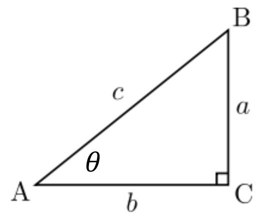
Vector Projection – Activities

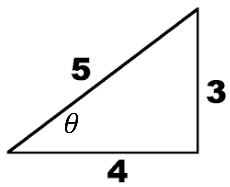
Objectives: Understand and create the projection vector $\boldsymbol{proj}_{\mathbf{u}}\,(\mathbf{v})$ where vector $\mathbf{v}$ is projected onto vector $\mathbf{u}$.
Find the distance from a point to a line using $\boldsymbol{proj}_{\mathbf{u}}\,(\mathbf{v})$.

Recall, from Trigonometry, for any right triangle

$$a^2 + b^2 = c^2, \quad \cos\theta = \frac{b}{c}\,, \text{ so } \boldsymbol{b} = c \cdot \cos\theta$$



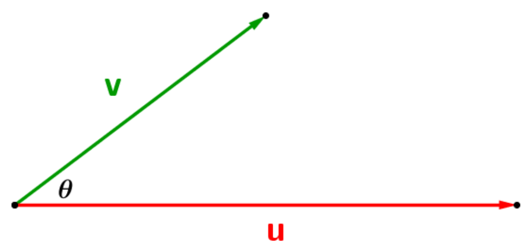
Example:



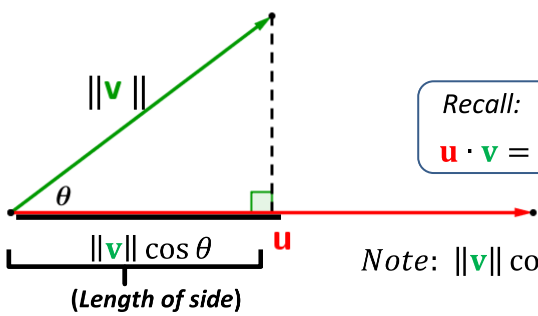
$$\cos\theta = \frac{4}{5}\,, \text{ so } \mathbf{4} = \mathbf{5} \cdot \cos\theta$$

Method to find the projection vector: $\boldsymbol{proj}_{\mathbf{u}}\,(\mathbf{v})$

Given two vectors **u** and **v.**



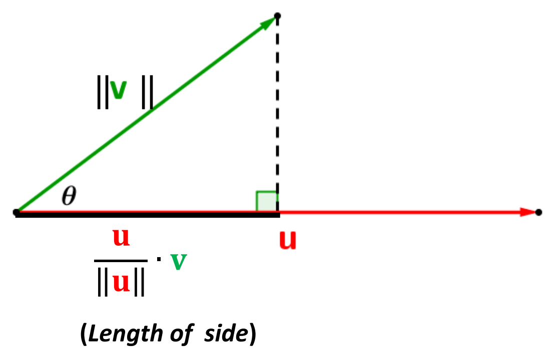
Using trigonometry



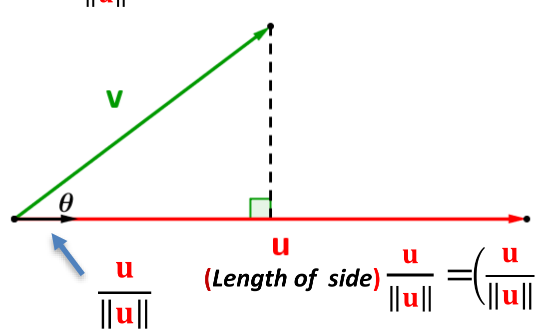
Recall:

$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos\theta$

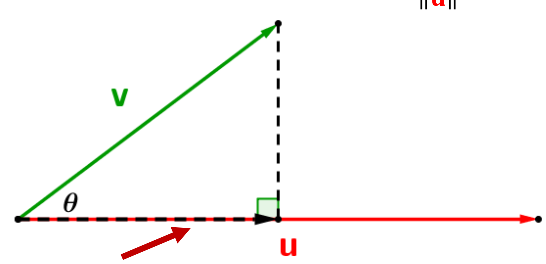
$$Note: \ \|\mathbf{v}\| \cos\theta = \frac{\mathbf{u}}{\|\mathbf{u}\|} \cdot \mathbf{v}$$



Recall: $\frac{\mathbf{u}}{\|\mathbf{u}\|}$ is the unit vector in the direction of $\mathbf{u}$. **Note:** One unit in the $\mathbf{u}$ direction is $\mathbf{1} \cdot \frac{\mathbf{u}}{\|\mathbf{u}\|}$.



