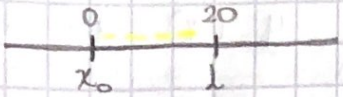
Sección 3,4

③

$m = 3\ kg$ $\quad x_0 = 0$ $\quad C = 0 \rightarrow$ No hay amortiguador

$l = 20\ cm$ $\quad V_0 = -10\ m/s$

$F = 15\ N$ $\quad A = ?$ $\quad T = ?$ $\quad w_0 = ?$



$mx'' + Cx' + Kx = F$

$3x'' + 75x = 0$

Homogénea

$K = F/L \rightarrow$ Ley de Hooke

$K = 15\ N / 0,2\ m$

$K = 75\ \frac{N}{m}$

$$w_0 = \sqrt{\frac{K}{m}}$$

$$w_0 = \sqrt{\frac{75}{3}} = \sqrt{25} = 5\ rad/s$$

$x'' + w_0^2 x = 0$

$x'' + 25x = 0$

$$T = \frac{2\pi}{w_0} = \frac{2\pi}{5} = 1,25\pi$$

$x(t) = A \cos w_0 t + B \operatorname{sen} w_0 t$

Solución general

$x'(t) = -w_0 A \operatorname{sen} w_0 t + w_0 B \cos w_0 t$

$x(0) = A \cos 5(0) + B \operatorname{sen} 5(0)$

$A = 0m$

$x'(0) = -5(0) \operatorname{sen}(0) + 5B \cos 5(0)$

$-10\ m/s = 5B$

$B = -2m/s$

$x(t) = -2 \operatorname{sen} 5t$

Solución particular

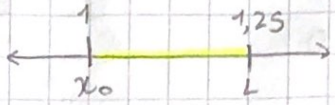
DD MM AA

(4)

$m = 250\ g \quad 0{,}25\ kg$ $\quad x_0 = 1\ m \quad v_0 = -5\ m/s \quad C = 0 \rightarrow$ No hay amortiguador

