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SECCION 1.5 HT2

27)

27) $(x + ye^y)\dfrac{dy}{dx} = 1$

$$x + ye^y = \frac{dx}{dy}$$

$$ye^y = \frac{dx}{dy} - 1 \cdot x$$

$$\frac{dx}{dy} - 1 \cdot x = ye^y$$

$$e^{-y}\left(\frac{dx}{dy} - 1 \cdot x\right) = e^{-y}(ye^y)$$

$$\frac{d}{dy}(e^{-y} \cdot x) = y$$

$$\int \frac{d}{dy}(e^{-y} \cdot x)\, dy = \int y\, dy$$

$$e^{-y} x = \frac{y^2}{2} + c$$

$$P(y) = e^{\int P(y)\,dy}$$

$$P(y) = e^{\int -1\,dy}$$

$$P(y) = e^{-y}$$

$$x(y) = e^y\left(\frac{1}{2}y^2 + c\right)$$

$$= \frac{1}{2}y^2 e^y + ce^y$$

30) 29)

30) $2x\frac{dy}{dx} = y\ 2x\cos x \ ; \quad y(1)=0$

$2x\frac{dy}{dx} - y = 2x(\cos x)$

$\frac{dy}{dx} - \frac{1}{2x}y = \cos x$

$x^{-1/2}\left[\frac{dy}{dx} - \frac{1}{2x}y\right] = x^{-1/2}\cos x$

$\frac{d}{dx}\left(x^{-1/2}y\right) = x^{-1/2}\cos x$

$d\left(x^{-1/2}y\right) = x^{-1/2}\cos x\, dx$

$\int d\left(x^{-1/2}y\right) = \int_1^x u^{-1/2}\cos u\, du + C$

$P(x) = e^{\int_1^x P(u)\,du}$

$P(x) = e^{\int_1^x \frac{-1}{2u}du}$

$P(x) = e^{[-1/2 \ln u]_1^x}$

$P(x) = e^{-1/2\ln x + 1/2\cancel{\ln 1}^{\,0}}$

$P(x) = e^{\ln x^{-1/2}} = x^{-1/2}$

$y(1) = 0: \ (1)^{-1/2}(0) = \int_1^1 u^{-1/2}\cos u\, du + C$

$0 = 0 + C$

$0 = C$

$\rightarrow y(x) = x^{1/2}\int_1^x u^{-1/2}\cos u\, du$

29) $\frac{dy}{dx} = 1 + 2xy$

$\frac{dy}{dx} - 2x\cdot y = 1$

$e^{-x^2}\left(\frac{dy}{dx} - 2x\cdot y\right) = e^{-x^2}$

$\frac{d}{dx}\left(e^{-x^2}y\right) = e^{-x^2}$

$d\left(e^{-x^2}y\right) = e^{-x^2}dx$

$\int d\left(e^{-t^2}y\right) = \int_0^x e^{-t^2}dt$

$$e^{-x^2}(y) = \int_0^x e^{-t^2}\,dt + C$$

$$y(x) = e^{x^2}\left(\int_0^x e^{-t^2}\,dt + C\right)$$

debemos reescribir $y(x)$ en términos de

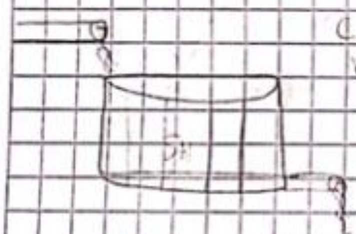
$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

$$y(x) = e^{x^2}\left(\frac{\sqrt{\pi}}{2}\cdot\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt + C\right)$$

$$y(x) = e^{x^2}\left(\frac{\sqrt{\pi}}{2}\operatorname{erf}(x) + C\right)$$

MODELO DE MEZCLAS

$V(0) = V_0 \rightarrow V(t) = V_0 + (r_e - r_s)t$



C_e: lb/gal

r_e: gal/min

$X(0) = X_0$

C_s: $\dfrac{X(t)}{V(t)}$

r_s: gal/min

$$\frac{dX(t)}{d(t)} = C_e r_e - C_s r_s$$

$$\frac{dx}{dt} = C_e r_e - \frac{X(t)}{V(t)} r_s$$

$$\frac{dx}{dt} = C_e r_e - \frac{r_s X(t)}{V_0 + (r_e - r_s)t}$$

36)