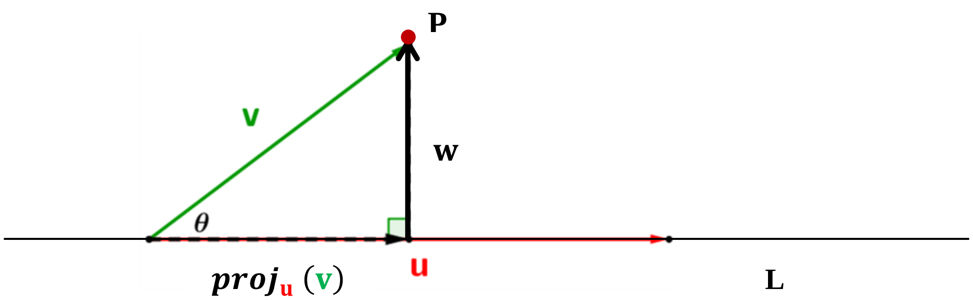
$$\textbf{(Length of side)}\frac{\mathbf{u}}{\|\mathbf{u}\|} = \left(\frac{\mathbf{u}}{\|\mathbf{u}\|} \cdot \mathbf{v}\right)\frac{\mathbf{u}}{\|\mathbf{u}\|} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \cdot \mathbf{u}}\right)\mathbf{u} = \boldsymbol{proj}_{\mathbf{u}}\,(\mathbf{v})$$



Note: We can use this knowledge to find the distance of a point **P** to a line **L**

Since $\boldsymbol{proj}_{\mathbf{u}}(\mathbf{v}) + \mathbf{w} = \mathbf{v} \;\rightarrow\; \mathbf{w} = \mathbf{v} - \boldsymbol{proj}_{\mathbf{u}}(\mathbf{v})$ So, $\|\mathbf{w}\| = \|\mathbf{v} - \boldsymbol{proj}_{\mathbf{u}}(\mathbf{v})\|$

which is the distance from point **P** to the line **L**

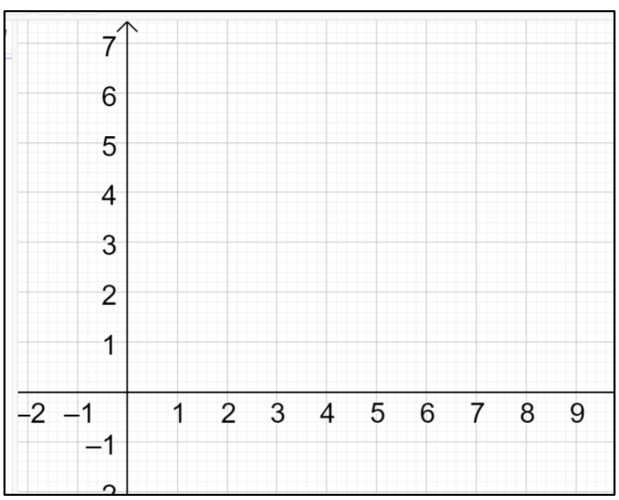


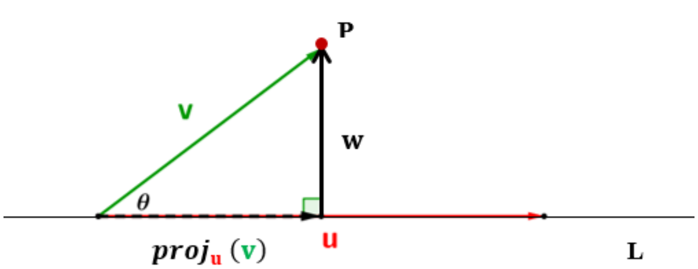
Using the above information, solve the following problems:

a. Let $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ Show all steps in finding the projection vector $\boldsymbol{proj}_{\mathbf{u}}(\mathbf{v})$

Draw and label all vectors.

Use the **Vector Projection** applet to verify your results.

