

Mechanics 2: Projectiles

Alan Windsor

In the questions below \mathbf{i} and \mathbf{j} are unit vectors in the vertical plane. \mathbf{i} is acting horizontally and \mathbf{j} vertically.

Take the numerical value for $g = 9.8\text{ms}^{-2}$.

Question 1 A ball is projected from A on level ground ($h=0$) with speed 24ms^{-1} . The ball is projected at an angle θ where $\sin \theta = \frac{4}{5}$. The ball moves freely under gravity until it strikes the ground at a point B, Find:

(a) the time of flight of the ball

(b) the distance from A to B.

[Answers a) 4.28 s 3sf, b) 120 m 3sf]

Question 2 A particle P is projected from the origin ($h=0$) with velocity $(15\mathbf{i} + 30\mathbf{j})\text{ms}^{-1}$, where \mathbf{i} is acting horizontally and \mathbf{j} vertically. The particle moves freely under gravity. Find:

(a) the position vector of P after 4 s

(b) the speed of P after 4 s

[Answers a) $(60\mathbf{i} + 41.5\mathbf{j})\text{m}$, b) 17.6ms^{-1} 3sf]

Question 3 A particle P is projected from a point O on a horizontal plane, ($h=0$) with speed 35ms^{-1} at an angle of elevation α . The particle just passes over a wall which is at a horizontal distance of 80 m from O and has a height of 35 m above the plane.

Find the two possible values of α , giving your answers to the nearest degree.

Hint:- you can use the formula $y = x \tan \alpha - \frac{gx^2}{2u^2}(1 + \tan^2 x)$

[Answers 61° and 52°]

Question 4 At time $t=0$, a particle P is projected from the top of a vertical cliff, 21 metres high, with a speed of 40ms^{-1} at an angle of elevation α , where $\tan\alpha = \frac{3}{4}$ with the horizontal, so that the particle rises above the height of the cliff.

- (a) Find the time taken for the particle to reach the ground, which is level with the base of the cliff.
- (b) Also at time $t=0$, another particle Q starts from rest from the base of the cliff, moves horizontally with constant acceleration, so as to intercept P at ground level. Find the acceleration required by the second particle.

[Answers a) 5.66 s b) 11.3ms^{-2} 3sf]

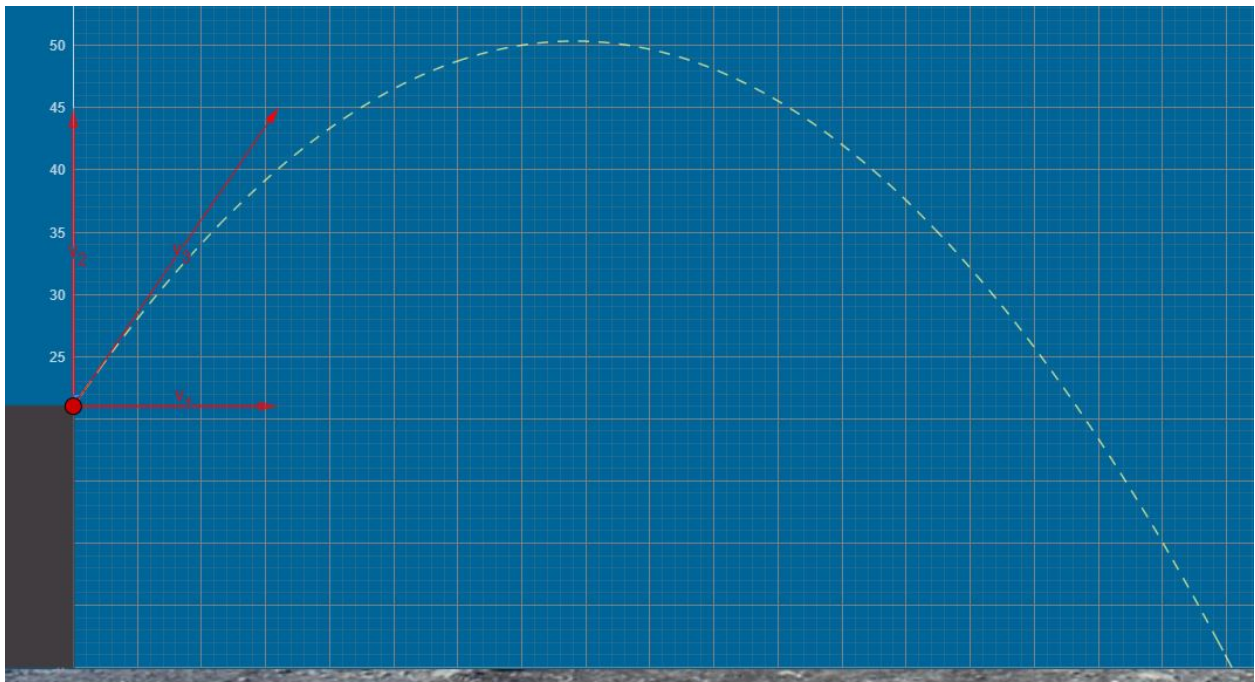


Figure 1: A Projectile from the top of a cliff.