36) 60 gal de H_2O pura :

$V(0) = 60$ gal

$x(0) = 0$ Lb

$C_e = 1 \frac{lb}{gal}$

$r_e = 2 \frac{gal}{min}$

$r_s = 3 \frac{gal}{min}$

$V(60) = 0$

$P(t) = e^{\int \frac{3}{60-t} dt}$

$P(t) = e^{3\ln|60-t|} \quad t < 60$

$P(t) = e^{-3\ln(60-t)}$

$P(t) = e^{\ln(60-t)^{-3}} = (60-t)^{-3}$

a) Debemos hallar $x(t)$:

$$\frac{dx}{dt} = C_e\, r_e - r_s \frac{x}{V_0 + (r_e - r_s)t}$$

$$\frac{dx}{dt} = (1)(2) - \frac{(3)x}{60 - 1t}$$

$$\frac{dx}{dt} = 2 - \frac{3}{60-t} x$$

$$\frac{dx}{dt} + \frac{3}{60-t} x = 2$$

$$(60-t)^{-3}\left[\frac{dx}{dt} + \frac{3}{60-t} x\right] = (60-t)^{-3}(2)$$

$$\frac{d}{dt}\left[(60-t)^{-3} x\right] = 2(60-t)^{-3}$$

$$\int \frac{d}{dt}\left[(60-t)^{-3} x\right] dt = \int 2(60-t)^{-3}\, dt$$

$$(60-t)^{-3} x = \frac{1}{(60-t)^2} + C$$

$$x(t) = \frac{(60-t)^2}{(60-t)^2} + c\,(60-t)^3$$

$$x(t) = 60 - t + c\,(60-t)^3$$

como $x(0) = 0$

$$(60-0) + c\,(60-0)^3 = 0$$

$$60 + c\,60^3 = 0$$

$$c = -\frac{1}{3600}$$

$$x(t) = (60-t) - \frac{1}{3600}(60-t)^3$$ cantidad de sal presente en el depósito en todo instante $t \in [0, 60]$ min.

b) hallamos el máximo de $x(t)$ en $t \in [0, 60]$

$$x'(t) = -1 - \frac{3}{3600}(60-t)^2(-1)$$

$$x'(t) = -1 + \frac{1}{1200}(60-t)^2 = 0$$

$$\frac{1}{1200}(60-t)^2 = 1$$

$$(60-t)^2 = 1200$$

$$60 - t = \pm 20\sqrt{3}$$

$60 - t = 20\sqrt{3}$ $\lor$ $60 - t = -20\sqrt{3}$

$60 - 20\sqrt{3} = t$ $\lor$ $60 + 20\sqrt{3} = t$

$t \approx 25.36$ min $\lor$ $t \approx 94.64$ min

$t \notin [0, 60]$ min

38)

$x(0) = 0\ lb$

$x(25{,}36) \approx 23{,}09\ lb = x_{max}$

$x(60) \approx 0.16$

38) $V_1(0) = 100\ gal$

$V_2(0) = 100\ gal$

$x(0) = 50\ lb$

$y(0) = 50\ lb$

$re_1 = rs_1 = 5\ \frac{gal}{min}$

$rs_2 = 5\ \frac{gal}{min}$

$ce_1 = 0$

$re_2 = rs_1 \qquad ce_2 = cs_1$

a) TANQUE A:

$$\frac{dx}{dt} = ce_1 \cdot re_1 - rs_1 \cdot cs_1$$

$$\frac{dx}{dt} = (0)(5) - (5)\frac{x(t)}{V_1(t)}$$

$$\frac{dx}{dt} = -5\ \frac{x}{V_2(0) + (re_1 - rs_1)t} \quad \text{como: } re_1 = rs$$