$F = 9\ N$

$L = 25\ cm \quad 0{,}25\ m$



$mx'' + Cx' + Kx = F$

$0{,}25x'' + 36x = 0$

Homogénea

$K = F/L \rightarrow$ Ley Hooke

$K = 9\ N / 0{,}25\ m$

$K = 36\ N/m$

$0{,}25r^2 + 36 = 0 \quad r = a \pm bj$

$r^2 = -144$

$r = \pm 12\,j \quad a = 0 \quad b = 12$

$x(t) = A\cos\omega_0 t + B\,sen\,\omega_0 t$

solución general

$x'(t) = -\omega_0 A\,sen\,\omega_0 t + \omega_0 B\cos\omega_0 t$

$x(t) = e^{at}(C_1\cos bt + C_2\,sen\,bt)$

$x(t) = e^{0t}(C_1\cos 12t + C_2\,sen\,12t)$

$T = \frac{2\pi}{12} = \frac{\pi}{6}\ s$

$x(0) = C_1\cos 12(0) + C_2\,sen\,12(0)$

$1 = C_1 = A$

$x'(t) = -12C_1\,sen\,12t + 12C_2\cos 12t$

$x'(0) = -12C_1\,sen\,12(0) + 12C_2\cos 12(0)$

$-5 = 12C_2$

$C_2 = \frac{-5}{12} = B$

$\alpha = 2\pi + \tan^{-1}(B/A)$

$\alpha = 2\pi + \tan^{-1}(-(5/12)/1) = 5{,}89$

$C = \sqrt{A^2 + B^2} = \sqrt{12 + (-5/12)^2} = 13/12$

$x(t) = \underbrace{13/12}_{\text{Amplitud}}\cos(12t - 5{,}89)$

(13) $m = 10$ $\quad x(0) = 0$

$C = 9$ $\quad x'(0) = 5$

$K = 2$

$mx'' + Cx' + Kx = F$

$10x'' + 9x' + 2x = 0$

$10r^2 + 9r + 2 = 0$

$(5r + 2)(2r + 1) = 0$

$5r + 2 = 0 \quad 2r + 1 = 0$

$r_1 = -2/5 \quad r_2 = -1/2$

$x(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

$x(t) = C_1 e^{-2t/5} + C_2 e^{-t/2}$

$x(0) = C_1 e^{-2(0)/5} + C_2 e^{-(0)/2}$

$0 = C_1 + C_2 \longrightarrow C_1 = -C_2$

$x'(t) = -2/5\, C_1 e^{-2t/5} - 1/2\, C_2 e^{-t/2}$

$x'(0) = -2/5\, C_1 e^{-2(0)/5} - 1/2\, C_2 e^{-(0)/2}$

$5 = -2/5\, C_1 - 1/2\, C_2$

Reemplazamos

$5 = 2/5\, C_2 - 1/2\, C_2$

$5 = -1/10\, C_2$

$C_2 = -50$

$C_1 = 50$

$x(t) = 50(e^{-2t/5} - e^{-t/2})$

$x'(t) = -20e^{-2t/5} + 25e^{-t/2}$

$5e^{-4t/10}(5e^{-t/10} - 4) = 0$

$\underbrace{5e^{-2t/5} = 0}$ No cumple propiedades

$5e^{-t/10} - 4 = 0$

$e^{-t/10} = 4/5$

$-t/10 = \ln|4/5|$

$t = -10 \ln|4/5| = 2{,}23$

$x(2{,}23) = 50e^{-2(2{,}23)/5} - 50e^{-2{,}23/2}$

$x(t) = 4{,}096$

(14) $m = 25$ $K = 226$ $x'(0) = 41$
$C = 10$ $x(0) = 20$

$mx'' + cx' + kx = F$
$25x'' + 10x' + 226x = 0$
$25r^2 + 10r + 226 = 0$

$$r = \frac{-10 \pm \sqrt{100^2 - 4(25)(226)}}{2(25)}$$

$r = -1/5 \pm 3j$ $r = a \pm bj$
$a = -1/5$ $b = 3$

$x(t) = e^{at}(C_1 \cos bt + C_2 \operatorname{sen} bt)$
$x(t) = e^{-t/5}(C_1 \cos 3t + C_2 \operatorname{sen} 3t)$

$x(0) = e^{-0/5}(C_1 \cos 3(0) + C_2 \operatorname{sen} 3(0))$
$20 = C_1$

$x'(t) = -\frac{1}{5} e^{-\frac{t}{5}}(C_1 \cos 3t + C_2 \operatorname{sen} 3t) + e^{-\frac{t}{5}}(-3C_1 \operatorname{sen} 3t + 3C_2 \cos 3t)$

$x'(t) = e^{-\frac{t}{5}}(-\frac{1}{5}20\cos 3t - \frac{1}{5}C_2 \operatorname{sen} 3t - 3(20)\operatorname{sen} 3t + 3C_2 \cos 3t)$

$41 = e^{-0/5}(-\frac{1}{5}20\cos 3(0) - \frac{1}{5}C_2 \operatorname{sen} 3(0) - 3(20)\operatorname{sen} 3(0) + 3C_2 \cos 3(0))$

$41 = -4 + 3C_2$
$C_2 = \frac{45}{3} = 15$

$x(t) = e^{-\frac{t}{5}}(20\cos 3t + 15 \operatorname{sen} 3t)$
$x(t) = A \cos w_0 t + B \operatorname{sen} w_0 t$

$A = e^{-t/5}20$ $B = e^{-t/5}15$ $C^2 = A^2 + B^2$

$C^2 = e^{-2t/5}(20)^2 + e^{-2t/5}(15)^2$
$C = \sqrt{e^{-2t/5}\ 625} = 25e^{-t/5}$

$\alpha = \tan^{-1}\left(\frac{B}{A}\right) = \tan^{-1}\left(\frac{15}{20}\right) = 0,644$ rad

