

Domaći rad - granična vrednost - Sara Milošević

1219. a)

$$\lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2} =$$

$$\lim_{n \rightarrow \infty} \frac{n^3 + 3n^2 + 3n + 1 - (n^3 - 3n^2 + 3n - 1)}{n^2 + 2n + 1 + n^2 - 2n + 1} =$$

$$\lim_{n \rightarrow \infty} \frac{n^3 + 3n^2 + 3n + 1 - n^3 + 3n^2 - 3n + 1}{2n^2 + 2} =$$

$$\lim_{n \rightarrow \infty} \frac{6n^2 + 2}{2n^2 + 2} = \lim_{n \rightarrow \infty} \frac{2(3n^2 + 1)}{2(n^2 + 1)} = \lim_{n \rightarrow \infty} \frac{3n^2 + 1 /: n^2}{n^2 + 1 /: n^2} =$$

$$\lim_{n \rightarrow \infty} \frac{3 + \frac{1}{n^2}}{1 + \frac{1}{n^2}} = \frac{3 + 0}{1 + 0} = 3$$

1219. v)

$$\lim_{n \rightarrow \infty} \left(\frac{2n^2}{2n+3} + \frac{1-3n^3}{3n^2+1} \right) =$$

$$\lim_{n \rightarrow \infty} \left(\frac{2n^2 \cdot (3n^2 + 1) + (1 - 3n^3) \cdot (2n + 3)}{(2n + 3)(3n^2 + 1)} \right) =$$

$$\lim_{n \rightarrow \infty} \frac{6n^4 + 2n^2 + 2n + 3 - 6n^4 - 9n^3}{6n^3 + 2n + 9n^2 + 3} =$$

$$\lim_{n \rightarrow \infty} \frac{-9n^3 + 2n^2 + 2n + 3 /: n^3}{6n^3 + 9n^2 + 2n + 3 /: n^3} =$$

$$\lim_{n \rightarrow \infty} \frac{-9 + 2 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}}{6 + 9 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}} = \frac{-9 + 2 \cdot 0 + 2 \cdot 0 + 3 \cdot 0}{6 + 9 \cdot 0 + 2 \cdot 0 + 3 \cdot 0} = -\frac{3}{2}$$

1220. b)

$$\lim_{n \rightarrow \infty} \frac{(n+2)! - (n+1)!}{(n+3)!} = \lim_{n \rightarrow \infty} \frac{(n+2)(n+1)! - (n+1)!}{(n+3)(n+2)(n+1)!} =$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)! \cdot (n+2-1)}{(n+3)(n+2)(n+1)!} = \lim_{n \rightarrow \infty} \frac{n+1}{n^2 + 3n + 2n + 6} =$$

$$\lim_{n \rightarrow \infty} \frac{n+1 /: n^2}{n^2 + 5n + 6 /: n^2} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{1}{n^2}}{1 + 5\frac{1}{n} + 6\frac{1}{n^2}} =$$

$$\frac{0+0}{1+5 \cdot 0+6 \cdot 0} = 0$$

1222. d)

$$\lim_{n \rightarrow \infty} \left(\frac{\overbrace{1+5+9+\dots+(4n-3)}^{S_n}}{2(n+1)} - n \right)$$

$$S_n = \frac{n}{2} \cdot (a_1 + a_n) = \frac{n}{2} \cdot (1 + 4n - 3) = \frac{n}{2} \cdot (4n - 2) =$$

$$= \frac{2n \cdot (2n - 1)}{2} = n \cdot (2n - 1) = 2n^2 - n$$

$$\lim_{n \rightarrow \infty} \frac{2n^2 - n - 2n \cdot (n+1)}{2 \cdot (n+1)} = \lim_{n \rightarrow \infty} \frac{2n^2 - n - 2n^2 - 2n}{2n + 2} =$$

$$\lim_{n \rightarrow \infty} \frac{-3n /: n}{2n + 2 /: n} = \lim_{n \rightarrow \infty} \frac{-3}{2 + 2\frac{1}{n}} = \frac{-3}{2 + 2 \cdot 0} = -\frac{3}{2}$$