$$\frac{dx}{dt} = -\frac{5x}{100}$$

$$\frac{dx}{x} = -\frac{5}{100}\,dt$$

$$\int \frac{dx}{x} = \int -\frac{1}{20}\,dt$$

$$\ln x = -\frac{1}{20}t + C_1$$

$$x(0) = 50$$

$$\ln 50 = -\frac{1}{20}(0) + C_1$$

$$\ln x = -\frac{1}{20}t + \ln 50 \qquad \ln 50 = C_1$$

$$e^{\ln x} = e^{-\frac{1}{20}t + \ln 50} = e^{-\frac{1}{20}t}\, e^{\ln 50}$$

$$x(t) = 50\, e^{-\frac{1}{20}t}$$

b) TANQUE B:

$$\frac{dy}{dt} = Ce_2\, re_2 - Cs_2\, rs_2$$

$$\frac{dy}{dt} = \frac{x(t)}{V_1(t)}(5) - \frac{y(t)}{V_2(t)}(5)$$

$$\underset{Cs_1}{\uparrow} \qquad \underset{rs_1}{\uparrow}$$

$$\frac{dy}{dt} = \frac{x}{V_1(0) + \cancel{(re_1 - rs_1)t}}\, 5 - \frac{y}{V_2(0) + \cancel{(re_1 - rs_1)}t}\, 5$$

$$\frac{dy}{dt} = \frac{5x}{100} - \frac{5y}{200}$$

$$\rightarrow \frac{dy}{dt} = \frac{5}{100}\left(50 e^{-\frac{1}{20}t}\right) - \frac{5}{200}y$$

$$\frac{dy}{dt} + \frac{1}{40}y = \frac{5}{2}e^{-\frac{1}{20}t}$$

$$e^{\frac{1}{40}t}\left[\frac{dy}{dt} + \frac{1}{40}y\right] = e^{\frac{1}{40}t}\frac{5}{2}e^{-\frac{1}{20}t}$$

$$\frac{d}{dt}\left(e^{\frac{1}{40}t}y\right) = \frac{5}{2}e^{-\frac{1}{40}t}$$

$$\int \frac{dy}{dt}\left(e^{\frac{1}{40}t}y\right)dt = \int \frac{5}{2}e^{-\frac{1}{40}t}dt$$

$$e^{\frac{1}{40}t}y = -100\, e^{-\frac{1}{40}t} + C_2$$

$$y(t) = e^{-\frac{1}{40}t}\left(-100\, e^{-\frac{1}{40}t} + c_2\right)$$

$$y(t) = -100\, e^{-\frac{1}{20}t} + c_2\, e^{-\frac{1}{40}t}$$

y como $y(0) = 50$

$$-100\, e^{0} + c_2\, e^{0} = 50$$

$$c_2 = 150$$

$$y(t) = -100\, e^{-\frac{1}{20}t} + 150\, e^{-\frac{1}{40}t}$$

c) $y'(t) = -100\, e^{-\frac{1}{20}t}\left(-\frac{1}{20}\right) + 150\, e^{-\frac{1}{40}t}\left(-\frac{1}{40}\right)$

$$y'(t) = 5\, e^{-1/20t} - \frac{15}{4}\, e^{-1/40t} = 0$$

$$5\, e^{-1/20t} = \frac{15}{4}\, e^{-1/40t}$$

$$\frac{20}{15} = e^{-1/40t}\, e^{1/20t}$$

$$\frac{20}{15} = e^{1/40t}$$

$$\ln\left(\frac{4}{3}\right) = \ln e^{1/4t}$$

$$\ln\left(\frac{4}{3}\right) = \frac{1}{40}t$$

$$40 \ln\left(\frac{4}{3}\right) = t$$

$$11.5\,\text{min} = t$$

$$y(t) = -100\, e^{-1/20t} + 150\, e^{-1/40t}$$

$y\ max \approx 56,25\ l_b$

cuando $t \approx 11,5$ min

44)