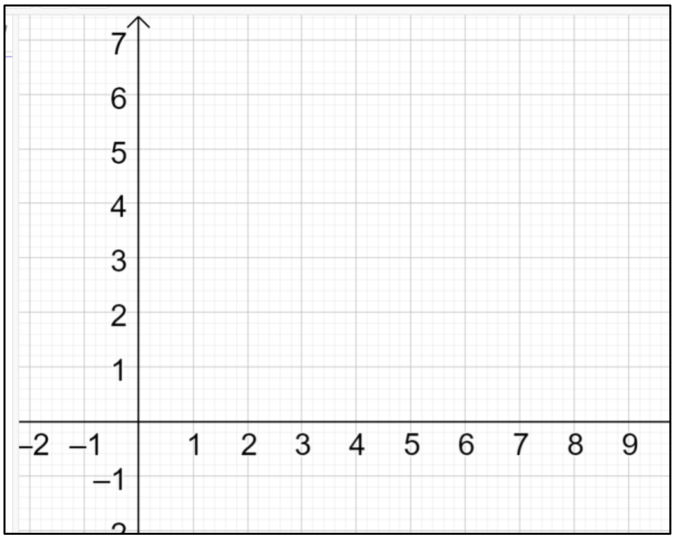


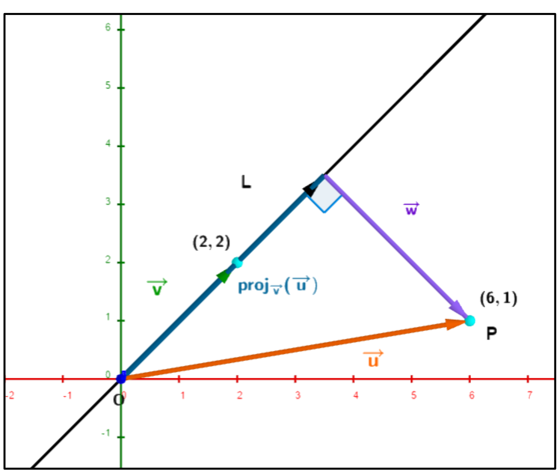


Find the distance between the point **P** and line **L**

b. Again, let $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ Show all steps in finding the projection vector $\boldsymbol{proj}_{\mathbf{v}}\,(\mathbf{u})$
Draw and label all vectors.

Use the **Vector Projection** applet to verify your results.
Note: To use this applet to do this change $\mathbf{u} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$



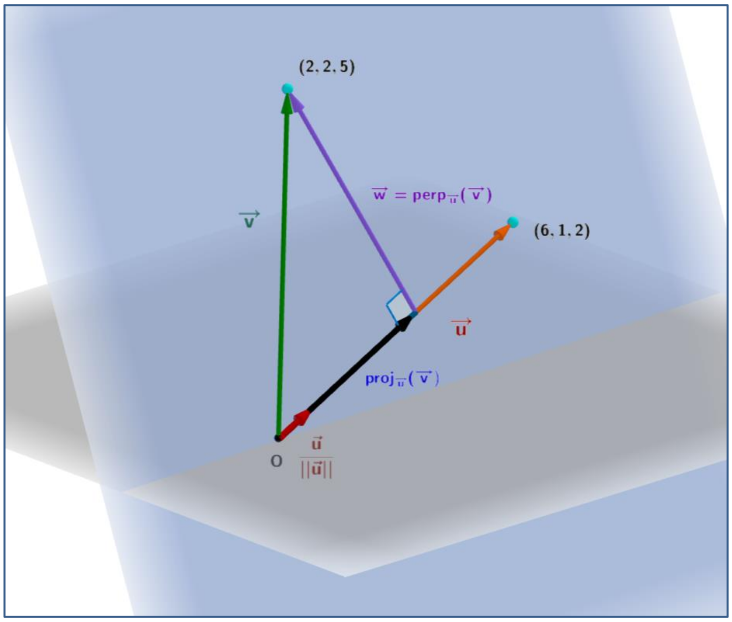


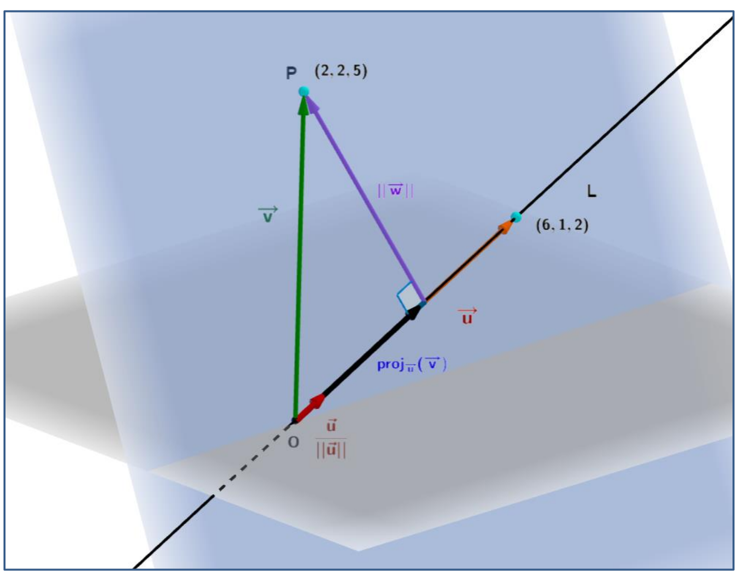
Find the distance between the point **P** and line **L**

A vector in any plane can be projected onto any other vector in that plane.

Let $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$ Show all steps in finding the projection vector $\boldsymbol{proj}_{\mathbf{u}}\,(\mathbf{v})$

Use the **Vector Projection** applet to verify your results.





Find the distance between the point **P** and line **L**