$x(t) = C\cos(w_0 t - \alpha) = 25e^{-t/5}\cos(3t - 0,644)$

(22) $w = 12\ lb$ $\quad x(0) = 1\ ft$ $\quad x'(0) = 0$

$l = 6\ in = 0{,}5\ ft$ $\quad \omega_0 = ?$ $\quad A = ?$

$C = 3\ lb$ $\quad \alpha = ?$

$m = w/g$

$m = \frac{12\ lb}{32\ ft/s^2} = 0{,}375\ slugs$

$K = w/L$

$K = 12\ lb / 0{,}5\ ft = 24\ lb/ft$

$mx'' + Cx' + Kx = 0$

$0{,}375x'' + 3x' + 24x = 0$

$0{,}375r^2 + 3r + 24 = 0$

$r^2 + 8r + 64 = 0$

$r = \frac{-8 \pm \sqrt{8^2 - 4(64)}}{2}$ $\quad r = -4 \pm 4\sqrt{3}\,j$ $\quad r = a \pm bj$

$a = -4$ $\quad b = 4\sqrt{3} = \omega_0$

$x(t) = e^{at}(C_1 \cos bt + C_2 \operatorname{sen} bt)$

$x(t) = e^{-4t}(C_1 \cos 4\sqrt{3}\,t + C_2 \operatorname{sen} 4\sqrt{3}\,t)$

$x(0) = e^{-4(0)}(C_1 \cos 4\sqrt{3}\,(0) + C_2 \operatorname{sen} 4\sqrt{3}\,(0))$

$1 \qquad 1 \qquad 0$

$C_1 = 1\ ft$ $\quad A = e^{-4t}$

$x'(t) = -4e^{-4t}(C_1 \cos 4\sqrt{3}\,t + C_2 \operatorname{sen} 4\sqrt{3}\,t) + e^{-4t}(-4\sqrt{3}C_1 \operatorname{sen} 4\sqrt{3}\,t + 4\sqrt{3}C_2 \cos 4\sqrt{3}\,t)$

$x'(0) = -4e^{-4(0)}(C_1 \cos 4\sqrt{3}\,(0) + C_2 \operatorname{sen} 4\sqrt{3}\,(0)) + e^{-4\cdot 0}(-4\sqrt{3}C_1 \operatorname{sen} 4\sqrt{3}\cdot 0 + 4\sqrt{3}C_2 \cos 4\sqrt{3}\cdot 0)$

$0 = -4 + 4\sqrt{3}\,C_2$

$C_2 = \frac{\sqrt{3}}{3}$ $\quad B = e^{-4t}\frac{\sqrt{3}}{3}$

$\alpha = \tan^{-1}(B/A) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

$C = \sqrt{A^2 + B^2} = e^{-4t}\frac{2}{3}\sqrt{3}$

$x(t) = e^{-4t}\frac{2}{3}\sqrt{3}\cos\left(4\sqrt{3}\,t - \frac{\pi}{6}\right)$

(23) $w = 3200\ lb$

$m = \frac{w}{g} = \frac{3200\ lb}{32\ ft/s^2} = 100\ slugs$

a) $f_0 = 80\ ciclos/min = \frac{4}{3}\ Hz$

$w_0 = f_0 2\pi = \frac{4}{3} 2\pi\ rad/s$

$w_0 = \sqrt{\frac{k}{m}} \longrightarrow k = m w_0^2 = 100 \cdot \left(\frac{8\pi}{3}\right)^2 = 7018\ lb/ft$

b)

$f_1 = 78\ ciclos/min = 1{,}3\ Hz$

$w_1 = f_1 2\pi = 2{,}6\pi\ rad/s$

$w_1 = \sqrt{\frac{k}{m}} \longrightarrow k = m w_1^2 = 100\,(2{,}6\pi)^2 = 6671\ lb/ft$

$w_1 = \sqrt{w_0^2 - \rho^2} \qquad \rho = \frac{c}{2m}$

$w_1^2 = w_0^2 - \frac{c^2}{4m^2}$

$(2{,}6\pi)^2 = \left(\frac{8}{3}\pi\right)^2 - \frac{c^2}{40000}$

$\sqrt{c^2} = \sqrt{\left(\left(\frac{8}{3}\pi\right)^2 - (2{,}6\pi)^2\right) 40000} \approx 372{,}31\ \frac{lb}{ft/s}$

$\rho = \frac{372{,}31}{2(100)} = 1{,}862$

$e^{-\rho t} = 1\%$

$-\rho t = \ln 10{,}011$

$t = \ln 10{,}oil\ /-1{,}862 = 2{,}4735$