$$y' = x + y$$

$$\frac{dy}{dx} - y = x \quad ; \quad \rho(x) = e^{\int p(x)dx}$$

$$\rho(x) = e^{\int -1 dx}$$

$$e^{-x}\left(\frac{dy}{dx} - y\right) = xe^{-x} \qquad \rho(x) = e^{-x}$$

$$\frac{d}{dx}(e^{-x}y) = xe^{-x}$$

$$\int xe^{-x}dx \quad u = x \quad dv = e^{-x}dx$$
$$du = dx \quad v = -e^{-x}$$
$$= -xe^{-x} - \int -e^{-x}dx$$
$$= -xe^{-x} + \int e^{-x}dx$$
$$-xe^{-x} - e^{-x} + c$$

$$d(e^{-x}y) = xe^{-x}\,dx$$

$$\int d(e^{-x}y) = \int xe^{-x}\,dx$$

$$e^{-x}y = -xe^{-x} - e^{-x} + C$$

$$y(x) = -x - 1 + Ce^{x}$$

Cuando $x \to -\infty$ $\quad y(x) \to -x-1$ asintoticamente pues $Ce^{x} \to 0$ conforme $x \to -\infty$

SECCION 1.6 HT2

29)

(29) Resolver la EDO

$$2x\,\text{sen}\,y\,\cos y\,\frac{dy}{dx} = 4x^2 + \text{sen}^2 y$$

$U = \text{sen}^2 y$

$\frac{d}{dx}(U) = \frac{d}{dx}(\text{sen}^2 y)$

$\frac{du}{dx} = 2\,\text{sen}\,y\,\cos y\,\frac{dy}{dx}$

$$x\,\boxed{2\,\text{sen}\,y\,\cos y\,\frac{dy}{dx}} = 4x^2 + \text{sen}^2 y$$

$$x\frac{du}{dx} = 4x^2 + u$$

$$x\frac{du}{dx} - u = 4x^2$$

$$\frac{du}{dx} - \frac{1}{x}u = 4x$$

(Ecuación lineal primer orden)

$\rho(x) = e^{\int p(x)dx}$

$\rho(x) = e^{\int -\frac{1}{x}dx}$

$\rho(x) = e^{-\ln x}$

$\rho(x) = e^{\ln x^{-1}} = x^{-1}$

$\rho(x) = \frac{1}{x}$

$$\frac{1}{x}\left(\frac{du}{dx} - \frac{1}{x}u\right) = \frac{1}{4}4x$$

$$\frac{d}{dx}\left(\frac{1}{x}u\right) = 4$$

$$d\left(\frac{1}{x}u\right) = 4\,dx$$

$$\int d\left(\frac{1}{x}u\right) = \int 4\,dx$$

$$\frac{1}{x}U = 4x + C$$

$$U = 4x^2 + Cx$$

$$\text{sen}^2 y = 4x^2 + Cx$$

30)

Ahora discutimos la idea de reducir el orden de una E.D.O de segundo orden

$$F(x, y, y', y'') = 0$$

$$F(x, y', y'') = 0$$

Para reducir el orden hacemos $U(x) = y'$

$$F(y, y', y'') = 0$$

Para reducir el orden hacemos $U(y) = y'$

(30) Resolver $(x + e^{y}) y' = x e^{-y} - 1$

$$(x + e^{y}) \frac{dy}{dx} + 1 - x e^{-y} = 0$$

$$(x + e^{y})\, dy + (1 - x e^{-y})\, dx = 0$$

$$(1 - x e^{-y})\, dx + (x + e^{y})\, dy = 0 \quad (1)$$

$$M(x,y)\, dx + N(x,y)\, dy = 0$$

donde $M_y = x e^{-y} \wedge N_x = 1$

$\Rightarrow M_y \neq N_x$ E.D.O (1) No es exacta

observamos que $\dfrac{N_x - M_y}{M} = \dfrac{1 - x e^{-y}}{1 - x e^{-y}} = 1$

$\Rightarrow$ Existe el Factor Integrante $\rho(y) = e^{\int \frac{N_x - M_y}{M} dy}$

