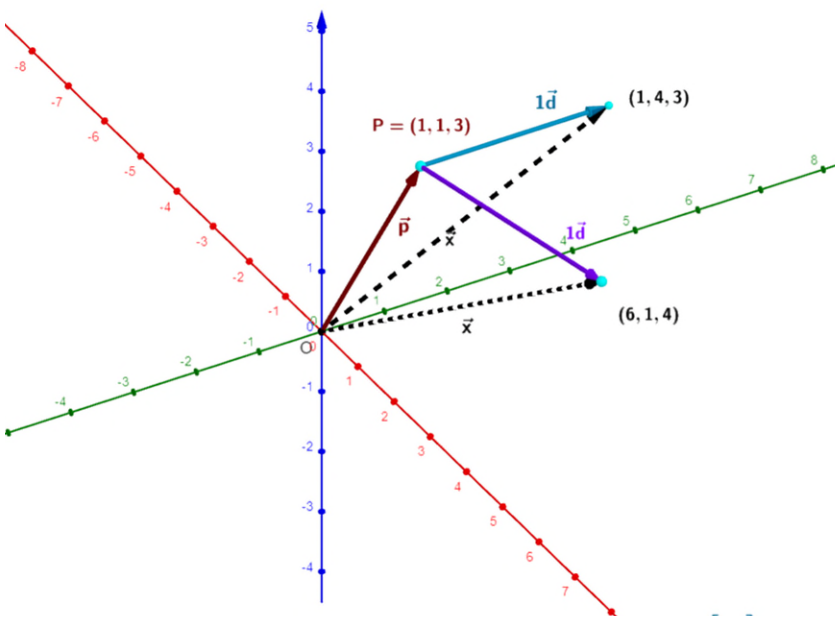


State the vector form of the plane defined by $\vec{d}$ and $\vec{d}$. Draw and label the coordinate numbers for both axes on the graph below. Draw and label the coordinate numbers for both axes on the graph below. Sketch and label the coordinate grid created from $\vec{d}$ and $\vec{d}$ which are the basis vectors for the plane. Lightly shade the plane created.

Vector form

$$\vec{x} = \vec{p} + s \cdot \vec{u} + t \cdot \vec{v}, \ -\infty < s, t < \infty$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} + s \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} + t \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



Plot the location $P = P_{\vec{d},\vec{d}} = (2, 1)$ on the plane. Write $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ as is a linear combination of the basis $\beta_{\vec{d},\vec{d}} = \{\vec{d}, \vec{d}\}$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix} =$

Create a matrix M where the first column is the basis vector $\vec{d}$ and the second column is the basis vector $\vec{d}$.

M =

Multiply M times the vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (i.e. the vector form of the point $P_{\vec{d},\vec{d}} = (2, 1)$)

$M * \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$

The result is the vector whose terminal point P is the <u>same location in the plane</u> as $P_{\vec{d},\vec{d}} = (\mathbf{2}, \mathbf{1})$ but defined by a different basis.

$$B_{\vec{e}_1,\vec{e}_2,\vec{e}_3} = \left\{ \vec{\boldsymbol{e}}_{\mathbf{1}} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \vec{\boldsymbol{e}}_{\mathbf{2}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix}, \; \vec{\boldsymbol{e}}_{\mathbf{3}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{bmatrix} \right\}$$

State the resulting vector resulting from M$*\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ as a linear combination representation of the basis vectors $B_{\vec{e}_1,\vec{e}_2,\vec{e}_3}$.

Using the new basis, state the point $P = P_{\vec{e}_1,\vec{e}_2,\vec{e}_3} =$