Figure 1 shows the path of the projectile starting from the top of a cliff.

Question 5 A projectile is projected on the Earth from a point O on level ground (make $h=0$) with an initial velocity of $\begin{pmatrix} 20 \\ 28.8 \end{pmatrix} \text{ms}^{-1}$. The particle just clears a vertical wall which is at a horizontal distance of 49 m from O.

- (a) Given that $g = 9.8 \text{ms}^{-2}$ Show that the height of the wall is 41 m to the nearest m.
- (b) The particle is now projected on the Moon where $g=1.62 \text{ms}^{-1}$. Given that the horizontal component of the initial velocity is 9ms^{-1} , find the initial vertical component of the velocity so that the particle just clears the same wall as in part (a). Give your answer correct to the nearest whole number.
- (c) Compare and make comments on the initial velocity on the Earth with that on the Moon.

(a) [Answer Show that]

(b) [Answer 12ms^{-1} 2sf]

(c) [The gravitational acceleration on the surface of the Moon is 16.5 percent of the gravitation acceleration on the Earth. Also the speed of projection on the Moon is less than that on the Earth, 15ms^{-1} compared to 35.1ms^{-1}]

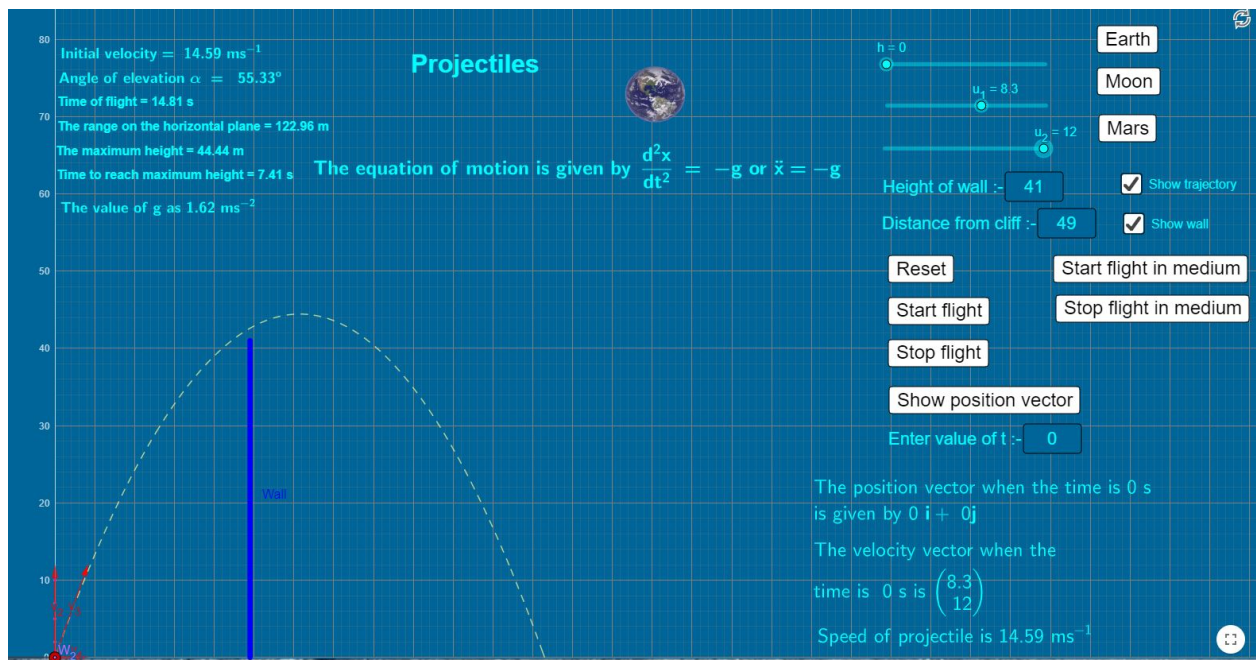


Figure 2: A Projectile on the Moon

Figure 2 shows the path of the particle on the moon.