$$\rho(y) = e^{\int 1\, dy}$$

$$\rho(y) = e^{y}$$

$$\Rightarrow (1 - xe^{-y})\,dx + (x + e^{y})\,dy = \emptyset$$

$$e^{y}(1 - xe^{-y})\,dx + e^{y}(x + e^{y})\,dy = \emptyset$$

$$(e^{y} - x)\,dx + (xe^{y} + e^{2y})\,dy = \emptyset$$

$$m(x,y)\,dx + n(x,y)\,dy = 0 \qquad \boxed{2}$$

observemos $m_y = e^{y} \wedge n_x = e^{y}$

$m_y = n_x$

Edo (2) es exacta

Existe una Funcion $F(x,y)$ tal que $f(x,y) = C$ es la Familia monoparametrica de soluciones de la E.D.O (2) donde

$$\frac{\partial F}{\partial x} = m(x,y) \quad \wedge \quad \frac{\partial F}{\partial y} = n(x,y)$$

$$\frac{\partial F}{\partial x} = e^y - x \quad \wedge \quad \frac{\partial F}{\partial y} = xe^y + e^{2y}$$

Integrar Parcialmente respecto a y

$$f(x,y) = \int \frac{\partial F}{\partial y}\, dy$$

$$f(x,y) = \int xe^y + e^{2y}\, dy$$

$$f(x,y) = xe^y + \frac{1}{2}e^{2y} + g(x)$$

derivo Parcialmente respecto a x

$$\frac{\partial F}{\partial x} = e^y + \emptyset + g'(x)$$

$$\Rightarrow e^y - x = e^y + g'(x)$$

$$-x = g'(x)$$

$$\int -x\, dx = g(x)$$

$$-\frac{1}{2}x^2 + C_1 = g(x)$$

entonces la solución a la EDO exacta

$$F(x,y) = C$$

$$xe^y + \frac{1}{2}e^{2y} - \frac{1}{2}x^2 + C_1 = C$$

$$xe^y + \frac{1}{2}e^{2y} - \frac{1}{2}x^2 = C_2 \quad (\text{donde } C_2 = C - C_1)$$

$$2xe^y + e^{2y} - x^2 = k \quad (\text{donde } K = 2C_2)$$

$$2\sqrt{x+y+1} - 2\ln(\sqrt{x+y+1} + 1) = x + C_1$$

$$x + C_1 = 2\sqrt{x+y+1} - 2\ln(\sqrt{x+y+1} + 1)$$

$$x = 2\sqrt{x+y+1} - 2\ln(\sqrt{x+y+1} + 1) + C$$

Solución implícita a la E.D.O

$$\int \frac{du}{1+\sqrt{u}} = \int \frac{2(w-1)}{w}\, dw$$

$$= \int \frac{2w}{w} - \frac{2}{w}\, dw$$

$$= \int 2 - \frac{2}{w}\, dw$$

$$= 2w - 2\ln|w| + C$$

$$= 2(1+\sqrt{u}) - 2\ln|1+\sqrt{u}| + C$$

$$= 2 + 2\sqrt{u} - 2\ln|\sqrt{u}+1| + C$$

$$= 2\sqrt{u} - 2\ln(\sqrt{u}+1) + C_1$$

$w = 1 + \sqrt{u} \Rightarrow w - 1 = \sqrt{u}$

$dw = \frac{1}{2\sqrt{u}}\, du$

$2\sqrt{u}\, dw = du$

$2(w-1)\,dw = du$

46)

46 Resolvemos para $xy'' + y' = 4x$; para reducir el orden

entonces si $U(x) = y'$

$$\frac{d}{dx}(U(x)) = \frac{d}{dx}(y')$$

$$U'(x) = y''$$

reemplazamos las expresiones $y' = U$ y $y'' = U'$ en la EDO tenemos que:

$$x\,U'(x) + U(x) = 4x$$

$$x\frac{dU}{dx} + U = 4x$$

$$\frac{dU}{dx} + \frac{1}{x}U = 4$$

$$x\left(\frac{dU}{dx} + \frac{1}{x}U\right) = 4x$$

$$\frac{d}{dx}(xU) = 4x$$

$$d(xU) = 4x\,dx$$

$$\int d(xU) = \int 4x\,dx$$

$$xU = 2x^2 + C_1$$

$$U = 2x + \frac{C_1}{x}$$

$$U(x) = 2x + C_1/x$$

Ecuación lineal

$$\rho(x) = e^{\int P(x)dx} = e^{\int 1/x\,dx}$$

$$\rho(x) = e^{\ln x} = x$$

$$\frac{dy}{dx} = 2x + C_1/x$$

$$dy = (2x + C_1/x)\,dx$$

$$\int dy = \int 2x + C_1/x \; dx$$

$$y(x) = x^2 + C_1 \ln x + C_2$$

49)

(49) Resolvemos para $yy'' + (y')^2 = yy'$ y para reducir el orden tenemos que: $u(y) = y'$

$\frac{d}{dx}(u(y)) = \frac{d}{dx}(y')$

$\frac{du}{dy} \cdot \frac{dy}{dx} = y''$

$\frac{du}{dy} \cdot y' = y''$

$\frac{du}{dy} u = y'$

Reemplazamos las expresiones $y'' = u\,\frac{du}{dy}$ y $y' = u$ en la EDO y obtenemos:

$y u \frac{du}{dy} + (u)^2 = yu$

$\frac{du}{dy} + \frac{1}{y} u = 1$

$y\left(\frac{du}{dy} + \frac{1}{y}u\right) = y$

$\frac{d}{dy}(uy) = y$

$d(uy) = y\,dy$

$\int d(uy) = \int y\,dy$

$uy = \frac{1}{2}y^2 + C_1$

$u(y) = \frac{1}{2}y + \frac{C_1}{y}$

$y' = \frac{1}{2}y + \frac{C_1}{y}$

$\frac{dy}{dx} = \frac{1}{2}y + \frac{C_1}{y}$

$dy = \left(\frac{1}{2}y + \frac{C_1}{y}\right)dx$

$dy = \frac{y^2 + C_2}{2y}\,dx$

$\frac{2y}{y^2 + C_2}\,dy = dx$

Ecuación lineal

$\frac{du}{dy} + P(y)\cdot u = Q(y)$

$\rho(y) = e^{\int P(y)dy}$

$\rho(y) = e^{\int 1/y\,dy} = e^{\ln y} = y$

$\ldots \int \frac{2y}{y^2 + C_2}\,dy = \int dx$

$\ln(y^2 + C_2) = x + C_3$

$e^{\ln(y^2 + C_2)} = e^{x + C_3} = e^x \cdot e^{C_3}$

$y^2 + C_2 = Be^x$

$y = +\sqrt{A + Be^x}$ y asumimos que $y > 0$

50)

Resolver: **$Y'' = (X + Y')^2$**

Se requiere realizar un cambio de variable.

$$U(x) = x + \frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = 1 + \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{du}{dx} - 1 = y''$$

$$y'' = (x + y')^2$$

$$\frac{du}{dx} - 1 = u^2$$

$$\frac{du}{dx} = 1 + u^2$$

$$du = (1 + u^2)\,dx$$

$$\frac{du}{1 + u^2} = dx$$

$$\int \frac{du}{1+u^2} = \int dx$$

$$\arctan u = x + c_1$$

$$\Rightarrow u(x) = \tan(x+c_1)$$

$$x + \frac{dy}{dx} = \tan(x+c_1)$$

$$\frac{dy}{dx} = \tan(x+c_1) - x$$

$$dy = (\tan(x+c_1) - x)\,dx$$

$$\int dy = \int \tan(x+c_1) - x \quad dx$$

$$y(x) = -\ln|\cos(x+c_1)| - \frac{x^2}{2} + c_2$$

$$y(x) = \ln\left|\frac{1}{\cos(x+c_1)}\right| - \frac{1}{2}x^2 + c_2$$

$$y(x) = \ln|\sec(x+c_1)| - \frac{1}{2}x^2 + c_2$$

$$\int \tan(x+c_1)\,dx$$

$$\int \frac{\operatorname{sen}(x+c_1)}{\cos(x+c_1)}\,dx$$

$$w = \cos(x+c_1)$$

$$dw = -\operatorname{sen}(x+c_1)\,dx$$

$$\int -\frac{dw}{w}$$

$$-\ln|w| + c_2$$

$$-\ln|\cos(x+c_1)| + c_2$$